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Dissertation

A Quantitative Approach to Tower Crane
Selection and Positioning on Construction Sites

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List of Abbreviations

AP	assignment problem
B & B	branch and bound
BIM	building information modeling
DP	dynamic programming
FLP	facility location problem
GA	genetic algorithm
GCBD	geometric covering by discs
IND-DOM-GRID	problem to find a minimum cardinality independent dominating set in a grid graph
IND-DOM-GRID-BIG	problem to find a minimum cardinality independent dominating set in a grid graph with more than four nodes
LB	lower bound
LSCP	location set cover problem
MIP	mixed-integer program
OR	operations research
SCP	set cover problem
TAC	total assignment cost
TCSP	tower crane selection and positioning problem
TCSP-GRID	tower crane selection and positioning problem with minimum distances and slewing ranges
TWPC	total weighted proximity cost
UB	upper bound

List of Symbols

Sets

$B_c = \{B_c^1, \dots, B_c^{ B_c }\}$	set of inter-structure sectors of candidate c
C	set of candidates
C_d	set of candidates that can cover pair (d, s_d) with respect to on-site structures
C^{ex}	in the B & B approach: set of candidates that are excluded from being selected due to branching decisions
C^{sel}	in the B & B approach: set of candidates that have been selected by branching decisions
C^*	in the B & B approach: set of candidates that have been selected in a lower bound solution
D	set of demand sites
F_t	set of forbidden areas for crane type t
$F = \cup_{t \in T} F_t$	union of all forbidden areas among all crane types
G_t	set of potential crane locations for cranes of type t in a grid
N_c	set of candidates that cannot be chosen due to minimum distances if candidate c is selected
N_c^{UB}	in the B & B approach: set of candidates that cannot be chosen due to minimum distances if candidate c is selected in the upper bound computation
P	set of pairs
P_c	set of pairs that can be covered by candidate c with respect to on-site structures
P^{cov}	in the B & B approach: set of pairs that are definitely covered by selected candidates due to branching decisions
P_c^{nco}, P_c^{ncl}	set of pairs in P_c that cannot be covered by candidate c using the counter-clockwise or clockwise, respectively, transport sector
$P_{(c,c')}$	set of pairs in P_c that candidate c cannot cover if candidate c' is chosen
$P_{(c,c')}^{co}, P_{(c,c')}^{cl}$	set of pairs that can be fully reached by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c'
$P_{(c,c')}^{nco}, P_{(c,c')}^{ncl}$	set of pairs that cannot be covered by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c'

$P_{(c,c',c'')}$	set of pairs that cannot be covered by candidate c when candidates c' and c'' are selected
$P_{(c,c',c'')}^{co}, P_{(c,c',c'')}^{cl}$	set of pairs that can be fully reached by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c' to candidate c''
S	set of supply sites
\mathcal{S}_c	in the B & B approach: set of sectors of candidate c 's operating area formed by blocking cranes and / or blocking on-site structures
T	set of crane types
T_c	set of tuples (c, c') with candidate c' blocking candidate c 's jib
T'_c	set of triples (c, c', c'') with candidates c' and c'' blocking candidate c 's jib
$T_f = \{t \in T \mid f \in F_t\}$	set of crane types which may not be located within forbidden area f
$W_s = \{w_d \mid d \in D, s_d = s\}$	set of maximum weight values associated with supply area $s \in S$

Parameters

ζ	side length of the site polygon (i. e. ζ times the smallest cranes maximum operating radius)
η	number of pairs on-site
θ	grid granularity (i. e. horizontal and vertical distance between the grid's intersection points)
Θ_p^i	angular coordinate of point p in the polar coordinate system with pole i and a horizontal polar axis defined by half-line l
κ	number of available crane types
λ	site density (i. e. average number of pairs per cell)
μ	type of forbidden areas on-site
π_c	cost of candidate c (i. e. of the crane type t associated with candidate c)
π_t	cost of crane type t
ρ_p^i	radial coordinate of point p in the polar coordinate system with pole i
σ	indicator for consideration of lifting weights
ϕ	indicator for consideration of demand area heights
$dist_{c,c'}$	Euclidean distance between the crane locations associated with candidates c and c'
(d, s_d)	pair of a demand site d and its associated supply site s_d
$D_{t,t'}^{min}$	minimum distance requirement between cranes of type t and t'
$D_{t,t'}^{min,UB}$	in the branch and bound procedure: modified minimum distance requirement between cranes of type t and t' for computing an upper bound
h_d	height of demand site d

h_f	height of forbidden area f
h_t	maximum operating height of a crane of type t
$i_{d,c}$	binary indicator; equals 1 if pair (d, s_d) can be covered by candidate c ; 0 otherwise
n_c	sufficiently large number, e. g. $n_c = N_c $
r_t^{max}	maximum operating radius of a crane of type t
$r_{t,w}$	maximum operating radius of a crane of type t for weight w
w_d	maximum weight to be lifted at demand site d

Variables

α_c	binary variable; equals 1 if candidate c is selected; 0 otherwise
$\beta_{c,d}$	binary variable; equals 1 if candidate c covers pair (d, s_d) ; 0 otherwise
γ_{c,B_c^i}	binary variable; equals 1 if candidate c covers inter-structure sector $B_c^i \in B_c$; 0 otherwise
$\delta_{(c,c')}^{co}, \delta_{(c,c')}^{cl}$	binary variable; equals 1 if candidate c slews counter-clockwise or clockwise, respectively, with respect to selected candidate c' ; 0 otherwise
$\epsilon_{(c,c',c'')}^{co}, \epsilon_{(c,c',c'')}^{cl}$	binary variable; equals 1 if candidate c slews counter-clockwise or clockwise, respectively, from selected candidate c' to selected candidate c'' ; 0 otherwise
τ_c^j	in the B & B approach: binary variable; equals 1 if candidate c covers its sector $j \in \mathcal{S}_c$; 0 otherwise
$\omega_{c,d}^{co}, \omega_{c,d}^{cl}$	binary variable; equals 1 if candidate c covers pair (d, s_d) by slewing counter-clockwise or clockwise, respectively; 0 otherwise

1 Introduction

Construction cranes, in general, cannot be regarded a mainstream topic in operations research (OR). There has been some effort on the topic, but mostly from engineering-oriented researchers applying quantitative methods. These efforts especially occurred in the mid-1980s to the mid-1990s. Nowadays, crane-related problems seem to experience a revival, but still in the engineering-oriented community. This indicates that the topic has quite some relevance. In the chapter at hand, we will have a closer look at the reasons for the topic's relevance (Section 1.1). Afterwards, the structure of the thesis will be outlined (Section 1.2).

1.1 Relevance of Tower Cranes in Construction Projects

Note: this chapter is based on Briskorn and Dienstknecht [10] and Briskorn and Dienstknecht [12].

Presumably everybody has come across tower cranes in his life. From small-scale construction projects like single residential buildings to large-scale construction projects such as infrastructure projects cranes can be seen looming over the sites. Whereas, for small-scale projects, often only a single crane operates, on large-scale projects, multiple cranes being more or less dispersed on-site can be observed. However, although tower cranes are one of the most characteristic pieces of construction equipment most people do not question their selection and on-site location, but seem to consider them as kind of given by nature, instead.

In fact, there is no such thing as *the* tower crane, but within this category various models can be distinguished: there are, e.g., rail-mounted cranes, climbing-base cranes or fixed-base cranes (see Shapira et al. [104] for a detailed overview on construction crane types). In this thesis, the focus is restricted to fixed-base flat-top tower cranes (which most people might have in mind when talking about construction cranes), i.e. cranes with a fixed base, a mast, a jib which can rotate by 360 degrees around the mast, a trolley which travels along the jib and, finally, the hook being attached to the trolley. One may ask why the scope is restricted to the seemingly simplest tower crane model available or why it is restricted to tower cranes, at all. From a practical perspective, tower cranes are quite often employed due to the rather limited space they require: the base is on-site and anything else happens overground. This makes tower cranes particularly interesting for congested sites which, nowadays, are quite common, especially in urban areas where space is a critical resource. From a theoretical perspective, planning fixed-base tower cranes already is a rather challenging task as will be seen in this thesis. The insights gained herein, however, may be utilized when tackling more complicated crane types.

Although, in everyday life, one may tend to take the cranes operating on-site as kind of naturally given they are, obviously, not. They have been put in their places by human beings and the questions arising are: why has this specific crane been selected and why has this very location been chosen for erecting a crane? As we will see, tower crane selection and location decisions are anything but trivial since numerous restrictions have to be taken into account.

Clearly, tower cranes are supposed to provide material transport in both horizontal and vertical direction. Consequently, cranes have to be able to reach both the source of material and its destination in terms of the operating radius and the operating height. The former not only depends on the specific crane, but on the weight to be lifted: heavy loads, in contrast to light loads, may only be carried closer to the crane's mast since they would cause instabilities otherwise.

But cranes may not be located arbitrarily just having coverage in mind. There are areas on-site where cranes with certain specifications may not be located, e.g. due to limited bearing capacities of the ground, insufficient accessibility for crane erection and dismantling, or safety clearance from power lines and other on-site structures. Similarly, cranes have to keep safety clearance from each other. Furthermore, a crane's operations are affected by given on-site structures and other cranes, as well, as a crane's jib may be blocked by an object of sufficient height. These considerations already indicate that decisions on crane selection and location are closely related and impact each other and, thus, are ideally addressed integratedly.

One may wonder if cranes are worth the effort. Not surprisingly, they are. The impact of crane selection and location traditionally includes both operational and economic aspects, but, nowadays, as sustainability becomes increasingly important even climate aspects of crane operations (CO₂ emissions) are emphasized. From an economic perspective, the costs imposed by operating cranes justify careful evaluation of their utilization. Tower cranes which are often provided by rental companies can easily impose costs of tens of thousands of dollars per month depending on a project's scale. As cranes are still the most important lifting equipment on-site their operational relevance is obvious.

Summarizing, the complex restrictions and interrelations of decisions in combination with the subject's practical relevance make it an interesting area for OR as decision support may be required and OR's methodological tool box may fill this need.

1.2 Thesis Outline

In this thesis, tower crane selection and location on construction sites will be considered. Roughly speaking, cranes with different specifications have to be selected and located on a polygonal construction site so that material transports between polygonal supply and demand areas can be established. Depending on the problem variant under research, several constraints have to be respected.

In order to fully introduce the topic, related literature will be presented first (Chapter 2). This literature review will consist of both a rather application-oriented perspective and a methodological part. In the former part, the focus is on crane-related planning issues and – since this thesis deals with locating cranes on-site – on construction site layout planning in general. As location planning has a long tradition in OR, the latter part is supposed to introduce concepts related to the problems considered

in the thesis at hand.

Once the literature has been reviewed, the planning problems at the core of this thesis can be focused. At first (Chapter 3), tower cranes will be located in the plane, i. e. in continuous space, but only few operational constraints are respected. The operational constraints included are load weight-dependent lifting radii, crane type-dependent infeasible areas for crane location and a rough approach to crane type-dependent operating heights. In this part, it will be proven that the by nature infinite set of potential crane locations can be boiled down to a finite set without loss of optimality which, in turn, allows to fall back on standard methods of OR. After this proof, a corresponding solution approach is presented and computationally tested in order to evaluate its performance capabilities and potential for practical application.

Afterwards (Chapter 4), more operational constraints will be considered, namely prescribed crane type-dependent minimum distances between cranes, interferences between single cranes and interferences of cranes and any on-site structures (buildings, obstacles). This enhanced perspective on crane operations, however, comes at the cost of switching from a continuous to a discrete perspective on space. In a first step, the problem will be modelled by four different mixed-integer programs (MIPs) that, as well, will be evaluated for their computational performance employing a standard solver. As the latter has been observed to be limited, in a second step, a branch & bound (B & B) procedure is developed and compared to the MIP-based approach with respect to computational performance.

Finally, the findings are summarized and an outlook on future research directions will be given in Chapter 5.

2 Related Literature

In this chapter, literature related to crane selection and location planning is reviewed. Note that it is restricted to publications presenting quantitative approaches. Consequently, research aiming primarily at visualizing or simulating processes or environments which is quite common in the area of construction engineering is excluded. As mentioned in Chapter 1.2, the review consists, on the one hand, of an application-oriented part (Chapter 2.1) in which both problems from the field of layout and location planning on construction sites and crane-related planning problems are presented and, on the other hand, of a methodological part (Chapter 2.2) which includes problems and problem aspects from the area of covering and location planning in general.

2.1 Application-Oriented Literature

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [11].

Quantitative approaches for tackling problems in construction engineering have gained some interest in the recent years. The problems focused are not limited to facility location problems (including cranes), but include numerous other problems such as, e. g., construction project scheduling, vehicle routing, contractor selection, as surveys like the ones by Chan et al. [16], Liao et al. [64], Sarker et al. [95] and Briskorn and Dienstknecht [11] show. However, as the focus in this thesis is restricted to the selection and location of tower cranes the literature review is restricted accordingly. In a rather rough and superficial approach, tower cranes can be regarded as any arbitrary type of facility to be located on-site. Thus, layout and location problems on construction sites in general are reviewed in Chapter 2.1.1. However, as already indicated in Chapter 1 the on-site location of tower cranes includes many more aspects than the location of any standard facility, e. g. operating areas, operating heights and interferences. These complicating considerations result in the requirement of putting crane location planning into a broader context, e. g. integrating location decisions and crane selection. For example, the decision where to optimally locate the single cranes depends on their covering capabilities, i. e. on the selected cranes, but the optimal selection of cranes, in turn, depends on their on-site location as this affects coverage. Thus, specific crane-related planning issues are reviewed in Chapter 2.1.2.

2.1.1 Layout and Location Planning on Construction Sites

In this part, research focused on the on-site location of not nearly specified construction facilities is reviewed. As will be seen there is quite a variety of objectives and constraints in these problems. In order to structure this part, the literature will be classified from a rather methodological perspective focusing on structural problem aspects. We will, thus, introduce some basic terms and concepts before reviewing the literature.

Most of the research considering the location of facilities on-site is an application of an assignment problem. Assignment problems have been studied in the area of OR for decades and numerous variants and interpretations exist (see Pentico [86]). For this thesis, let us simply state that objects have to be assigned to positions. “Usually, a set of objects to be positioned is given and restrictions regarding the positioning have to be considered, e. g. objects must not overlap and must be placed within a given area. It should be emphasized that this chapter only covers research concerned with placing objects *within* the boundaries of a single construction site. Note that some authors employ concepts for facility location planning in order to tackle such problems. These approaches, consequently, are summarized in the chapter at hand, as well. Often an objective function is given implying that not only a feasible positioning, but an optimal one or at least a good one is desired.

In the literature different objective functions have been proposed in order to evaluate a given assignment. Two common ones are instantiated by the quadratic assignment problem (AP) and the linear AP, respectively.

- The quadratic AP employs distances between positions, amount of material to be transported between objects, and – potentially – a cost factor. The effort for transport from one object to another equals the amount to be transported times the distance between the assigned locations times the cost factor. The objective of the quadratic AP is to minimize total effort for transport. The quadratic AP is NP-hard, that is it is hard to solve, and it cannot even be approximated within a constant factor in polynomial time (see Burkard [13]). Nevertheless, since it is one of the most intensively analyzed optimization problems there are many solution methods available in the literature, see Loiola et al. [69].

In construction engineering there is a variety of concepts regarding the objective above. Most of them rely on the distance between two objects as a first factor. The distance is multiplied by a second factor depending on the pair of objects. The interpretation of this second factor varies among different papers. It may represent, e. g., amount of material transported, safety factors, preferences, or simply be an abstract value. Sometimes, a third factor, mostly reflecting variable costs is employed. Note that all these different interpretations do not influence the objective function’s structure. In order to emphasize these structural commonalities and unify the phrasing we refer to this component of objective functions as *total weighted proximity cost* (TWPC).

- The linear AP employs assignment costs for each object and each position. A layout is then evaluated by the total cost of chosen assignments. Again, various interpretations can be found, e. g. set-up costs, associated risk or utility when installing a facility in a certain position. We

refer to this component of objective functions as *total assignment cost* (TAC). As opposed to the quadratic AP, the linear AP can be solved in polynomial time and is, therefore, used either as a simplifying problem capturing the main characteristics or as sub-problem in order to tackle problems in numerous applications.”

“Regarding the term *position*, it can be broadly distinguished between discrete approaches where a predefined finite set of available locations is given and continuous approaches where any point on the construction site that is not occupied by any existing structure is available for placing an object. More precisely, we refer to a model or an approach as continuous if there are two different locations available so that each location in between these two is available, as well. It should be noted that most researchers discretize a continuous space by laying a grid over the site. Hence, we differentiate with respect to the original problem description rather than to the model and categorize papers according to their problem description rather than the model and solution procedure developed. If a paper considers a truly continuous solution space, this will be explicitly stated. Another distinction can be made with respect to time. In static approaches, a single layout is planned and considered to be valid throughout the planning horizon. A dynamic approach, in contrast, respects requirements changing over time. For example, a storage place for bricks is needed maybe prior to and definitely during building the walls, but afterwards it can be removed from the site and its position is free for other equipment. Most of these dynamic approaches respect the time dimension by subdividing the whole construction life cycle into periods or phases that are planned successively. Andayesh and Sadeghpour [7] correctly point out, that this is more of a phased perspective rather than a dynamic one. However, this type of approach is considered dynamic in this review, since the dynamic nature of the problem has been recognized and is reflected.

Most of the reviewed papers are classified in dynamic and static as well as discrete and continuous approaches. According to this classification, they are listed in Table 2.1 and are presented in more detail in the corresponding Chapters 2.1.1.1 to 2.1.1.4. Additionally, within these categories, we distinguish single-objective and multi-objective problems. The modelling variety in discrete approaches is much smaller than in continuous approaches. We therefore lay the emphasis on outlining structural commonalities when discussing these in Chapters 2.1.1.1 and 2.1.1.2. When reviewing continuous approaches in Chapters 2.1.1.3 and 2.1.1.4 we provide more details about the model and the actual application.”

“Some publications are rather related to facility location problems (FLPs), which constitute a traditional area in OR (see Klose and Drexl [52]). These are presented in Chapter 2.1.1.5. Similar to the layout problems from Chapters 2.1.1.1 to 2.1.1.4, FLPs can be categorized into discrete and continuous and static and dynamic problem variants, as well. While the categorization with respect to time does not differ from the one for layout problems, we briefly outline the difference between discrete and continuous FLPs in the following.

In the discrete version, there is a set of customers with given demands of a single product and there is a set of potential facilities with given supply capacities. Opening a facility is charged with a facility-dependent fixed cost and gives the opportunity to supply customers from this facility. Supplying a customer from a facility yields transportation cost per unit depending on both customer and supplier. The objective is to decide which facilities to open and which quantities to ship so that customer demand

		discrete		continuous	
		single-obj.	multi-obj.	single-obj.	multi-obj.
layout	static	[20], [53], [54], [55], [60], [61], [62], [65], [72], [116], [119], [125], [126], [130]	[80]	[26], [30], [38], [41], [45], [73], [85], [94], [133], [135]	[27], [28], [37], [50], [122]
	dynamic	[81], [82]	[82], [121]	[6], [29], [31], [32], [36], [91], [93], [134]	[90], [92], [123], [124]
location	static	[43], [116], [117], [118]		[116]	
	dynamic	[18], [116], [117], [118]			

Table 2.1: Classification of layout- and location-related research

is fulfilled at minimal total cost. In contrast to the assignment problems discussed in Chapters 2.1.1.1 and 2.1.1.2, discrete FLPs are not concerned with assigning facilities to predefined locations, but to select given facilities (or facility locations) on-site. Additionally, transportation does not occur between facilities to be located, but between facilities to be located and given customers (i. e. there are no inter-facility flows).

In continuous FLPs, a given number of facilities has to be located in the plane. If only a single facility has to be located, the problem is known as the Weber problem; if more than one facility has to be located (and if it has to be decided which demand point is served completely by which facility), this is referred to as the multi-source Weber problem. Again, transportation does not occur between the single facilities, but between facilities and given customers.

Especially for the layout-related research, there is a considerable variety of practically motivated constraints and goals. When defining the basic concepts for our categorization, we already mentioned different practical interpretations of the classical OR objectives TAC and TWPC like location-dependent set-up costs, risk or utility (for TAC) and transportation effort, safety risk or layout preferences (for TWPC). As can be seen in the following parts, most papers rely on these concepts, but there are quite specific objectives due to special applications and environments that, in turn, necessitate specific formulations, as well. These include, e. g., noise pollution, illuminance (when locating lighting equipment) as well as aviation safety and airport security (at an airport construction site).

APs traditionally include constraints requiring at most one facility per position and – in the generalized version – constraints allowing only a subset of positions for having assigned a certain facility. In construction engineering these constraints are rather common as well. However, there are additional constraints that are respected in several papers, e. g., prescribed maximum and/or minimum distances between facilities, constraints requiring or prohibiting the (joint) location of certain facilities in specific site areas or the need to determine a facility’s orientation on-site.”

2.1.1.1 Static Discrete Layout Planning Approaches

“In static discrete problems the goal is to identify an assignment of facilities to positions so that each facility gets a position and no position gets more than one facility. In the basic variant, each facility can be assigned to any position. In a generalized problem version each facility can be assigned only to a subset of positions. We first review papers that focus on the basic variant and consider the generalized variant afterwards. With one exception (Ning and Lam [80]) all papers in this section focus on a single objective.

The basic variant is equivalent to the quadratic AP if the objective is to minimize TWPC. Lam et al. [54], Lam et al. [55], Liang and Chao [62], and Lien and Cheng [65] employ an ant colony optimization, a hybrid of GA and max-min ant system, tabu search, and a particle bee algorithm, respectively. Lam et al. [53] and Li and Love [60] develop GAs. In addition to their GA, Lam et al. [53] present fuzzy techniques and the entropy technique for determining the proximity weights. Wong et al. [119] restrict their scope to an area on a construction site where pre-cast concrete is produced. They basically consider the quadratic AP, propose a GA and a mathematical program that is solved by a standard solver.

Warszawski [116] presents a mathematical program that basically models an AP with the objective to minimize TWPC plus TAC. For the same problem, Mawdesley and Al-Jibouri [72] and Yeh [125] present a GA and a solution approach that combines simulated annealing with artificial neural networks, respectively.

Other authors tackle an AP with the objective to maximize TWPC plus TAC. A hybrid approach of simulated annealing and artificial neural networks (Yeh [126]), a particle bee algorithm (Cheng and Lien [20]) and tabu search (Liang and Chao [62]) are employed.

The generalized variant of the problem where some objects cannot be assigned to each position turns out to be less popular in the literature. Li and Love [61] extend their approach in Li and Love [60] by including unequal-size facilities. Zhang and Wang [130] consider the objective to minimize the sum of TWPC and the TAC and solve the corresponding problem by particle swarm optimization.

Ning and Lam [80] are the only ones to perform multi-objective optimization. For the generalized version of the assignment problem, they propose an ant colony optimization. From a structural perspective, both objectives – minimization of transportation effort and minimization of safety risks – express TWPC.”

2.1.1.2 Dynamic Discrete Layout Planning Approaches

“In dynamic discrete problems the goal is to identify an assignment of facilities to positions over time. We have a discretized time horizon partitioned into a number of periods. Each facility may occupy the assigned position for multiple consecutive periods. No position can get more than one facility in any period. Again, we start with single-objective approaches and consider multi-objective ones afterwards.

Ning et al. [81] propose a sort of ant colony optimization for a problem that is a quadratic AP over T periods with the objective of minimizing the weighted sum of transportation effort and safety risk over

the whole planning horizon. Both transportation effort and safety risk are TWPC from a structural perspective. The problem is altered by Ning et al. [82] in two ways: first, unequal-size facilities are included and second, besides the weighted objective function, true multi-objective optimization is employed. For both approaches, ant colony optimization is used and the final decision is supported by fuzzy techniques.

Multi-objective optimization in a construction project that spans over T periods is also considered by Xu and Li [121]. They employ a multi-objective particle swarm optimization algorithm in order to minimize the net present value of costs – comprising both deterministic and stochastic elements – on the one hand and safety risks related to the positions of high-risk and high-protection facilities on the other hand.”

2.1.1.3 Static Continuous Layout Planning Approaches

“In continuous layout problems, there are no predefined locations for placing objects, but the whole site area – excluding areas of fixed objects – is available. Often, this continuous space is discretized by laying a grid over the site. In any case, for each object the coordinates have to be determined. Some papers additionally focus on an object’s orientation, which is usually limited to determining whether an object is placed horizontally or vertically in a coordinate system. The objectives are similar to those in discrete approaches and are generally subject to the constraints of non-overlapping of objects and placing objects completely within the site boundaries. Subsequently, we first present single-objective and afterwards multi-objective approaches.

Most papers propose quantitative approaches for minimizing TWPC. Imam and Mir [45] consider a continuous site that can be decomposed into rectangles where a number of rectangular facilities of given sizes has to be placed. Hegazy and Elbeltagi [41] use a GA for placing facilities of arbitrary shapes on a given construction site of arbitrary shape that is modelled as a grid. This approach is extended in Elbeltagi and Hegazy [30] by developing a decision support system that first identifies the necessary facilities and their sizes via an expert system, then employs fuzzy techniques to determine the proximity weights between facilities and finally solves the problem from Hegazy and Elbeltagi [41]. Zouein et al. [135] consider a site on which a number of rectangular facilities with given sizes and known material flows has to be placed, i. e. location and orientation have to be determined in a continuous space. In addition to the above mentioned general constraints, prescribed minimum and maximum distances between facilities as well as relative positions of facilities have to be respected. A GA is proposed for solving the problem. Osman et al. [85] employ a GA for planning the layout of a site of arbitrary shape modelled as a grid. Rectangular facilities of different sizes have to be placed. Easa and Hossain [26] formulate a mathematical program for the problem to position a number of facilities on a site that is divided into several rectangular areas. Within these areas, facilities can be positioned arbitrarily with regard to several constraints comprising minimum and maximum distances between facilities and prohibited areas for certain facilities. Sanad et al. [94] tackle a similar problem with a given construction site of arbitrary shape, given facilities of given sizes and shapes. Minimum and maximum distances are required between facilities. The authors discretize the site using a grid and

propose a GA to solve the problem. Hammad et al. [38] consider a rectangular site with rectangular barriers on it. Rectangular facilities have to be placed and oriented on-site overlapping neither each other nor barriers (i. e. barriers render the feasible region for facility placement non-convex). The goal is to minimize TWPC. As barriers are infeasible regions for transportation movements, distances among facilities have to account for these regions. This problem setting is formulated as a mathematical program. However, in order to simplify the setting the continuous region is discretized using a grid. For the discretized version, two variants of problem setting are proposed. In the first one, there may be at most one facility per cell, in the second one, multiple facilities may share a cell. Both variants are modelled as MIPs. For the first variant, an exact approach – a cutting plane algorithm – is proposed and tested favourably against a standard solver.

Mawdesley et al. [73] consider a rectangular construction site that is modelled as a grid and a number of rectangular facilities of different sizes. There are prohibited areas for facilities and distance constraints between facilities. The problem to minimize TAC plus TWPC is approached using a GA.

Zhou et al. [133] consider a construction site of arbitrary shape with rectangular or circular zones where no facility can be placed and rectangular facilities have different sizes. They develop a GA that respects both hard constraints, e. g. non-overlapping facilities, and soft constraints, e. g. preferred orientation of a facility, when locating facilities in continuous space.

A number of papers aim at multi-objective optimization with rather specific applications and, thus, specific components for layout evaluation. El-Rayes and Khalafallah [28] consider a construction site of a given shape that can be decomposed into rectangles, where a number of rectangular facilities with different sizes has to be located so that safety is maximized and TWPC is minimized. They propose a GA to solve this problem. El-Rayes and Hyari [27] develop a GA for illuminating a highway construction site. The objective is to determine number, type, configuration and location of lighting equipment so that on the one hand average illuminance and lighting uniformity are maximized and on the other hand lighting costs and glare are minimized. Khalafallah and El-Rayes [50] propose a GA for multi-objective optimization on an airport construction site. A number of rectangular facilities has to be placed in order to maximize construction safety, aviation safety and airport security and to minimize layout costs. Xu and Song [122] propose multi-objective optimization via particle swarm optimization for placing given facilities on a given continuous site. The objectives are minimization of TWPC and maximization of both site utilization and logistics relevancy. In Hammad et al. [37], the construction site is partitioned into several rectangular areas within which rectangular facilities have to be located continuously. Multiple facilities may be located in the same area, but facilities must not overlap and must not reach out of the respective area. The first objective function is to minimize TWPC while the second objective function seeks to minimize noise pollution (measured as the maximum noise level recorded at given on-site receivers). The problem is modelled as a mixed-integer non-linear program. In the model, a location-dependent distance measure rather than the classical Euclidean or Manhattan distance is used. Trade-off solutions are identified by employing the ϵ -constraint method.”

2.1.1.4 Dynamic Continuous Layout Planning Approaches

“Just as in Chapter 2.1.1.2, the goal here is to identify an assignment of facilities to positions over time. Again, we have a discretized time horizon and, first, present single-objective and afterwards multi-objective approaches. Within these parts, we distinguish between the problems that allow relocation of facilities and those that do not.

Elbeltagi et al. [31, 32] extend the paper by Elbeltagi and Hegazy [30] discussed in Chapter 2.1.1.3 to a multi-period problem. Andayesh and Sadeghpour [6] consider a construction site of arbitrary shape. Circular facilities that remain on-site for given time intervals have to be placed so that TWPC is minimized. The authors discretize neither the construction site nor the planning horizon and propose a concept from physics called minimum total potential energy for layout optimization. Minimizing TWPC in a multi-period problem is also considered by Hammad et al. [36]. The construction site is partitioned into several rectangular areas. Rectangular facilities have to be positioned continuously within these areas. There may be more than one facility per area, but a facility must not reach out of an area and must not overlap with another facility. The problem is formulated as a mathematical program. Additionally, the paper has two special features regarding the calculation of TWPC (i.e. the sum of the products of inter-facility distances and travel frequencies): first, travel frequencies are derived from work schedule and building information modeling (BIM) information. Second, distances are computed depending on the facilities’ locations like in Hammad et al. [37].

Zouein and Tommelein [134] propose an approach that combines mathematical programming and a customized heuristic for the following problem: in each period, a number of rectangular facilities has to be placed in terms of location and orientation on a given continuous construction site so that TWPC and total relocation cost are minimized over all periods. As constraints, both minimum and maximum distances between facilities and relative positions of facilities have to be respected as well as restrictions regarding the assignment of facilities to areas. El-Rayes and Said [29] focus on a rectangular construction site – modelled as a grid – over a number of planning periods. Rectangular facilities have to be placed in terms of location and orientation in order to minimize relocation cost and TWPC over the whole planning horizon. There are constraints regarding minimum and maximum distances between facilities and area constraints (i.e. certain facilities have to be or must not be placed in certain areas of the site). The problem is solved by approximate dynamic programming (DP). An alternative solution procedure, namely a GA, is proposed in Said and El-Rayes [93]. In Said and El-Rayes [91] a similar problem is considered. A special feature of this paper is the integration of procurement planning. Some of the facilities are storage areas; their sizes depend on the inventory levels which result from the procurement decisions. The authors develop a GA to minimize the sum of relocation cost, TWPC, stock-out cost, capital and ordering cost.

Few authors perform multi-objective optimization. Yahya and Saka [124] consider a construction project of multiple periods. In each period, a number of rectangular facilities has to be placed on the site – modelled as a grid – with regard to location and orientation. The objectives are minimization of TWPC and minimization of safety risk – from a structural perspective, a kind of TWPC – in each period subject to the fact that some facilities have to be placed within the radius of a tower crane. The optimization is performed by a multi-objective artificial bee colony algorithm.

In contrast to Yahya and Saka [124], the following papers allow relocation of facilities. Said and El-Rayes [90] consider a special type of construction project – a critical infrastructure project. The problem is similar to the one in El-Rayes and Said [29], but now security is an additional issue. Therefore, relocation cost and TWPC have to be minimized while security is maximized. With a GA, non-dominated solutions are found. The paper by Said and El-Rayes [91] is extended by Said and El-Rayes [92] in such a way that areas within the building structure can be used as storage areas as well and the construction schedule can be altered in order to make areas within the building available at certain times. Two objectives, namely changes in the schedule and the sum of relocation cost, TWPC, stock-out cost, capital and ordering cost, are to be minimized. A GA is employed to find non-dominated solutions. Xu and Song [123] consider a multi-period layout problem where rectangular facilities have to be located on a rectangular construction site. There are two objective functions, namely the minimization of distances between the facilities' centers and the minimization of the sum of TWPC and relocation cost over the complete planning horizon. In the problem, the cost factor for calculating TWPC accounts for uncertainty. A mathematical program is formulated and a particle swarm algorithm is proposed for solving the problem.”

2.1.1.5 Location Problems

“In Warszawski [116], a static discrete FLP is modelled with customers representing demand points on the construction site and facilities representing feasible supply locations. Warszawski and Peer [118] consider the special case of locating only a single supply point. Both papers present exact and heuristic solution procedures. An extended model that considers multiple supply locations and materials is presented in Warszawski [117] and Warszawski and Peer [118] along with several exact and heuristic solution approaches. Huang et al. [43] formulate a mathematical program for the construction of a high rise. Each storey implies demand for multiple materials. Some storeys which are divided into cells can be used for storing materials and supplying others. Such cells correspond to facilities in the FLP while storeys with demand correspond to customers.

Some papers consider dynamic discrete FLPs over multiple periods. Warszawski [116] presents an extended mathematical program that considers multiple periods and materials and respects period-, material- and location-specific operating costs of supply points. Similarly, Warszawski [117] and Warszawski and Peer [118] introduce a model considering a single material and multiple periods and propose a number of exact and heuristic solution procedures. Chau [18] formulates a mathematical program for a multi-echelon distribution of material. It can be transported directly from a supply to a demand point or via a transshipment center. There is a given set of available transshipment centers. Opening, operating and closing a transshipment center is charged with costs. For each period it has to be decided which transshipment centers are used and which material flows occur. The objective is to minimize total costs over the planning horizon. A two-stage solution approach is proposed. While a GA searches for good configurations of transshipment centers, a transshipment problem is solved in order to evaluate such a configuration.

Warszawski [116] considers a static continuous version of the FLP where demand points with given

Field of application	Literature
Crane selection	[2], [3], [39], [40], [96], [98], [99], [101], [120]
Crane location	[44], [46], [66], [88], [89], [108], [109], [113], [131], [132]
Lift planning	[4], [5], [17], [48], [49], [56], [57], [58], [59], [68], [84], [87], [106], [110], [111], [114], [127], [128], [129]
Miscellaneous	[1], [35], [51], [67], [71], [100], [102], [103], [105], [112]

Table 2.2: Classification of crane-related research

coordinates and demands have to be supplied by a given number of supply points that have to be located.”

2.1.2 Construction Crane-Related Planning Problems

Once the on-site location of arbitrary facilities has been considered, the location of a specific type of facility, i. e. of cranes, is focused now. As pointed out in the introductory part of Chapter 2.1 crane location decisions are ideally made integratedly with other crane-related decisions which affect location planning. Thus, the literature reviewed here is not limited to crane location planning, but includes the most prominent fields of crane-related planning problems. These fields comprise crane selection, crane location and lift planning and are reviewed in Chapters 2.1.2.1, 2.1.2.2 and 2.1.2.3, respectively. “Finally, approaches that do not fit into this classification scheme or that integrate different fields of application are presented in Chapter 2.1.2.4. In each part, approaches for mobile cranes and those for tower cranes are distinguished. The literature and its categorization are summarized in Table 2.2.

While the modelling variety in crane selection and lift planning problems is rather small, crane location problems are more diverse. Regardless of the considered crane type, they differ with respect to the perspective of crane location which is either discrete or continuous, i. e. there is either a finite set of locations to be chosen from or an infinite set. Additionally, it can be distinguished between approaches locating a single crane or several cranes – in the latter case, it can be further differentiated between approaches for homogeneous and heterogeneous cranes. With regard to objectives, there are three broad categories: time-, cost-, and safety-oriented objectives. Time-oriented objectives focus on minimizing (average) transportation times by locating cranes. Depending on the cost to be considered, cost-oriented objectives are closely related to time-oriented objectives. This is the case when transportation amounts are weighted by a cost factor instead of a rate for transportation time. Sometimes, costs include – maybe location-specific – charges for setting up and dismantling cranes, as well. Safety-oriented approaches usually focus on collision potential among cranes and, thus, have to operationalize this goal, e. g. by the extent to which the cranes’ operating areas overlap.

Restrictions are often not clearly defined or are implied in some assumptions. Most approaches just require the coverage of certain objects (these can be geometric objects like points or polygons or even the whole site). Different interpretations of coverage can be found in the literature: an object may be covered jointly by several cranes or it has to be covered completely by at least one crane.”

2.1.2.1 Crane Selection Problems

“Crane selection is concerned with the selection of the appropriate crane types, cranes, and crane configurations for a construction project. Most decision support approaches tackling this type of problem have their focus on laying the ground for the application of OR approaches, e. g. by identifying important drivers, rather than on formulating models or applying algorithms. However, a classic OR problem capturing a flavour of crane selection certainly is the set cover problem, see Caprara et al. [14], as a set of requirements regarding different types of capabilities has to be met by the selection of cranes. At first, we present approaches not restricted to a particular type of crane. Afterwards, the focus is on mobile cranes and, finally, tower cranes are considered.

Shapira and Goldenberg [98] emphasize the importance of soft factors in equipment selection in general (not restricted to cranes) and present an analytical hierarchy process-based approach for the quantification of soft factors which then can be combined with hard factors to compute a score for each equipment. The same authors identify 27 important soft factors in equipment selection (not restricted to, but mainly focused on cranes) and criticize their lack of consideration in both practice and research (Shapira and Goldenberg [99]). Furthermore, they find that there is no structured equipment selection process in practice and thus propose one that respects both hard and soft factors. Hanna and Lotfallah [39] use fuzzy techniques to select the right crane type for a given construction project based on both quantitative and qualitative factors. Sawhney and Mund [96] develop a tool called IntelliCranes that helps the user in selecting the right crane type and crane for a given project. A neural network is employed to find the best crane type. Finally, an expert system finds feasible crane models.

Shapira and Schexnayder [101] conduct interviews with experts in order to identify and rank relevant factors in mobile crane selection. Al-Hussein et al. [2, 3] propose algorithms for selecting mobile cranes for a given construction project. An iterative procedure to filter all technically feasible mobile crane configurations from a database for a given lift operation is introduced by Wu et al. [120].

For a given construction project, Hasan et al. [40] present a framework for tower crane selection that respects simulation results, the crane’s productivity and carbon emission.”

2.1.2.2 Crane Location Problems

“In crane location planning, the location of a single crane or several cranes has to be determined considering various site constraints. Reference problems from OR depend on the exact problem variant under consideration, i. e. the features of cranes captured by the problem setting. In a simple variant, cranes are handled like any other facility on the site. Consequently, this can be regarded as a specific application of a layout or location problem from Chapter 2.1.1 where cranes (facilities) have to be placed in order to supply certain areas with material. A less simplifying problem variant focuses on a two-dimensional perspective where cranes (discs) have to be placed in some area (the construction site) so that other areas (buildings, e. g.) are covered. Covering problems, in general, have received considerable attention in the OR literature (see Farahani et al. [34]). We would like to emphasize that the problem described has a set cover flavour and we even have a geometric interpretation of subsets to

be chosen (buildings being covered when a particular crane is placed in a particular position) so that each element of the groundset is covered. Problems of this type are referred to as geometric set cover problems, see Clarkson and Varadarajan [23]. If discs – i. e. working areas of cranes – are preferred not to overlap, we may have a flavour of circle packing problems, see Castillo et al. [15].

For a given construction project, Tantisevi and Akinci [113] present a simulation-based procedure to identify possible mobile crane locations that avoid spatial conflicts and minimize the number of crane relocations when performing a number of lift tasks. Safouhi et al. [89] propose an algorithm to determine collision-free areas for positioning a given mobile crane.

Location of tower cranes in the literature is mostly concerned with single cranes only. Rodriguez-Ramos and Francis [88] consider a problem where a tower crane serves a set of construction facilities with known locations. Servicing a facility with its specific demand causes transportation cost depending on the demand and the distance between facility and crane. The authors treat this problem as a single facility location problem with rectilinear distances and solve it by means of graph theory. In Zhang et al. [131], a set of supply points and a set of demand points are given. Transportation amounts for each pair of supply and demand point are known. Additionally, a set of feasible crane locations is given. A simulation based approach is used to find the optimal crane location, i. e. the location with the minimal average transportation time. Tam et al. [109] develop a GA in order to solve another problem concerning tower cranes. For a given set of demand points, a given set of possible supply points, known transportation amounts between supply and demand points, given transportation cost rates and a set of possible crane locations, the objective is to select supply points and a crane location so that total transportation cost is minimized. In a follow-up paper, Tam and Tong [108] extend the proposed approach by adding the prediction of hoisting times via an artificial neural network. Huang et al. [44] formulate an MIP for a more general tower crane problem where more than one type of material is considered.

Only a few papers focus on the location of multiple tower cranes. Zhang et al. [132] consider a setting similar to the one in Zhang et al. [131]. Here, however, there is a group of cranes, but transportation between two points cannot be conducted jointly by more than one crane. Irizarry and Karan [46] propose a graphically supported planning approach to locate tower cranes on a construction site and reduce collision potential. Lien and Cheng [66] consider a tower crane problem with given supply and demand points where the supply capacity of each supply point is limited and each demand point has a certain given demand. For a given set of cranes and a given set of possible crane locations, a particle bee algorithm is proposed to determine transportation amounts between supply and demand points and to select the crane locations that minimize total costs, that comprise transportation and crane costs.”

2.1.2.3 Lift Planning Problems

“Lift planning problems are concerned with the transport of objects by means of cranes. There are two types of typical OR problems related to this area. The problem to sequence a set of transport requests to be conducted by some mean of transport is captured by the pickup and delivery problem,

see Berbeglia et al. [9]. There are even variants where pickup and delivery locations are positioned on a circle which comes very close to the moving characteristics of many crane types on construction sites. The second typical OR problem stems from computational geometry and is referred to as path or motion planning. In a basic variant, a robot – i. e. a construct of links and joints – has to move from a given starting position to a given destination in two-dimensional space under the presence of polygonal obstacles. A feasible or even a shortest path from the starting position to the destination has to be found (see de Berg et al. [25]).

Zavichi et al. [127] formulate an MIP for the crane service sequencing problem. A single tower crane with a given location is surrounded by a set of supply and demand points with known coordinates. Transportation amounts between supply and demand points and hook travel times between locations are known, as well. The goal is to find a time-minimal service sequence.

Lift path planning problems aim at finding a feasible – i. e. collision free – path on which a given crane located at a given position can move a lifting object from a pick position to its destination on a given construction site. Most – but not all – approaches for lift path planning are based on concepts from kinematics / robotics.

For a mobile crane lift, Reddy and Varghese [87] propose a heuristic search procedure to find a lift path. Tantisevi and Akinci [114] use a cyclic coordinate descent-based procedure to generate a sequence of mobile crane moves to lift an object from its pick point to its destination. Zhang and Hammad [128] employ rapidly-exploring random trees algorithms to first plan a lift path of a hydraulic crane and later adapt the plan when new information is available. For a single mobile crane, Lei et al. [56, 57] develop methods for checking whether there is a feasible lift path for a mobile crane. Olearczyk et al. [84] propose an algorithm for finding a smooth feasible lift path for a mobile crane. A special type of mobile crane that is allowed to move during the lift is considered by Lin et al. [68]. They aim at finding a short feasible lift path by employing a rapidly-exploring random trees algorithm.

Kang and Miranda [48] and Kang and Miranda [49] first develop several search algorithms for finding and optimizing a lift path for a single tower crane that transports a given object from its pick point to its destination and then introduce an iterative procedure to coordinate multiple tower cranes to avoid collisions. A framework that allows for coordination of multiple cranes is also provided by AlBahnassi and Hammad [4]. Paths are planned initially via rapidly-exploring random trees algorithms and can be adapted in short time when new information is available. The coordination of cranes is achieved by prioritization of cranes. The higher a crane's priority, the earlier it is planned, and is then considered as a dynamic obstacle for cranes with lower priority. Zhang and Hammad [129] propose a multi-agent system for coordinating multiple cranes.

Some authors study cooperative lifts of multiple cranes. Sivakumar et al. [106] apply an A* algorithm and hill climbing for finding a lift path for two identical mobile cranes lifting a given object. A GA is used by Ali et al. [5] to find a collision free lift path for an object that is lifted by multiple cranes. Chang et al. [17] rely on the probabilistic roadmap method for planning the lift path of a given object that is moved either by a single crane or two cranes cooperatively.

The prediction of hoisting times for both mobile and tower cranes is another topic treated in the literature. For tower cranes, Leung and Tam [58], Leung et al. [59] and Tam et al. [110] identify

factors impacting hoisting times and apply multiple regression and artificial neural network models for prediction. Similarly, Tam et al. [111] propose different artificial neural network models for predicting mobile crane hoisting times.”

2.1.2.4 Integrated Problems and Other Crane-Related Problems

We, first, focus on problems integrating multiple of the decisions discussed in Chapters 2.1.2.1, 2.1.2.2, and 2.1.2.3. The decisions integrated are the ones regarding crane selection and location since these are long-term decisions (in contrast to the rather short-term issue of lift planning) which are closely related as pointed out previously.

“Furusaka and Gray [35] employ an MIP-based approach and DP for crane selection and location so that the construction site is fully covered. The objective is to minimize total costs comprised of hire, assembly and dismantling costs over the length of the construction project. Lin and Haas [67] propose an MIP-based approach in order to minimize the number of crane relocations while respecting safety measures.” Finally, Marzouk and Abubakr [71] use a combination of an analytical hierarchy process and a GA in order to identify the best-suited tower crane type for a given construction project and identify the single cranes’ on-site locations. Available locations are given by laying a grid over the site and restricting feasible crane locations to the grid’s intersection points. The objective is to minimize fixed crane cost while providing full site coverage respecting the cranes’ operating ranges.

“Furthermore, there are several approaches in the literature tackling problems that hardly fit in Chapters 2.1.2.1, 2.1.2.2, or 2.1.2.3.

Kim et al. [51] present an iterative simulation-based approach for determining a cost-optimal foundation for a given tower crane. Several researchers focus on safety aspects. Tam and Fung [112] conduct a survey on tower crane safety, whereas Al-Humaidi and Tan [1] use fuzzy set approaches to link the clearance of a mobile crane to an overhead powerline with the probability of an electrocution accident. In order to evaluate safety related to the operations of tower cranes on a given construction site, Shapira and Lyachin [100] first interview industry experts to identify factors affecting tower crane safety. Then, weights of the factors are determined (Shapira and Simcha [102]) and – for two of the identified factors – the measurement of a factor on a given site and the derivation of the associated risk are presented (Shapira and Simcha [103]). Based on that, Shapira et al. [105] finally calculate a site safety index.”

2.2 Methodological Literature

The previous part of this literature review has been application-oriented. Nevertheless, this application-oriented literature has been related to problems from OR. Now, the corresponding OR literature will be studied in more detail. However, since the related problems mentioned so far have a long tradition in OR countless papers have been published. Hence, this methodological review has to and will be limited to aspects covered in Chapters 3 and 4. As indicated in Chapter 1.2 these aspects belong to the field of location planning problems and, within this field, to the area of covering problems. Roughly speaking,

covering location problems are concerned with locating facilities in order to serve given demand sites. However, each facility has a limited operating area, i. e. whether a specific demand site can be served by a facility in a certain location depends on the distance between demand site and facility location and the facility's operating area. There has been extensive research in this field as can be seen in, e. g., the survey by Farahani et al. [34]. The review presented in the following will thus be restricted to the branches of literature related to the problems considered in this thesis.

As outlined in Chapter 1.2, in this thesis, cranes have to be located on a polygonal construction site in order to establish transport relations between polygonal supply and demand sites. Obviously, a crane's ability to establish such a relation depends on its lifting capacity which, in turn, is related to the operating range of a crane due to the law of the lever. Thus, from a methodological perspective, covering location problems are considered in this thesis where facilities are tower cranes and demands to be covered are represented by polygonal areas. The covering characteristics of a crane depend on the precise problem under consideration. The following parts are thus dedicated to the specific problem variants.

2.2.1 Continuous Covering Location Planning with Circular Coverage Areas

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [10].

As already pointed out in Chapter 1.2, in a first approach, cranes will be located in continuous space with only few operational constraints being respected (Chapter 3). Particularly, there will be no crane interdependencies and no interferences of a crane's jib with any on-site object. This, in combination with a tower crane's moving characteristics, allows to represent a crane's operating area by a disc centered at the crane's location. Hence, this part of the literature review is dedicated to covering objects in the plane by discs.

“With respect to covering objects in the plane by discs, research has predominantly focused on covering points. As covering points in the plane by discs has been shown to be NP-complete (Johnson [47]), a lot of research has dealt with heuristics and approximation algorithms. The problems dealt with in the literature can be divided into problems where potential disc positions are given and the actual discs can be chosen only from them and problems where discs can be positioned freely in a plane (with some restrictions). Since there is a huge amount of literature, we refer the reader to the papers by Liao and Hu [63] and Chen et al. [19] providing extensive reviews on related literature. In their review, Chen et al. [19] cite several papers that make use of the observation that a disc covering at least two points has (at least) two of them on its border. We will develop a generalization of this observation and make use of it in our approach.

Covering edges by discs that may have arbitrary positions in the plane is considered only by Mao et al. [70]. They study two problems of which one is concerned with placing homogeneous discs in the plane so that every point on each edge of a given graph is covered by at least k discs. Note that this does not mean that each point on an edge has to be covered by the same discs.

For covering two-dimensional polygons by discs, there has been a considerable amount of research. We distinguish between a rather theory-driven branch of research and a rather application-driven branch. The theoretical branch deals with thinnest coverings, i.e. a given polygon has to be covered by a given number of discs whose positions and radii have to be determined. Most researchers consider identical discs, so the objective is to find the positions of the discs' centers that allow for minimal radius ensuring full coverage of the polygon. Research includes thinnest coverings of rectangles by five discs and – for certain aspect ratios of the rectangle – by seven discs (Heppes and Melissen [42]) which is later extended by Melissen and Schuur [74] who present thinnest coverings by seven discs and – again, for certain aspect ratios of the rectangle – by six discs. Nurmela [83] presents results (conjectured to be optimal) for covering an equilateral triangle by up to 36 identical discs. Das et al. [24] present an algorithm for finding a thin (not necessarily a thinnest) covering of an arbitrary convex polygon, Stoyan and Patsuk [107] do so for any polygon. Finally, Banhelyi et al. [8] consider a variant of the thinnest covering: for a given arbitrary polygon and a given number of discs with known centers, the radius of each disc has to be determined so that the sum of the squared radii is minimal while covering the whole polygon. In this setting, the discs' radii do not need to be identical. In any case, it suffices if a polygon is covered by multiple discs jointly. The application-driven branch of covering polygons by discs is mainly motivated by the insight that in facility location planning in continuous space the representation of demand sites as points is oversimplifying in certain settings (e.g. Murray and Tong [76], Murray et al. [78], Murray et al. [79] and Murray and Wei [77]).”

2.2.2 Discrete Covering Location Planning with Facility Interdependencies

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

As soon as interdependencies among cranes and interferences between cranes and on-site objects are taken into account considerations become much more complicated. This step will be taken in Chapter 4 by requiring safety clearances (i.e. minimum distances) between cranes and respecting interferences both between cranes and between cranes and on-site structures. These interferences result in the cranes' operating areas being obstructed, i.e. a crane may not be able to reach all points within its operating radius. Due to these interdependencies cranes may not be located independently and, hence, a key characteristic from Chapter 3 is lost. Thus, the continuous perspective on space is dropped and, instead, potential crane locations are given in advance by laying a grid over the construction site. Employing such a grid is a quite common technique for discretizing a by nature continuous space with potential loss of optimality, see, e.g., Furusaka and Gray [35] and Marzouk and Abubakr [71].

A combination of covering location planning and such peculiar facility interdependencies does not exist in the literature to-date. However, single facets of the problem have been studied so far. Thus, the related literature is briefly reviewed in the following.

Locating facilities with limited operating areas on a finite set of potential locations in order to serve a finite set of customers represented by points has been introduced as the location set cover problem (LSCP) by Toregas et al. [115]. Here, “a finite set of demand points is given and a minimum subset of

them has to be selected for locating fire stations. Each fire station has a given operating area, i. e. it can serve all demand points within this operating area.” In the problem considered in Chapter 4, “the set of potential facility (i. e. crane) locations is given by the grid’s intersection points and the potential operating area of a crane is given by its operating radius. However, in contrast to the LSCP, we cover polygons – or, more precisely, pairs of polygons – instead of points and the actual operating areas of the cranes to be located are not known in advance as cranes can obstruct each other.”

As minimum distances between cranes will be respected in Chapter 4, as well, there is a flavour of facility dispersion problems. “For distributing (or dispersing) facilities, there are two branches in the literature: the p -dispersion problem and the anti-covering location problem. In the discrete version of the p -dispersion problem, there is a given finite set of points from which a subset of p points has to be selected so that the minimum distance between any two of these points is maximized (Erkut et al. [33]). Contrastingly, the anti-covering location problem is concerned with selecting a maximum subset from a given finite set of points so that any two of the selected points keep at least a given minimum distance (Murray and Church [75]). These two problems are closely related. Each feasible solution to an anti-covering location instance with at least p chosen points certifies a lower bound for the corresponding p -dispersion instance (with the same point set and identical distance matrix). In turn, each feasible solution to a p -dispersion instance with a minimum distance of at least d certifies a lower bound of p for the corresponding anti-covering location instance (with distance requirement of d). This relationship can be exploited as shown in Sayah and Irnich [97].” Note that in the problem studied in Chapter 4 “the minimum distance is given, but it is not known in advance how many points (i. e. crane locations) have to be selected. Instead, the number of points to be selected depends on the requirement of covering polygons located in the plane and, furthermore, is not necessarily maximized.”

3 Tower Crane Selection and Location in the Plane

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [10].

In the current chapter, the tower crane selection and positioning problem (TCSPP) is presented. It aims at a cost-minimal selection and on-site location of tower cranes. Tower cranes have to be selected from a given set of crane types with certain specifications, e. g. cost per crane, load weight-dependent maximum lifting radii and maximum operating heights, and may be located at arbitrary points within the construction site which are not occupied by any existing on-site structures. When locating cranes the coverage of on-site supply and demand areas with specific lifting requirements has to be guaranteed. For this problem, a solution approach will be presented which allows to boil down the by nature infinite set of potential crane locations to a finite set without losing optimality.

The problem sketched above will be precisely defined in Chapter 3.1 including a proof of complexity and a brief statement regarding the contribution to the scientific literature. Afterwards, it will be proven that the by nature infinite set of potential crane locations can be boiled down to a finite set without losing optimality which allows to reduce it to the classic set cover problem (SCP) and, thus, to rely on standard OR methods (Chapter 3.2). The resulting solution approach is computationally tested and its performance is analyzed in Chapter 3.3. Finally, some problem extensions that can easily be incorporated in the approach are presented (Chapter 3.4).

3.1 Problem Definition, Computational Complexity and Contribution

In this part, a formal problem description including a discussion of the assumptions made is given (Chapter 3.1.1) and, afterwards, a proof of complexity is presented (Chapter 3.1.2). Finally, a brief statement regarding the contribution to the academic literature is given (Chapter 3.1.3).

3.1.1 Problem Definition: TCSPP

We consider a construction site represented by a simple polygon. On this construction site, a set of demand areas D and a set of supply areas S are located each of these areas being represented by a simple polygon fully contained within the site polygon. From an application-oriented perspective, demand areas can be thought of as buildings or floors of buildings where material is to be received for

completing construction tasks. Materials are supplied by on-site storage locations, i. e. supply areas. “Each demand site $d \in D$ is supplied by exactly one supply site $s_d \in S$. [...] Note that a supply site may very well supply multiple demand sites.” We refer to (d, s_d) as the supply-demand-pair, ‘pair’ for short, related to d . These pairs are completely predetermined, i. e. shape, size, and position of each demand site and each supply site as well as the transport relations between supply and demand areas are given. Each demand site $d \in D$, furthermore, has a maximum weight $w_d \in \mathbb{N}$ to be lifted and height $h_d \in \mathbb{N}$.

Tower cranes are employed for establishing all material transports from supply to demand areas. There is a set T of crane types with each crane type $t \in T$ having fixed cost π_t (e. g. setup cost and/or rental cost, the latter being a fixed amount if the duration of the construction project is fixed), maximum operating height h_t and maximum operating radius $r_{t,w}$ for each weight $w \in \mathbb{N}$. “If crane type t cannot lift weight w at any operating radius we set $r_{t,w} = -1$.” Cranes of a certain type $t \in T$ may not be located arbitrarily on-site, e. g. due to safety clearance from on-site structures or the ground’s bearing capacity. “The set of forbidden areas where cranes of type t cannot be placed is denoted by F_t .” Consequently, $F = \cup_{t \in T} F_t$ is the set of all forbidden areas and each forbidden area $f \in F$ has a set $T_f \subseteq T$ of crane types that cannot be positioned in f . Each $f \in F$ is represented by a simple polygon fully contained within the site polygon.

“In Figure 3.1, we give an illustrative example with $|T| = 2$, $S = \{s_1, s_2, s_3\}$, $D = \{d_1, \dots, d_{11}\}$ and $F = \{f_1, f_2\}$ with $T_{f_1} = T_{f_2} = T$. Assume that s_1 supplies $\{d_1, d_2, d_3\}$, s_2 supplies $\{d_4, d_5, d_6\}$ and, finally, s_3 supplies $\{d_7, \dots, d_{11}\}$. There are two different maximum weights w and w' to be lifted at the single demand sites, $w < w'$. Demand areas with maximum weight w are represented by solid polygons, demand areas with maximum weight w' are represented by dashed polygons, that is we have $w_{d_1} = w_{d_3} = w_{d_4} = w_{d_5} = w_{d_8} = w_{d_{11}} = w$ and $w_{d_2} = w_{d_6} = w_{d_7} = w_{d_9} = w_{d_{10}} = w'$. Consequently, we differentiate between two maximum lifting radii for each crane type t , namely $r_{t,w}$ and $r_{t,w'}$ with $r_{t,w} > r_{t,w'}$, represented by solid and dashed circles, respectively. Additionally, there are two heights of demand areas, $h < h'$. Assume that the smaller of the two crane types available can operate at height h only, while the larger crane type can operate at height h' , as well. Further assume that $h_{d_4} = h_{d_5} = h_{d_8} = h'$ while the remaining demand sites have height h . Given these data, in Figure 3.1, the small crane located at k_3 does neither cover the pair (s_3, d_7) nor the pair (s_3, d_8) : d_7 does not lie within the crane’s working radius for the high weight and d_8 ’s height is larger than the small crane’s maximum operating height. Instead, pairs (s_3, d_7) and (s_3, d_8) are covered by the large crane located at k_2 .

A solution is a number $k_t \in \mathbb{N}^0$ for each crane type $t \in T$ and a set P_t of k_t points within the construction site for each $t \in T$. Such a solution implies that k_t cranes of type t are positioned with their centers at the respective points.

For a given solution, we say that a crane of type t centered at point p covers pair (d, s_d) if each point in d and each point in s_d has Euclidean distance to p of no more than r_{t,w_d} and $h_t \geq h_d$. That is, the crane can lift the maximum weight associated with demand site d at each point in d and at each point in d ’s supply site s_d and its maximum operating height is sufficient. Since we are interested in Euclidean distances we can represent a crane centered at point p as a set of discs (one for each potential

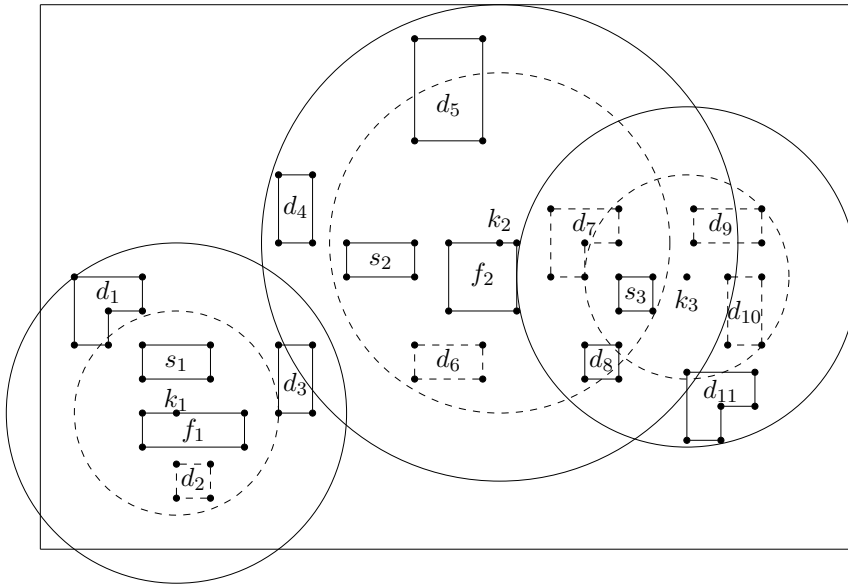


Figure 3.1: Illustrative construction site

weight) centered in p . The crane of type t with sufficient height centered at point p covers pair (d, s_d) if the disc with radius r_{t,w_d} centered at point p fully covers the polygons of d and of s_d .

A solution is feasible if and only if pair (d, s_d) is covered by at least one crane for each demand site $d \in D$ and no crane type $t \in T$ has a point $p \in P_t$ so that p is in any $d \in D$, $s \in S$, or $f \in F_t$. Less formally, a solution is feasible if each demand site can be supplied and each crane's position is feasible for the respective crane, that is no crane is positioned in a supply site, a demand site, or an area forbidden for the respective crane type.

We like to emphasize that this definition of feasible solutions incorporates required minimum distances between demand sites and supply sites and cranes since these can be easily represented by forbidden areas.

We associate total cost of $\sum_{t \in T} k_t \pi_t$ with such a solution and the tower crane selection and positioning problem (TCSPP) is to find a feasible solution with minimum total cost among all feasible solutions. [...]

Finally, we like to summarize and shortly discuss the key assumptions the problem definition is based on.

1. Pairs are given. This assumption can be justified as there may be a given plan for storing materials on-site and demands can be derived from construction plans which, in turn, allows to assign supply areas to demand areas.
2. Each demand site is supplied completely by one supply site. This is certainly a simplifying assumption. We accept it for the time being, but can easily drop it as discussed in Chapter 3.4.
3. Capacities of cranes in terms of available time or number of lifts are not considered. This, again, is a simplifying assumption, but certain types of capacity constraints can easily be incorporated

as we will discuss in Chapter 3.4.

4. We consider a static construction site, i. e. there are no changes in layout over time, and, thus, once crane locations are determined, they can be kept until the end of the project. This is a simplifying assumption, as well, but in Chapter 3.4 we show how to account for certain dynamic concepts.
5. We assume that the maximum weight of a pair has to be lifted at any point of the specific pair. As we can adjust the granularity of demand and supply areas (by dividing them in a number of smaller polygons instead of representing them as a single polygon each), this assumption is not a restriction.
6. We do not consider interdependencies among cranes, i. e. a crane's location has no impact on the other cranes' locations. This is a restricting assumption and the issue will be considered more closely in Chapter 4.
7. We do not account for obstacles limiting the cranes' moves. In particular, the jib's (i. e. the crane's operating arm's) rotation is not limited by obstacles like buildings or other cranes within the jib's range. This also is a restricting assumption and the issue will be considered more closely in Chapter 4, as well.
8. Each pair has to be covered completely by at least one crane, i. e. a pair is only said to be covered if each point of the supply area and each point of the demand area are covered by the same crane. Again, since we can adjust the granularity of demand and supply areas, this assumption is not a restriction."

3.1.2 Computational Complexity of TCSPP

"The decision version of TCSPP asks whether there is a feasible solution with total cost not exceeding a given threshold.

Theorem 1. *The decision version of TCSPP is strongly NP-complete even if $|T| = 1$, $F = \emptyset$, each demand site consists of a single point only and coincides with its supply site.*

Proof. The special case of TCSPP pretty obviously generalizes the problem of geometric covering by discs (GCBD), see Johnson [47]. Here, we have a set X of points with integer-coordinates in the plane. The question is, whether all points in X can be covered by placing at most a given number Y of discs of radius r in the plane.

We obtain an instance of GCBD by letting $w_d = h_d = 0$ for each $d \in D$ and $h_t = 0$ and $r_{t,w} = r$ for each $w \in \mathbb{N}^0$ and the only crane type t .

Finally, membership in NP of the decision version of TCSPP can be seen rather easily taking into account that we do not need more than $|D|$ cranes." □

3.1.3 Research Gap

The research presented in this chapter adds to the application-oriented engineering literature in a way that, in contrast to most of the dedicated tower crane-related research activities, crane selection and location are integrated and that it is the first approach “to locate cranes optimally in continuous space by generating a finite set of candidate locations without loss of optimality.” Furthermore, regarding the methodological OR literature concerning facility location and covering problems several shortcomings which have been identified by Farahani et al. [34] are addressed.

- “Demand is often point-based. We, however, consider polygons to be covered. The idea of generating a finite set of candidate locations without losing optimality has been introduced by Murray and Tong [76], but their approach does not respect any of the subsequently mentioned problem features.
- Usually, homogeneous facilities – i. e. facilities with identical covering radii – are assumed. Since we consider multiple crane types and lifting weight-dependent working radii [...] we have heterogeneous facilities and demand- and facility-dependent covering radii.
- There is limited research on covering problems in the plane. We locate cranes in continuous space with the additional restriction that cranes have to be located on-site (i. e. within a bounded area). On top, we consider areas where cranes cannot be placed.”

3.2 Reduction to the Set Cover Problem

“As can be concluded from the problem definition in Chapter 3.1.1, there usually is an infinite number of points where a crane can be located. In this part, we first prove that the crane location problem can be restricted to a finite set of candidate locations without losing optimality (Chapter 3.2.1). Given this finite candidate set, we can generate an instance of the classic SCP. Note that we have a special case of SCP here, namely the geometric SCP. However, as mentioned in Clarkson and Varadarajan [23], applying techniques developed for SCP is a common approach with respect to approximating geometric versions of the SCP. We present the method generating the SCP instance corresponding to an instance of TCSP in Chapter 3.2.2.

3.2.1 Finite Sets of Position Candidates

In this part, we will show that we can restrict ourselves to a finite set of candidate locations for each crane type $t \in T$. Since these sets can be achieved independently we focus on a single crane type t^* only in the following. We will refer to both types of geometric objects, circle and disc, in order to represent cranes in specific positions depending on the context. We, furthermore, make two simplifying assumptions when developing our central insights. We will later on show that similar techniques following the very same ideas can be applied without these assumptions.

1. Crane type t^* can cover each demand site with respect to height, i. e. $h_{t^*} \geq \max_{d \in D} \{h_d\}$.

2. All demand sites have identical maximum weights w , i. e. $w_d = w$ for each $d \in D$. Consequently, there is only one relevant maximum operating radius, i. e. we have $r_{t^*,w} = r^{max} > 0$ for each weight $w \in \mathbb{N}$.

For simplicity we make two more assumptions. As opposed to those assumptions above, however, we argue that we can easily modify each instance so that the following assumptions are met.

3. We assume that the instance of TCSPP has no isolated vertices (a demand site with zero dimensions coinciding with its own supply site) with distance of more than r^{max} to any edge and distance of more than $2r^{max}$ to any other vertex. If there are any, we can trivially identify each isolated vertex in a pre-processing step, choose an appropriate position for a crane exclusively covering a single isolated vertex, and drop them from further consideration.
4. We assume that there is no demand site that can be covered by a crane of type t^* in any arbitrary feasible position. If there are any, we, again, can identify each such demand site in a pre-processing step and transform them into a forbidden area f with $T_f = \{t^*\}$. If the corresponding supply site has no demand site left, then we transform it, too, into a forbidden area f with $T_f = \{t^*\}$. Finally, if no demand site remains after all of them have been handled, then we choose an arbitrary position for a single crane which constitutes the only position to be considered for cranes of type t^* .

Murray and Tong [76] prove that a polygon is included in a circle of given radius when all its nodes are inside the circle's border. Based on that, they develop the idea to draw circles of the given radius with the polygon's nodes as their center points in order to identify an area where each of the nodes lies within the considered radius (i. e. they generate an area of overlapping circles). Each point within the circles' overlapping area is equally good, so it is concluded that a finite set constituted of the circles' intersection points can be used as location candidates without loss of optimality. This idea covers a special case of the setting considered in this paper. However, we will see that restricting ourselves to circles' intersection points is not enough and we will show how to generalize the candidate set accordingly.

We, first, develop a geometric property concerning edges and circles and their relative position in a plane. Consider an arbitrary circle c with center m and radius r and an arbitrary edge e with vertices v and v' . It is easy to see that among all points on e either v is the unique point with maximum distance to m , v' is the unique point with maximum distance to m , or both, v and v' are the only points with maximum distance to m . If no point on e has distance larger r to m , that is edge e is fully covered by the disc implied by c , then only v or v' can have minimum distance to c among all points on e . We summarize this observation in the following Corollary 1.

Corollary 1. *For each circle c and each edge e where the disc implied by c fully covers e , only vertices of e can have minimum distance to c .*

With Corollary 1 we are prepared to prove the following theorem stating that we have to consider specific discrete points in the plane only as candidate locations for cranes of type t^* .

Theorem 2. *Under assumptions 1. and 2. there is an optimum solution to TCSPP so that for each crane of type t^* its center*

1. *has distance of exactly r^{max} to at least two vertices of polygons in $D \cup S$ or*
2. *lies on an edge of the construction site polygon or a polygon in $D \cup S \cup F_{t^*}$ and has distance of exactly r^{max} to at least one vertex of polygons in $D \cup S$.*

In order to improve readability, we will keep the proof as informal as possible without risking ambiguity.

Proof. We will modify an arbitrary optimum solution step by step to an optimum solution with each crane's center positioned according to the theorem. We do so by moving a crane's position continuously until a certain condition is fulfilled. When conducting such moves the relative position of polygons and their edges and the circle corresponding to the crane being moved changes.

The following observation is central to the proof and applies to all kinds of continuous moves. We consider an edge e covered by the disc implied by a circle c . According to Corollary 1, at the first time a point p on e lies on c while moving c , point p is a vertex of e and each point on e is covered by the corresponding disc. That means, edge e is fully covered by the corresponding disc if no vertex of e lied on c while moving c .

Consider an arbitrary optimum solution Z^* for an instance of the TCSPP and a crane represented by a circle c with a center position m_1 not according to the theorem. We consider an arbitrary half-line l starting at m_1 and shift c away from m_1 so that its center is on l as long as no vertex of a pair that has been covered by the corresponding disc prior to the shift lies on c . After conducting this shift, a vertex v of a pair that has been covered prior to the shift lies on c . Note that according to Corollary 1 each pair that has been covered prior to the shift is still covered after the shift.

Let m_2 be the center's position of c after the shift. Note, furthermore, that m_2 is not necessarily a feasible position for the crane since m_2 may lie in a polygon in $D \cup S \cup F_{t^*}$ or outside the polygon representing the construction site. We distinguish three cases with regard to m_2 in the following.

1. If the resulting center point m_2 is not in a polygon in $D \cup S \cup F_{t^*}$, then we found a feasible center position for the crane having distance of exactly r^{max} to v . We then consider a circle c' with radius r^{max} and center v . Note each circle with radius r^{max} and its center on c' contains v . We proceed by moving c so that its center moves clockwise on c' (we could move counterclockwise just as well) until
 - a second vertex v' of a pair that has been covered by the corresponding disc prior to the shift lies on c ,
 - the center of c lies on the edge of a polygon in $D \cup S \cup F_{t^*}$, or
 - the center of c lies on the edge of the polygon representing the construction site

and all pairs that have been covered prior to the shift are still covered. Note that this condition will be met due to Corollary 1 and assumptions 3 and 4. Clearly, the new center m_3 of c is a

feasible position for the crane under consideration, we obtain a feasible solution when moving the crane to m_3 since it covers the same set of pairs as before, and m_3 is in accordance with the theorem.

2. If the resulting center point m_2 is in a polygon q , $q \in D \cup S \cup F_{t^*}$, then we consider the maximum set $P(q)$ of polygons in $D \cup S \cup F_{t^*}$ so that $q \in P(q)$ and for each pair of polygons (q', q'') in $P(q)$ there is a sequence of q_1, \dots, q_α so that (i) $q' = q_1$, $q'' = q_\alpha$ and (ii) q_β and $q_{\beta+1}$ are overlapping for each $\beta = 1, \dots, \alpha - 1$. Consider the union of polygons in $P(q)$ which is itself a polygon. We refer to this super-polygon as Q . Half-line l has at least two intersection points with edges of Q . Consider the intersection point p with minimum distance to c 's original center point m_1 .

We locate c so that its center point coincides with p and move c so that its center point moves on edges of polygon Q in a 'counterclockwise orientation' (clockwise would be fine, as well) as long as no vertex of a pair that has been covered prior to the shift lies on c . Note that eventually a vertex of a pair that has been covered prior to the shift lies on c since there is a vertex of Q having distance of more than r^{max} to v (m_2 lies in Q). Clearly, the new center m_4 of c (after the shift) is a feasible position for the crane under consideration, we obtain a feasible solution when moving the crane to m_4 since it covers the same set of pairs as before (due to Corollary 1), and m_4 is in accordance with the theorem.

3. If the resulting center point m_2 is outside the polygon representing the construction site, then half-line l has at least one intersection point with edges of the polygon representing the construction site. Consider the intersection point p with minimum distance to c 's original center point m_1 . We locate c so that its center point coincides with p and move c so that its center point moves on edges of the polygon representing the construction site in a 'clockwise orientation' (counterclockwise would be fine, as well) as long as no vertex of a pair that has been covered prior to the shift lies on c . Note that eventually a vertex of a pair that has been covered prior to the shift lies on c due to assumption 4. Clearly, the new center m_5 of c is a feasible position for the crane under consideration, we obtain a feasible solution when moving the crane to m_5 since it covers the same set of pairs as before (due to Corollary 1), and m_5 is in accordance with the theorem.

So taking all of the preceding arguments into consideration, we can apply the procedure above to each crane and ultimately obtain a solution according to the theorem. □

In order to ease comprehension, we now illustrate the moves of circles described in the proof. In Figure 3.2, there is a pair of polygons, say a demand site with its associated supply site, and a crane with center position m_1 and a given maximum operating radius. Figure 3.2a illustrates the initial move of the circle in an arbitrary direction according to half-line l resulting in center position m_2 of the circle as depicted in Figure 3.2b. Note that v lies on c now and m_2 is a feasible position for the crane, that is the depicted example corresponds to the first case considered in the proof. In Figure 3.2b we see circle c' along which circle c 's center is moved in the following until a second vertex v' lies on c , as well, implying c 's final center position m_3 as shown in Figure 3.2c.

In Figure 3.3a, we find a different initial setting and see the initial move of the circle in an arbitrary

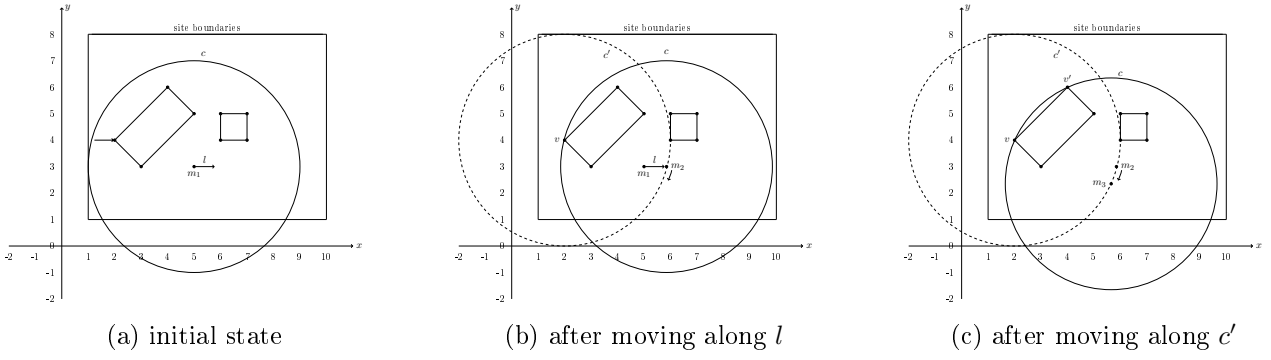


Figure 3.2: Moving the crane according to the first case

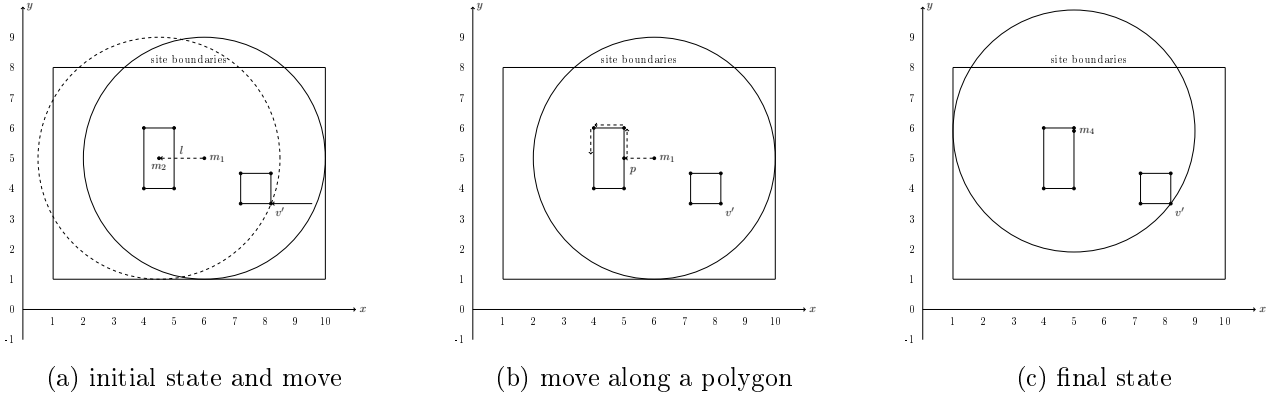


Figure 3.3: Moving the crane according to the second case

direction from its original position (depicted in solid drawing) to the resulting position (depicted in dashed drawing). Note that m_2 is not a feasible position for the crane since it is in a polygon in $D \cup S \cup F_{t^*}$, that is the depicted example corresponds to the second case considered in the proof. Figure 3.3b depicts the path along which the circle's center is moved around the polygon until a vertex v' lies on c implying c 's final center position m_4 as shown in Figure 3.3c.

Now that we have established Theorem 2 under assumptions 1. and 2. we discuss how dropping one or both of them affects our result. Dropping assumption 1. does not change the result in Theorem 2 at all. Note that we can represent demand site d with $h_{t^*} < h_d$ as a forbidden area since (d, s_d) cannot be covered by a crane of type t^* , but no crane can be placed in a polygon of d or s_d nevertheless. If for any supply site $s \in S$ each corresponding demand site d has $h_{t^*} < h_d$, then we represent s by a forbidden area, as well. Since these forbidden areas have the same shape as the demand sites and supply sites, respectively, they represent, Theorem 2 remains true even after dropping assumption 1. However, when dropping assumption 2. our result must account for different weights as stated in Theorem 3.

Theorem 3. *There is an optimum solution to TCSPP so that for each crane of type t^* its center*

1. *has distance of exactly r_{t^*, w_d} to at least one vertex v of any pair (d, s_d) covered by the crane and has distance of exactly $r_{t^*, w_{d'}}$ to at least one vertex $v', v \neq v'$, of any pair $(d', s_{d'})$ covered by the crane or*

2. lies on an edge of the construction site polygon or a polygon in $D \cup S \cup F_{t^*}$ and has distance of exactly r_{t^*,w_d} to at least one vertex v of any pair (d, s_d) covered by the crane.

Since we can apply exactly the same ideas we abstain from giving a formal proof here. When moving circles in the proof of Theorem 2 we act on a maxim predicting that no vertex of a pair covered by the crane corresponding to the circle can leave the crane's sphere of influence. This maxim we still can follow after dropping assumption 2. Then, however, the resulting distances between the center of a circle and vertices v and v' on the verge of the crane's coverage depend on the weight associated with v and v' , respectively. Theorem 3 accounts for that.

For this modification of the proof we provide an illustration in Figure 3.4 with a single supply site s supplying two demand sites d and d' . We assume that $w_d > w_{d'}$ and, thus, $r_{t^*,w_d} < r_{t^*,w_{d'}}$. The different operating radii of a crane related to d (dashed) and d' (solid) are reflected by different radii of circles (dashed/solid) with center m_1 .

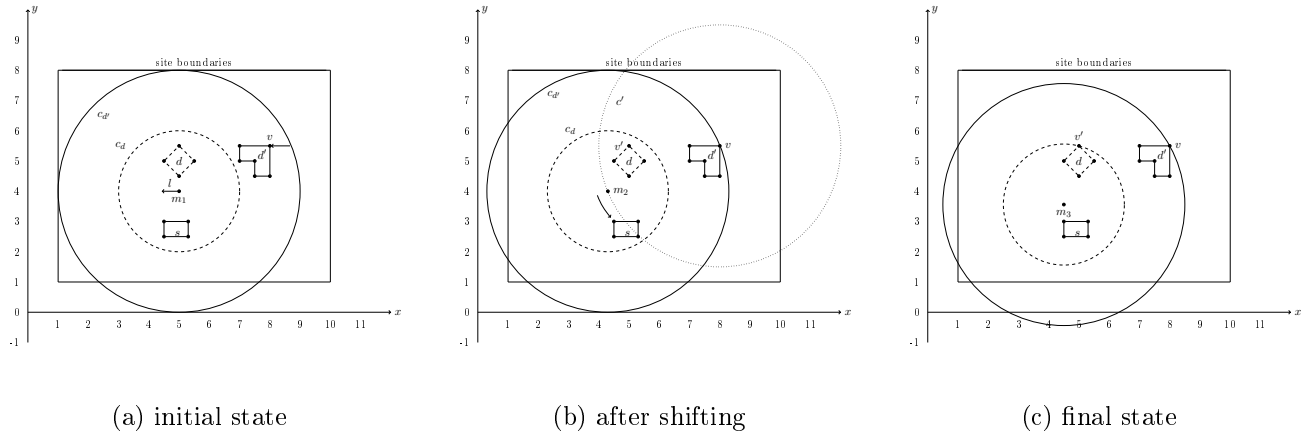


Figure 3.4: Covering demand areas with different weights

Figure 3.4a illustrates the initial move according to half-line l resulting in the circles' position depicted in Figure 3.4b. Note that v belongs to d' and lies on the corresponding circle with center position m_2 . Since m_2 is a feasible position the depicted example corresponds to the first case considered in the proof. In Figure 3.4b, we see circle c' along which circle $c_{d'}$'s (and c_d 's) center is moved in the following until a second vertex v' lies on c_d , as well, implying the circles' final center position m_3 as shown in Figure 3.4c. Note that the radius of circle c' equals $r_{t^*,w_{d'}}$ since the second moving operation is supposed to keep v on $c_{d'}$. Note furthermore that v' belongs to d and, therefore, lies on the corresponding circle with center position m_3 .

3.2.2 Set Cover Instances

Our main result in Chapter 3.2.1 stated in Theorem 3 suggests an obvious reduction of the TCSP to the weighted SCP. The set of pairs to be covered in the TCSP corresponds to the set to be covered in the weighted SCP. Each candidate position p of a crane of type t corresponds to a subset of pairs covered when a crane of type t is positioned in p . We refer to a pair of crane type t and a corresponding

candidate position as candidate in the following. The crane type and position associated with candidate c are denoted by t_c and p_c . Choosing a candidate comes at a price of π_{t_c} . Now, our problem is to choose candidates so that total cost is minimized while each pair is covered.

It remains to detail how the set of position candidates is generated. As mentioned in Chapter 3.2.1 we can do so for each crane type independently. In the following we restrict ourselves to a single crane type t^* , consequently. Potentially, each pair of vertices of polygons in $D \cup S$ and each pair of a vertex of a polygon in $D \cup S$ and an edge of any polygon can give rise to multiple candidates. Recall that vertices of polygons in S may be associated with multiple weights since a supply site may supply multiple demand sites (each with a unique maximum weight). Let W_s be the set of maximum weight values associated with $s \in S$, that is $W_s = \{w_d \mid d \in D, s_d = s\}$. In the following, we outline the set of position candidates in detail.

1. For each pair of vertices v and v' of two (not necessarily distinct) demand sites d and d' , respectively, we have at most two points with distance of r_{t^*,w_d} to v and distance of $r_{t^*,w_{d'}}$ to v' .
2. For each pair of vertices v and v' of demand site d and supply site s , respectively, and each weight $w \in W_s$ we have at most two points with distance of r_{t^*,w_d} to v and distance of $r_{t^*,w}$ to v' .
3. For each pair of vertices v and v' of two (not necessarily distinct) supply sites s and s' and each pair of weights $w \in W_s$ and $w' \in W_{s'}$, respectively, we have at most two points with distance of $r_{t^*,w}$ to v and distance of $r_{t^*,w'}$ to v' .
4. For each pair of a vertex v of a demand site d and an edge e of any polygon in $D \cup S \cup F_{t^*}$ or the polygon describing the construction site we have at most two points on e with distance of r_{t^*,w_d} to v .
5. For each pair of a vertex v of a supply site s and an edge e of any polygon in $D \cup S \cup F_{t^*}$ or the polygon describing the construction site and each weight $w \in W_s$ we have at most two points on e with distance of $r_{t^*,w}$ to v .

Obviously, each of these points can qualify as a candidate only if it is not in any polygon in $D \cup S \cup F_{t^*}$ but in the polygon describing the construction site. Additionally, we have some further rules enabling us to reduce the number of candidates to be considered.

- a) A vertex of a polygon in $D \cup S$ needs to be considered in 1. to 5. above only if the interior angle is below 180° . We refer to such vertices as spanning vertices in the following. Note that the convex hull of the set of spanning vertices of a polygon coincides with the convex hull of the polygon itself. Furthermore, it is easy to see that a disc covers a polygon if and only if it covers its convex hull. Since spanning vertices are the only vertices of the polygon corresponding to the convex hull we can restrict ourselves to them for generating candidates.
- b) Strengthening 3. above, for each pair of vertices v and v' of (the same) supply site s we consider a point only if it has distance $r_{t^*,w}$ to both, v and v' , for any weight $w \in W_s$. That is, we do not consider points having distance according to different weights in W_s . Using the same ideas as in

the proof of Theorem 3 we can easily see that this restriction does not prevent us from finding the optimum solution.

- c) A point generated according to 1. above needs to be considered only if pairs (d, s_d) and $(d', s_{d'})$ are covered by a crane positioned in the point. Analogously, a point generated according to 2. or 4. above needs to be considered only if pair (d, s_d) is covered by a crane positioned in the point.
- d) A point generated according to 3. above needs to be considered only if at least one pair (d, s) with $w_d = w$ and at least one pair (d', s') with $w_{d'} = w'$ is covered by a crane positioned in the point. Analogously, a point generated according to 2. or 5. above needs to be considered only if at least one pair (d', s) with $w_{d'} = w$ is covered by a crane positioned in the point.

For the sake of clarity we outline the mixed-integer programming fomulation. We have a binary variable α_c for each candidate c . Let C be the set of candidates. Given the crane type t_c and position p_c of candidate c we can easily derive parameter $i_{d,c}$ signaling whether candidate c covers pair (d, s_d) from the set of all pairs P ($i_{d,c} = 1$) or not ($i_{d,c} = 0$). We, then, can formulate the following model.”

$$\text{Min } Z = \sum_{c \in C} \alpha_c \cdot \pi_{t_c} \tag{3.1}$$

$$\sum_{c \in C} \alpha_c \cdot i_{d,c} \geq 1 \quad \forall (d, s_d) \in P \tag{3.2}$$

$$\alpha_c \in \{0, 1\} \quad \forall c \in C \tag{3.3}$$

3.3 Computational Study

“The discretization scheme outlined in Chapter 3.2 enables us to solve TCSPP using any method suitable for solving SCP instances. We employ two methods in the following. First, we employ standard solver CPLEX 12.6.3 in order to solve the SCP instances exactly. Second, we implemented the greedy approach proposed by Chvatal [21] in order to find heuristic solutions in a short amount of time in case time is a critical factor. The implementation has been done in Java 8 using the Eclipse development environment. All computational studies have been performed on a computer with 32GB RAM and an i7-4790 CPU @ 3.6GHz. An analysis of performance is presented in Chapter 3.3.2. The evaluation is based on a set of instances whose generation is described in Chapter 3.3.1.

3.3.1 Test Set Generation

For our tests, we consider four different crane types 1, 2, 3, and 4, each with a specific cost, maximum working height and maximum operating radius depending on the maximum weight to be lifted. We

have five different maximum weights. We do not use specific weights and heights but classes of weights and heights instead. We distinguish four different height classes 1, 2, 3, and 4 (corresponding to the maximum operating height of the four types of cranes) where a higher class number represents a higher operating height. For weights we consider five classes 1 to 5. Consequently, maximum operating radii are given depending on crane type and weight class in Table 3.1. Furthermore, Table 3.1 gives the cranes' costs.

Crane type	Weight class					Cost per crane
	1	2	3	4	5	
1	5	10	10	10	10	1,000
2	6	12	20	20	20	1,500
3	7	14	22	30	30	3,000
4	8	16	24	32	40	4,500

Table 3.1: Maximum operating radii and crane cost

We consider a square construction site because the site's shape has an impact on the number of (feasible) candidates and we assume for the sake of simplicity that all demand sites, supply sites, and forbidden areas are represented by polygons of four or six vertices with their edges parallel to those of the construction site. Note that this gives rectangles and L-shaped areas. Furthermore, we restrict ourselves to two sizes of rectangles and one size of L-shaped area and rotate the respective shape randomly by 0, 90, 180, or 270 degree when placing them on the construction site.

We place supply and demand areas so that they do not overlap. Forbidden areas and safety zones (i. e. minimum distances), however, may overlap with any other area. In order to control the density of sites on the construction site, we partition the site by a grid laid over the construction site. The side length of a cell of this grid equals twice the smallest crane type's maximum operating radius for the heaviest weight. We position a supply area or a demand area completely within such a cell. A forbidden area may be placed over several cells. A cell may be empty, but if it is not, there is one supply area that supplies all demand areas in that cell. So basically, the number of demand areas in a cell determines the number of pairs in it (we work with at most two demand areas per cell). The overall number of cells is set depending on the number $|D|$ of pairs we consider in the instance at hand and a density parameter $\lambda \in \{0.4, 0.8, 1.2\}$ describing the average number of pairs per cell. The number of cells is then defined as $\lceil \sqrt{|D|/\lambda} \rceil^2$.

We distinguish between different sets of instances varying the following parameters:

- $\kappa \in \{1, 2, 3, 4\}$ is the number of crane types available
- $\eta \in \{100, 300, 500, 700, 900, 1100\}$ is the number of pairs
- $\lambda \in \{l, m, h\}$ is the density (l for low (0.4), m for medium (0.8), h for high (1.2))
- $\mu \in \{no, sim, min\}$ indicates whether there are forbidden areas (*no* if there are none, *sim* if there are simple forbidden areas – i. e. no minimum distances –, *min* if forbidden areas reflect simple forbidden areas and minimum distances as well)

- $\sigma \in \{t, f\}$ indicates whether demand sites differ in their maximum weights (t for true, f for false)
- $\phi \in \{t, f\}$ indicates whether demand sites differ in their heights (t for true, f for false)

With the given parameters, an instance is created using uniformly distributed random numbers. First, κ crane types are drawn from the crane type list. Then, demand sites are randomly assigned to the grid's cells. The procedure continues until the total number of demand areas to be placed is reached. In a next step, exact (non-overlapping) positions for supply and demand areas are randomly determined for each cell. The resulting pairs are randomly assigned a weight and a height (not above the maximum height class among the chosen crane types) in case different weights and different heights, respectively, are to be considered. Finally, shapes of forbidden areas are determined and they are, afterwards, placed on-site.

3.3.2 Results

In the first part of our computational study, we want to evaluate the impact of the individual parameters presented in Chapter 3.3.1 on both candidate generation and solution of the SCP instance (Chapters 3.3.2.1 and 3.3.2.2). For this purpose, it suffices to set $\eta \in \{100, 300, 500\}$. We generate 25 instances for each combination of parameters which gives us a total of 378 parameter combinations (called scenarios, in the following) with 9,450 instances. We encode a scenario by a sequence of letters and numbers of the scheme $\kappa - \eta - \lambda - \mu - \sigma - \phi$. Note that, for the sake of simplicity, we assume $T_f = T$ for all $f \in F$, i. e. each forbidden area is a forbidden area for all crane types.

Table 3.2 summarizes the results. We outline both average and maximum values of

- the number of feasible candidates generated,
- the duration of candidate generation in seconds, and
- the total time (in seconds) of the procedure, i. e. candidate generation and exact solution via CPLEX 12.6.3

for each scenario. In a second run, we will analyze computational limitations by pushing CPLEX to the limits (3.3.2.2).

3.3.2.1 Candidate Generation Analysis

According to Chapter 3.2.2, we expect both the number of feasible candidates and the time for candidate generation (and checking) to be non-decreasing in the number of cranes, vertices and edges. This is supported by our study as the entries in Table 3.2 show: for given values of μ , σ and ϕ , increasing the number κ of crane types or the number η of pairs results in an increase of both, the maximum and the average number of candidates, and the time needed to generate the candidates. The same holds true for the density parameter λ , but here, a critical factor may be the ratio of crane radii and distance between the single pairs of supply and demand sites. However, it should be noted, that the above mentioned patterns look consistent, but there are considerable differences among single instances as

	# candidates avg. /max.	generation time (sec) avg. /max.	total time (sec) avg. /max.	# candidates avg. /max.	generation time (sec) avg. /max.	total time (sec) avg. /max.	# candidates avg. /max.	generation time (sec) avg. /max.	total time (sec) avg. /max.
1-100-		l-		m-			h-		
no-f-f	1,347 /2,787	1.18 /1.83	1.31 /2.06	2,237 /4,794	1.81 /3.69	2.03 /4.08	2,757 /5,442	2.28 /4.78	2.55 /5.25
sim-f-f	1,435 /2,929	1.26 /1.94	1.43 /2.30	2,304 /4,900	1.89 /3.84	2.12 /4.23	2,815 /5,533	2.38 /5.00	2.64 /5.47
min-f-f	1,575 /3,139	1.20 /1.97	1.35 /2.23	2,441 /4,913	1.81 /3.74	2.04 /4.14	2,892 /5,539	2.26 /4.97	2.52 /5.46
no-t-f	826 /1,496	0.86 /0.96	0.95 /1.08	1,323 /2,479	1.05 /1.47	1.18 /1.67	1,716 /3,255	1.44 /1.99	1.63 /2.25
sim-t-f	886 /1,579	0.94 /1.07	1.04 /1.19	1,372 /2,559	1.11 /1.54	1.23 /1.74	1,760 /3,298	1.51 /2.09	1.69 /2.35
min-t-f	992 /1,772	0.87 /1.04	0.97 /1.18	1,496 /2,686	1.05 /1.54	1.19 /1.75	1,855 /3,249	1.43 /1.96	1.63 /2.22
1-300-		l-		m-			h-		
no-f-f	4,441 /9,159	9.91 /15.57	11.20 /17.86	7,390 /16,027	15.97 /36.60	17.74 /40.35	9,343 /20,593	22.80 /55.62	25.12 /60.81
sim-f-f	4,698 /9,649	10.88 /17.06	12.25 /19.40	7,608 /16,390	16.75 /38.26	18.57 /42.07	9,529 /20,898	23.48 /57.20	25.86 /62.54
min-f-f	5,123 /10,199	10.48 /17.51	11.93 /20.00	7,978 /16,562	16.43 /37.45	18.34 /41.38	9,623 /20,134	22.07 /52.41	24.44 /57.30
no-t-f	2,564 /4,511	7.38 /7.78	8.26 /8.75	4,000 /7,817	9.20 /12.19	10.38 /13.88	5,178 /9,976	12.36 /16.35	14.00 /18.53
sim-t-f	2,735 /4,798	8.03 /8.63	8.93 /9.65	4,160 /8,049	9.72 /12.97	10.95 /14.71	5,321 /10,191	12.83 /16.94	14.51 /19.17
min-t-f	3,039 /5,222	7.69 /8.24	8.70 /9.36	4,521 /8,374	9.20 /12.67	10.48 /14.48	5,551 /10,222	11.90 /16.28	13.56 /18.59
1-500-		l-		m-			h-		
no-f-f	7,227 /14,663	26.50 /39.47	30.27 /45.84	12,112 /25,690	43.25 /92.89	48.54 /102.65	16,652 /36,050	69.46 /170.09	76.94 /186.38
sim-f-f	7,648 /15,370	28.56 /43.44	32.39 /51.19	12,480 /26,313	45.38 /96.50	50.73 /106.49	16,965 /36,514	71.42 /174.03	78.84 /190.49
min-f-f	8,361 /16,481	27.52 /44.98	31.51 /52.02	13,113 /27,045	44.70 /96.81	50.33 /107.33	16,908 /34,737	65.95 /157.10	73.19 /171.41
no-t-f	4,228 /7,406	19.71 /21.05	22.03 /23.66	6,801 /12,649	28.22 /32.24	32.08 /36.72	9,254 /18,658	37.64 /51.43	42.70 /58.17
sim-t-f	4,518 /7,886	21.81 /23.28	24.28 /26.05	7,072 /13,095	28.27 /34.13	31.98 /38.77	9,494 /19,025	39.09 /53.46	44.26 /60.32
min-t-f	5,037 /8,620	20.42 /22.33	23.09 /25.40	7,662 /13,725	26.79 /33.83	30.59 /38.70	9,789 /18,634	35.73 /49.76	40.72 /56.50
2-100-		l-		m-			h-		
no-f-f	2,674 /4,259	2.34 /3.55	2.62 /3.99	4,361 /7,204	4.13 /7.89	4.53 /8.48	5,393 /8,336	5.85 /10.77	6.34 /11.50
sim-f-f	2,836 /4,462	2.57 /3.86	2.86 /4.21	4,492 /7,387	4.33 /8.21	4.75 /8.83	5,506 /8,472	6.03 /11.02	6.54 /11.75
min-f-f	3,107 /4,760	2.56 /3.96	2.87 /4.63	4,756 /7,539	4.39 /8.30	4.82 /8.93	5,649 /8,441	5.97 /10.67	6.48 /11.40
no-t-f	2,297 /3,778	1.80 /2.96	2.05 /3.39	3,776 /6,420	3.30 /6.59	3.65 /7.14	4,610 /7,271	4.36 /8.10	4.76 /8.72
no-t-f	1,661 /2,412	1.63 /2.02	1.76 /2.21	2,574 /3,934	2.19 /3.12	2.42 /3.42	3,404 /5,261	2.91 /4.62	3.19 /5.04
no-t-t	1,344 /1,972	1.22 /1.63	1.34 /1.78	2,086 /3,222	1.56 /2.35	1.74 /2.60	2,721 /4,173	2.05 /3.24	2.28 /3.57
sim-f-t	2,434 /3,964	2.02 /3.28	2.29 /3.60	3,889 /6,587	3.34 /6.68	3.68 /7.21	4,707 /7,398	4.52 /8.41	4.93 /9.04
sim-t-f	1,777 /2,571	1.80 /2.22	1.95 /2.42	2,671 /4,050	2.31 /3.27	2.54 /3.58	3,492 /5,346	3.02 /4.75	3.31 /5.18
sim-t-t	1,434 /2,089	1.34 /1.78	1.46 /1.94	2,165 /3,322	1.64 /2.47	1.84 /2.73	2,789 /4,241	2.12 /3.34	2.36 /3.69
min-f-t	2,663 /4,256	2.00 /3.38	2.29 /3.73	4,112 /6,737	3.43 /6.71	3.80 /7.25	4,825 /7,427	4.43 /8.22	4.84 /8.84
min-t-f	1,987 /2,812	1.69 /2.17	1.86 /2.39	2,912 /4,309	2.24 /3.35	2.49 /3.69	3,657 /5,430	2.94 /4.65	3.24 /5.08
min-t-t	1,599 /2,319	1.25 /1.70	1.41 /2.08	2,350 /3,581	1.59 /2.49	1.78 /2.77	2,905 /4,334	2.04 /3.26	2.30 /3.61
2-300-		l-		m-			h-		
no-f-f	8,950 /14,068	20.98 /33.24	23.24 /37.18	14,486 /22,602	38.82 /70.31	42.32 /75.65	18,307 /29,506	56.95 /113.06	61.49 /120.24
sim-f-f	9,465 /14,778	23.12 /36.52	25.55 /40.56	14,915 /23,187	40.83 /73.83	44.45 /79.25	18,670 /29,966	58.90 /116.77	63.52 /124.21
min-f-f	10,272 /15,772	23.21 /37.66	25.81 /42.08	15,592 /23,890	41.19 /75.62	44.91 /81.16	18,832 /29,160	57.10 /109.31	61.74 /116.37
no-f-t	7,761 /12,801	16.00 /28.52	18.03 /32.17	12,672 /20,243	30.25 /57.77	33.28 /62.46	15,962 /26,249	44.29 /91.26	48.22 /97.67
no-t-f	5,243 /7,676	13.93 /17.14	15.11 /18.78	8,169 /12,001	18.39 /25.80	20.21 /28.37	10,546 /15,806	23.71 /36.52	26.05 /40.12
no-t-t	4,221 /6,309	10.09 /13.59	11.05 /14.94	6,643 /9,985	13.21 /19.97	14.69 /22.17	8,560 /12,982	16.79 /27.02	18.70 /29.83
sim-f-t	8,208 /13,478	17.63 /31.32	19.75 /34.87	13,048 /20,789	31.80 /60.39	34.09 /65.22	16,271 /26,677	45.82 /94.03	49.83 /100.40
sim-t-f	5,602 /8,186	15.32 /19.06	16.56 /20.82	8,487 /12,429	19.50 /27.30	21.39 /29.97	10,835 /16,151	24.78 /38.17	27.19 /41.69
sim-t-t	4,508 /6,750	11.15 /15.03	12.20 /16.48	6,899 /10,351	13.98 /21.17	15.52 /23.45	8,787 /13,285	17.50 /28.15	19.47 /31.03
min-f-t	8,900 /14,397	17.64 /32.35	19.88 /36.21	13,606 /21,484	32.01 /62.15	35.27 /67.06	16,380 /25,987	44.13 /89.25	48.17 /95.60
min-t-f	6,198 /8,870	14.32 /18.55	15.69 /20.46	9,131 /13,138	19.12 /27.68	21.15 /30.50	11,259 /16,361	24.18 /37.30	26.69 /40.83
min-t-t	4,981 /7,342	10.18 /14.33	11.32 /15.92	7,392 /11,029	13.47 /21.13	15.13 /23.65	9,071 /13,516	16.75 /27.36	18.79 /30.29
2-500-		l-		m-			h-		
no-f-f	14,638 /22,950	55.54 /86.35	66.92 /142.59	24,189 /39,654	105.28 /209.09	119.01 /225.81	32,691 /52,619	175.33 /352.74	190.86 /374.59
sim-f-f	15,487 /24,100	61.39 /95.05	74.97 /158.00	24,924 /40,589	111.08 /218.79	125.04 /235.11	33,305 /53,408	180.96 /362.53	196.38 /384.25
min-f-f	16,883 /25,881	61.77 /99.69	73.43 /173.06	26,080 /41,296	112.30 /218.98	127.73 /235.45	33,193 /52,362	172.39 /342.00	188.72 /362.95
no-f-t	12,607 /19,553	41.71 /67.86	49.25 /87.03	21,114 /34,688	81.36 /164.25	91.35 /177.47	28,665 /46,887	137.42 /283.83	149.93 /302.91

no-t-f	8,582 /12,589	38.48 /45.94	41.82 /50.37	13,668 /20,621	51.76 /73.28	56.96 /80.63	18,353 /28,856	68.26 /111.83	75.12 /122.18
no-t-t	6,838 /9,952	27.71 /34.91	30.47 /38.59	11,039 /16,476	36.79 /52.89	41.03 /58.87	14,929 /22,969	48.00 /76.99	53.60 /85.43
sim-f-t	13,342 /20,589	46.11 /74.87	54.35 /100.11	21,749 /35,549	85.60 /171.99	96.06 /185.65	29,197 /47,638	141.63 /292.02	154.57 /311.09
sim-t-f	9,165 /13,401	42.68 /51.07	46.24 /55.77	14,211 /21,331	55.08 /77.86	60.54 /85.47	18,831 /29,473	71.19 /116.45	78.23 /126.97
sim-t-t	7,303 /10,639	30.69 /38.72	33.61 /42.64	11,472 /17,090	39.01 /56.00	43.44 /62.10	15,309 /23,481	49.93 /80.19	55.69 /88.71
min-f-t	14,538 /22,233	46.18 /78.35	54.94 /99.19	22,718 /36,394	86.18 /173.04	96.99 /186.89	29,059 /46,935	134.13 /277.64	146.75 /296.42
min-t-f	10,197 /14,688	39.73 /50.01	43.60 /55.17	15,327 /22,451	53.24 /78.17	58.95 /86.16	19,391 /29,443	68.92 /111.93	76.12 /122.41
min-t-t	8,111 /11,701	27.76 /37.01	30.84 /41.32	12,335 /18,164	36.93 /55.91	41.56 /62.37	15,673 /23,579	47.65 /77.14	53.57 /85.79
3-100-		l-				m-		h-	
no-f-f	3,876 /4,989	3.89 /5.02	4.25 /5.42	6,242 /8,589	7.15 /11.00	7.72 /11.76	7,720 /10,434	10.16 /15.51	10.87 /16.41
sim-f-f	4,110 /5,297	4.28 /5.52	4.66 /5.95	6,425 /8,842	7.49 /11.54	8.08 /12.28	7,876 /10,614	10.51 /16.02	11.23 /16.92
min-f-f	4,506 /5,770	4.36 /5.81	4.79 /6.48	6,813 /9,190	7.75 /12.01	8.37 /12.84	8,090 /10,626	10.50 /15.58	11.23 /16.48
no-f-t	3,097 /4,038	2.65 /3.52	2.97 /3.95	5,052 /6,963	4.84 /7.46	5.28 /8.02	6,185 /8,361	6.94 /10.52	7.47 /11.25
no-t-f	2,407 /3,004	2.57 /2.95	2.77 /3.18	3,714 /5,007	3.53 /4.71	3.83 /5.09	4,884 /6,587	4.75 /6.79	5.13 /7.29
no-t-t	1,750 /2,204	1.66 /1.98	1.83 /2.24	2,719 /3,499	2.14 /2.65	2.36 /2.91	3,545 /4,711	2.88 /4.06	3.19 /4.44
sim-f-t	3,283 /4,253	2.95 /3.93	3.31 /4.35	5,200 /7,164	5.08 /7.83	5.52 /8.41	6,312 /8,513	7.16 /10.83	7.70 /11.57
sim-t-f	2,579 /3,210	2.84 /3.25	3.06 /3.50	3,855 /5,181	3.72 /4.95	4.02 /5.34	5,007 /6,753	4.93 /7.08	5.33 /7.60
sim-t-t	1,872 /2,356	1.83 /2.19	2.01 /2.41	2,821 /3,597	2.26 /2.80	2.49 /3.08	3,635 /4,848	2.92 /4.12	3.22 /4.50
min-f-t	3,596 /4,637	2.96 /4.04	3.34 /4.46	5,509 /7,466	5.22 /8.09	5.69 /8.69	6,488 /8,552	7.15 /10.65	7.71 /11.39
min-t-f	2,877 /3,523	2.71 /3.17	2.95 /3.45	4,201 /5,489	3.73 /5.01	4.07 /5.43	5,254 /6,957	4.92 /7.13	5.35 /7.66
min-t-t	2,079 /2,610	1.70 /2.11	1.90 /2.32	3,059 /3,816	2.21 /2.79	2.46 /3.09	3,790 /5,014	2.92 /4.19	3.26 /4.60
3-300-		l-				m-		h-	
no-f-f	12,979 /16,181	35.75 /44.97	39.23 /50.45	20,920 /27,250	70.06 /101.27	75.37 /107.80	26,378 /34,592	104.63 /155.11	111.21 /163.67
sim-f-f	13,712 /17,090	39.43 /49.82	43.08 /55.82	21,531 /28,007	73.78 /106.57	79.15 /113.29	26,907 /35,232	108.41 /161.07	115.16 /169.70
min-f-f	14,883 /18,442	40.43 /51.81	44.27 /56.63	22,510 /29,069	75.53 /109.90	81.23 /117.28	27,124 /34,895	106.01 /155.49	112.80 /164.12
no-f-t	10,428 /13,246	23.66 /30.54	26.33 /33.60	17,120 /22,345	47.73 /69.37	51.85 /74.52	21,543 /27,993	71.29 /103.88	76.57 /110.62
no-t-f	7,669 /9,238	22.16 /24.89	23.89 /26.86	12,029 /15,854	31.98 /41.97	34.72 /45.51	15,390 /19,528	41.15 /54.68	44.53 /58.92
no-t-t	5,511 /6,565	13.58 /15.72	14.89 /17.36	8,803 /11,561	19.00 /25.15	21.05 /27.66	11,210 /13,964	24.02 /31.35	26.57 /34.41
sim-f-t	11,029 /13,974	26.16 /33.85	28.99 /37.24	17,624 /22,979	50.23 /72.94	54.44 /78.24	21,971 /28,523	73.72 /107.42	79.06 /114.29
sim-t-f	8,180 /9,875	24.57 /27.71	26.39 /29.82	12,490 /16,462	33.93 /44.56	36.78 /48.22	15,807 /20,018	43.06 /57.01	46.55 /61.34
sim-t-t	5,881 /6,975	15.03 /17.44	16.41 /19.15	9,141 /11,989	20.16 /26.72	22.29 /29.32	11,509 /14,315	25.10 /32.78	27.72 /36.07
min-f-t	11,989 /15,182	26.74 /35.30	29.86 /39.34	18,413 /23,680	51.34 /74.35	55.72 /79.81	22,162 /28,176	72.04 /105.17	77.43 /111.87
min-t-f	9,058 /11,012	23.59 /27.55	25.60 /29.92	13,429 /17,511	33.27 /45.75	36.23 /49.53	16,417 /20,669	42.73 /57.36	46.35 /61.83
min-t-t	6,517 /7,772	14.02 /16.53	15.54 /18.43	9,789 /12,751	19.70 /26.85	21.96 /29.77	11,904 /14,705	24.38 /32.00	27.11 /35.44
3-500-		l-				m-		h-	
no-f-f	21,240 /26,727	95.09 /120.27	113.80 /165.35	34,911 /46,324	190.30 /284.86	220.30 /305.99	46,975 /61,239	323.80 /481.32	350.03 /508.01
sim-f-f	22,473 /28,248	105.23 /133.71	124.15 /162.07	35,973 /47,576	200.67 /298.20	229.78 /319.85	47,861 /62,283	334.89 /497.03	363.75 /524.23
min-f-f	24,464 /30,703	108.13 /141.65	128.44 /181.04	37,621 /49,123	205.87 /306.72	236.89 /337.27	47,746 /62,138	323.96 /490.56	350.39 /518.89
no-f-t	16,996 /21,202	62.30 /80.65	73.81 /108.53	28,409 /38,086	128.07 /195.11	142.46 /210.97	38,700 /50,391	223.20 /329.46	239.49 /349.46
no-t-f	12,551 /15,404	59.80 /66.91	64.34 /72.32	19,930 /26,014	85.00 /110.55	92.21 /119.75	26,605 /35,142	116.77 /167.11	126.34 /179.69
no-t-t	8,927 /10,782	36.72 /42.73	40.10 /46.74	14,383 /18,040	50.04 /62.17	55.35 /68.71	19,431 /24,787	67.95 /92.70	75.05 /101.73
sim-f-t	18,007 /22,479	69.12 /89.90	81.12 /116.27	29,278 /39,146	135.06 /204.99	149.35 /221.41	39,426 /51,262	230.60 /339.15	247.06 /359.32
sim-t-f	13,397 /16,478	66.41 /74.86	71.22 /80.63	20,708 /26,947	99.57 /117.76	98.05 /127.30	27,300 /35,948	121.80 /174.02	131.61 /186.86
sim-t-t	9,538 /11,509	40.77 /47.48	44.36 /51.76	14,948 /18,715	53.24 /66.26	58.74 /73.12	19,935 /25,368	70.83 /96.31	78.07 /105.57
min-f-t	19,626 /24,505	70.71 /93.90	83.64 /121.98	30,610 /40,648	138.24 /211.42	154.18 /228.50	39,326 /50,972	221.75 /330.47	238.44 /351.85
min-t-f	14,883 /18,193	63.70 /73.82	69.05 /80.24	22,306 /28,871	89.89 /121.40	97.88 /131.58	28,113 /36,699	119.90 /173.04	129.99 /186.22
min-t-t	10,604 /12,782	38.02 /45.40	42.00 /49.98	16,073 /20,040	51.58 /66.39	57.45 /73.69	20,436 /25,856	68.36 /93.79	75.84 /103.14
4-100-		l-				m-		h-	
no-f-f	5,179 /5,469	5.75 /6.13	6.26 /6.57	8,369 /9,056	11.12 /12.38	11.86 /13.15	10,324 /10,957	16.13 /17.41	17.10 /18.38
sim-f-f	5,496 /5,784	6.35 /6.81	6.89 /7.27	8,616 /9,311	11.68 /13.08	12.44 /13.91	10,537 /11,163	16.71 /17.98	17.70 /19.08
min-f-f	6,025 /6,380	6.59 /7.09	7.16 /7.58	9,130 /9,780	12.25 /13.63	13.04 /14.50	10,829 /11,355	16.91 /17.97	17.91 /19.08
no-f-t	3,911 /4,189	3.38 /3.70	3.77 /4.18	6,424 /7,058	6.65 /7.66	7.17 /8.23	7,828 /8,490	9.42 /10.89	10.08 /11.62
no-t-f	3,209 /3,510	3.63 /3.85	3.89 /4.18	4,967 /5,701	5.25 /6.17	5.65 /6.61	6,492 /7,355	7.21 /8.50	7.72 /9.08
no-t-t	2,148 /2,340	1.95 /2.11	2.17 /2.45	3,338 /3,729	2.71 /2.94	2.99 /3.28	4,319 /4,947	3.57 /4.20	3.96 /4.81
sim-f-t	4,153 /4,429	3.74 /4.06	4.16 /4.55	6,613 /7,270	6.98 /8.08	7.52 /8.67	7,991 /8,650	9.71 /10.83	10.38 /11.56

sim-t-f	3,437 /3,753	4.02 /4.27	4.30 /4.65	5,154 /5,875	5.55 /6.47	5.97 /6.93	6,658 /7,575	7.50 /8.93	8.02 /9.52
sim-t-t	2,296 /2,511	2.17 /2.32	2.41 /2.68	3,459 /3,842	2.86 /3.10	3.16 /3.46	4,428 /5,101	3.72 /4.37	4.11 /4.77
min-f-t	4,550 /4,849	3.87 /4.22	4.32 /4.78	7,001 /7,619	7.29 /8.37	7.87 /8.99	8,219 /8,719	9.84 /10.89	10.52 /11.62
min-t-f	3,836 /4,219	3.93 /4.26	4.24 /4.58	5,619 /6,395	5.68 /6.78	6.14 /7.27	6,995 /7,780	7.63 /8.95	8.19 /9.56
min-t-t	2,548 /2,808	2.05 /2.22	2.34 /2.68	3,741 /4,100	2.85 /3.13	3.17 /3.50	4,610 /5,224	3.67 /4.42	4.10 /5.03
4-300-		l-			m-			h-	
no-f-f	17,350 /18,390	53.96 /57.59	58.64 /62.68	28,089 /29,125	111.01 /118.05	118.41 /125.59	35,392 /37,245	168.82 /178.88	177.94 /189.53
sim-f-f	18,333 /19,435	59.68 /63.74	64.66 /69.56	28,912 /29,992	117.15 /124.72	124.69 /132.12	36,099 /37,985	174.81 /184.96	184.23 /196.31
min-f-f	19,907 /21,235	62.36 /67.50	67.65 /73.39	30,244 /31,274	121.64 /127.56	129.47 /135.72	36,419 /37,834	172.83 /180.54	182.11 /190.22
no-f-t	13,204 /14,165	31.60 /35.44	35.02 /38.91	21,928 /23,326	67.35 /75.50	72.45 /80.93	27,610 /28,861	102.29 /110.68	108.97 /117.55
no-t-f	10,194 /10,820	31.55 /33.27	33.77 /35.58	15,906 /17,246	46.58 /51.31	50.06 /55.18	20,381 /22,329	63.02 /71.38	67.49 /76.36
no-t-t	6,714 /7,087	16.69 /17.20	18.31 /18.98	10,714 /11,823	23.79 /26.26	26.31 /29.03	13,672 /14,646	31.07 /33.25	34.22 /36.69
sim-f-t	13,979 /14,981	35.12 /39.47	38.68 /43.25	22,572 /24,003	71.07 /79.57	76.31 /85.09	28,157 /29,403	106.12 /114.82	112.91 /121.76
sim-t-f	10,871 /11,509	34.98 /36.68	37.35 /39.14	16,522 /17,931	49.52 /54.75	53.15 /58.77	20,936 /22,983	66.04 /75.31	70.64 /80.43
sim-t-t	7,164 /7,587	18.61 /19.22	20.35 /21.03	11,120 /12,262	25.34 /27.81	27.94 /30.93	14,035 /15,041	32.52 /34.83	35.75 /38.31
min-f-t	15,212 /16,443	36.77 /41.10	40.59 /45.41	23,594 /24,949	73.83 /82.05	79.32 /87.81	28,412 /29,646	104.76 /113.35	111.58 /120.55
min-t-f	12,033 /12,714	34.24 /36.18	36.88 /39.10	17,805 /19,175	50.62 /56.11	54.53 /60.40	21,760 /23,904	66.46 /76.23	71.23 /81.51
min-t-t	7,932 /8,413	17.68 /18.54	19.56 /20.52	11,908 /13,040	25.20 /27.89	27.97 /30.96	14,496 /15,563	32.02 /34.56	35.34 /38.09
4-500-		l-			m-			h-	
no-f-f	28,401 /29,511	143.99 /149.60	171.58 /217.04	46,780 /49,021	301.21 /326.53	344.13 /379.02	63,189 /65,145	526.05 /546.83	558.23 /585.00
sim-f-f	30,044 /31,146	159.60 /165.21	189.57 /337.24	48,196 /50,375	318.23 /344.05	359.64 /405.11	64,376 /66,320	544.58 /567.17	575.78 /598.72
min-f-f	32,725 /33,786	167.20 /173.19	196.30 /264.00	50,426 /52,318	331.12 /352.85	376.54 /449.04	64,208 /65,942	537.15 /555.35	570.20 /589.31
no-f-t	21,448 /22,298	83.01 /87.18	97.84 /120.28	36,225 /38,731	179.69 /200.58	194.95 /216.62	49,725 /51,538	323.10 /337.67	343.60 /358.27
no-t-f	16,731 /18,003	85.35 /90.27	91.33 /96.76	26,559 /28,409	127.09 /136.76	136.56 /146.94	35,588 /38,922	182.42 /208.67	195.27 /222.75
no-t-t	10,860 /11,370	44.89 /47.15	49.01 /51.17	17,597 /18,368	63.50 /66.01	70.02 /72.96	23,962 /25,998	89.75 /399.11	98.52 /108.87
sim-f-t	22,729 /23,612	92.58 /97.28	108.88 /132.91	37,328 /39,824	190.29 /212.26	205.87 /228.88	50,658 /52,474	334.61 /351.12	355.36 /372.40
sim-t-f	17,861 /19,198	94.94 /100.22	101.29 /106.98	27,602 /29,467	135.63 /145.68	145.45 /156.24	36,515 /39,890	190.78 /218.33	203.96 /232.78
sim-t-t	11,600 /12,149	50.00 /51.46	54.33 /55.77	18,283 /19,061	67.72 /70.41	74.47 /77.75	24,577 /26,612	93.86 /103.72	102.87 /113.88
min-f-t	24,823 /25,790	97.02 /102.45	113.48 /137.43	39,049 /41,442	197.85 /218.47	214.18 /235.48	50,508 /51,958	327.20 /342.72	347.64 /363.13
min-t-f	19,855 /21,257	92.92 /99.06	100.00 /106.56	29,754 /31,783	138.88 /150.85	149.50 /162.20	37,611 /40,605	190.22 /214.83	203.78 /229.42
min-t-t	12,903 /13,475	47.40 /48.65	52.18 /53.50	19,641 /20,494	67.48 /71.01	74.71 /78.85	25,156 /26,924	91.69 /100.12	100.93 /110.44

Table 3.2: First computational study for TCSP

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the sometimes quite huge differences between maximum and average values already indicate. E. g., for the 3-100-h-no-f-f scenario the maximum number of candidates (10,434) exceeds the minimum number for the 4-100-h-no-f-f scenario (9,699).

When setting $\mu = sim$, the number of edges is higher than for $\mu = no$ and, thus, we expect the time for candidate generation (and checking) to be higher, as well. Additionally, as any $f \in F$ may overlap with any $d \in D$, $s \in S$ or $f' \neq f \in F$ the time for feasibility checking should increase (because now a candidate lying on an edge may well be within a polygon). With regard to the number of feasible candidates, there are two opposing effects: on the one hand, increasing the number of edges should increase the number of candidates. But on the other hand, there are more infeasible areas on-site now. Comparing the respective entries in Table 3.2 for scenarios of type $\kappa\text{-}\eta\text{-}\lambda\text{-no-}\sigma\text{-}\phi$ and $\kappa\text{-}\eta\text{-}\lambda\text{-sim-}\sigma\text{-}\phi$ shows us that – at least for our test set – both the number of feasible candidates and the candidate generation time increase.

The effect of introducing minimum distances around supply and demand sites ($\mu = min$) is expected to be small in comparison to instances with $\mu = sim$ since i) minimum distances can be regarded as a kind of forbidden area, so the arguments from the preceding paragraph hold and ii) for candidate generation, there are neither new vertices nor does the number of edges increase, since edges of supply and demand areas are not relevant anymore and are substituted by the edges of the minimum distance areas. The data in Table 3.2 supports this expectation with regard to the number of feasible candidates as the values for $\kappa\text{-}\eta\text{-}\lambda\text{-sim-}\sigma\text{-}\phi$ and $\kappa\text{-}\eta\text{-}\lambda\text{-min-}\sigma\text{-}\phi$ only differ by relatively small amounts. An interesting observation is that introducing minimum distances increases both average and maximum number of candidates in all scenarios with low and medium density whereas for high-density scenarios either the maximum number of candidates (1-300-h-sim-f-f vs. 1-300-h-min-f-f) or even both, maximum and average number of candidates, (1-500-h-sim-f-f vs. 1-500-h-min-f-f) may decrease. This is due to the fact that minimum distances enlarging the areas where no crane can be positioned are related to supply sites and demand sites. On the one hand, this increases the probability for the existence of candidates generated by intersecting circles and edges. On the other hand, however, it increases the probability of an arbitrary candidate to be infeasible, as well. Note that the latter effect grows stronger with higher density. Regarding candidate generation times, there is no clear effect: For scenarios with low or medium density, introducing minimum distances sometimes increases average and maximum generation times, but sometimes decreases them. An increase might support the explanation that more intersection points are generated and checked. For high-density scenarios, however, average and maximum times usually drop by a sometimes considerable amount (2-500-h-sim-f-f vs. 2-500-h-min-f-f). Generally, such a decrease might be due to the procedure of candidate feasibility checking. We stated in Chapter 3.2.2 that a candidate is sorted out when it is infeasible or does not cover the pair(s) generating it. As soon as a candidate is infeasible, the checking routine can be terminated. Introducing minimum distances increases the chance of infeasibility and, thus, may reduce the time spent on feasibility checking. In high-density scenarios there may be more candidates that do not pass feasibility checking compared to low- or medium-density scenarios – or infeasibilities occur and are detected sooner.

The impact of introducing weights to the basic setting, i. e. dropping $w_d = w$ for all $d \in D$, is hard

to predict as the effect of changing the circles' radii is unclear. Comparing the respective entries for $\kappa\text{-}\eta\text{-}\lambda\text{-}\mu\text{-f-f}$ and $\kappa\text{-}\eta\text{-}\lambda\text{-}\mu\text{-t-f}$ in Table 3.2 shows that both the number of feasible candidates and the candidate generation time drop significantly after imposing weights.

Finally, introducing heights ($\phi = t$), should reduce the number of nodes to consider for candidate generation (cf. Chapter 3.2.1) and, thus, both the number of feasible candidates and the time for generating candidates. Based on the entries for $\kappa\text{-}\eta\text{-}\lambda\text{-}\mu\text{-f-f}$ and $\kappa\text{-}\eta\text{-}\lambda\text{-}\mu\text{-f-t}$ in Table 3.2, we can confirm that both the average and maximum number of feasible candidates decrease when imposing heights. The same holds true for the maximum and average generation times in the single scenarios.

3.3.2.2 Set Cover Solution Analysis

Once the candidates are generated, the SCP instance can be solved. We focus on CPLEX in the following. We will summarize our findings employing the greedy approach only briefly since run times are very short and, thus, generating the candidates is the most challenging part. In the first part of our study, optimal solutions for each scenario are obtained within computational times reaching from about one second (1-100-l-no-t-f) to about 10 minutes (4-500-h-sim-f-f). An interesting finding for practitioners might be that the most realistic setting with forbidden areas, minimum distances, weights and heights has been solved optimally within less than two minutes (4-500-h-min-t-t).

The average time it takes CPLEX to solve a scenario's instance to optimality can be derived as the difference between total time and generation time in Table 3.2. It is not surprising that small set cover instances are solved by far faster than larger instances. From an application-oriented perspective, this gives us the number of cranes and pairs as drivers of solving time. The influence of the density on the solution times is less clear. The solution time tends to increase with higher density, but we can observe exception from that. For example, the three longest solution times (41, 43 and 45 seconds, respectively) do not occur for high-density scenarios, but for medium-density scenarios (4-500-m-sim-f-f, 4-500-m-no-f-f and 4-500-m-min-f-f, respectively), although these medium-density scenarios have a significantly smaller number of candidates than the high-density scenarios with the next-highest solution times (4-500-h-sim-f-f, 4-500-h-no-f-f and 4-500-h-min-f-f, respectively).

Since all instances in the first part of the study could be solved to optimality, we will run a second study to push CPLEX to the limits. In order to do so, we first evaluate which scenario takes on average the longest time to compute an optimal solution. We identify scenario type 4-500-h-sim-f-f. Now, there are several options to increase the computational effort, e.g. increasing the density or the number of pairs. Since we identified vertices and edges – imposed by both pairs and obstacles – as the most reliable drivers of the computational effort, previously, we will increase the number of pairs. Remember that this increases the number of forbidden areas as well since we use a fixed ratio of the site's size – depending on the number of pairs – for determining the number of forbidden areas. We can, then, start a second computational run with the number of pairs set to 700, 900, and 1,100, respectively. Again, for each scenario, 25 instances are generated. Additionally, the total computational time – i.e. for candidate generation and SCP solution – is limited to one hour. The results are reported in Table 3.3. It can be seen that for up to 700 pairs plus forbidden areas all instances could be solved optimally.

η	# forb. areas	# candidates	gen. time (sec)	total time (sec)	MIP gap (%)
		avg. /max.	avg. /max.	avg. /max.	avg. /max.
100	20	10,537 /11,163	16.71 /17.98	17.70 /19.08	0.00 /0.00
300	64	36,099 /37,985	174.81 /184.96	184.23 /196.31	0.00 /0.00
500	100	64,376 /66,320	544.58 /567.17	575.78 /598.72	0.00 /0.00
700	144	90,876 /93,417	1,072.15 /1,123.51	1,776.20 /3,015.41	0.00 /0.00
900	182	116,413 /118,745	1,760.92 /1,823.37	3,600.00 /3,600.00	3.76 /5.53
1,100	225	140,783 /142,904	2,565.91 /2,629.15	3,600.00 /3,600.00	5.83 /7.26

Table 3.3: Second computational study for TCSPP: analysis of scenario 4- η -h-sim-f-f

For 900 and 1,100 pairs not a single instance could be solved to optimality within one hour, but the optimality gap reported by CPLEX is quite small.

Usually, run time is not a critical factor when solving the TCSPP. Just in case it is, it might be suitable to employ heuristics. We implemented the standard greedy approach for SCP proposed by Chvatal [21] and tested it during the first part of the computational study. Clearly, the greedy approach is by far faster than CPLEX (about 1.5 seconds in the worst case for any instance) and finds good or even optimal solutions for some instances. But the average percentage deviation from the exact solutions in any scenario from study one was worse than the average lower bound gap CPLEX achieved in study two. Thus, terminating an exact procedure might be the better choice with regard to solution quality for practical application.”

3.4 More General Problem Variants

“The problem described and defined in Chapter 3.1.1 captures several essential requirements for tower crane selection and positioning. However, there are countless further aspects that could be taken into account. After thoroughly analyzing the TCSPP in Chapter 3.2 and evaluating the potential of the reduction to the weighted SCP in Chapter 3.3 we, therefore, outline how to incorporate some of these aspects in the problem setting and how to adapt the reduction mechanism if necessary.

3.4.1 Multiple Supply Sites per Demand Site

It is easy to imagine a construction site where one or more demand sites are provided with material by multiple supply sites. We, then, can consider two types of requirements with respect to pairs of demand sites and their supply sites: either each pair of a demand site can be covered by an individual crane or all pairs of one demand site have to be covered by a single crane.

In both cases, Theorem 3 still applies. In the first case, we can simply apply the very same procedure as detailed in Chapter 3.2.2 by introducing a copy of demand site d for each pair it is involved in. Each copy then gets a unique supply site. In the second case, we have to slightly adapt our procedure. Reduction rules c) and d) do not account for demand sites with multiple supply sites. However, they

are easy to adapt by requiring a candidate covering a demand site covers each of its supply sites, too. Note that covering groups of demand areas that have to be supplied by one or more supply areas with a single crane can be incorporated in a similar way, as well.

3.4.2 Capacitated Tower Cranes

According to the problem as defined in Chapter 3.1.1, we assume that an arbitrary number of pairs can be served by a single crane if it is only located feasibly with respect to geometry. However, when the load of a crane's capacity imposed by a pair is considerable this might not be true. We can enrich the problem setting by introducing capacity C_t for each crane type $t \in T$ and load of capacity imposed by a pair (d, s_d) as a_d for each demand site $d \in D$.

A solution, then, is not only the number of cranes of a certain type and their respective positions but also an assignment of pairs to cranes. A solution is feasible if it is feasible with respect to geometry (as before) and to capacity constraints, that is for each crane the total workload of pairs assigned does not exceed the crane's capacity. Again, Theorem 3 applies even for this generalization of the problem setting. However, we have to generalize the concept of candidates and adapt the procedure for generating them. A capacitated candidate implies as usual a crane type t and a position but also a set of pairs being served with total load of capacity not exceeding C_t . Note that in the original problem setting the set of pairs being served coincides with the set of pairs being covered since we assume $C_t = \infty$. It is not hard to see that for each candidate position for a crane type t we can simply generate a capacitated candidate for type t in the respective position and each subset of pairs being covered being maximal with respect to total load of capacity.

3.4.3 Time Dynamic Construction Sites

The problem defined in Chapter 3.1.1 does not have any temporal dimension. In particular for large-scale construction sites demand sites, supply sites, and forbidden areas may be relevant only for a limited time interval within the planning horizon. Consequently, we may associate each demand site $d \in D$ and each forbidden area $f \in F$ with a start time and an end time. The respective time interval is the time interval where this demand site has to be supplied and where this forbidden area cannot be used for placing a crane. There is such an interval for each supply site, as well. It is given as the union of intervals supplied by this supply site. We consider rental cost π'_t per time unit for a crane of type t . Notably, we do not consider a fixed charge for setting up a crane.

A solution, then, is not only the number of cranes of a certain type t and their respective position p but also a time window for a crane of type t to be located in p . A solution is feasible if it is feasible with respect to geometry (defined as before) at each point of time, that is each pair (d, s_d) is covered at each point of time between d 's start time and end time. It is not hard to see that (due to the particular cost structure) the problem decomposes with respect to time. For each maximum time window with no start time or end time of any polygon, we can solve an instance of TCSP."

4 Tower Crane Selection and Location with Mutual Interference

In Chapter 3, tower cranes of given types, i. e. with given specifications, had to be selected and located on a polygonal construction site in order to cover pairs of polygonal supply and demand areas at minimum cost. Cranes could be located arbitrarily on the site (except from certain infeasible areas) and a pair was considered to be covered by a crane if all points of the polygons constituting the pair were within a sufficiently high crane's operating radius for the maximum weight to be lifted at the respective pair. This problem captures basic characteristics of tower crane selection and location, but leaves out interrelations between cranes and between cranes and fixed on-site structures. Such interrelations will be under research in the chapter at hand. However, the incorporation of these considerations comes at the cost of dropping the continuous location model and, instead, relying on an artificial discretization of space by a grid. This step is necessary since the concepts presented in Chapter 3 were based on the assumption that cranes could be placed independently which is, obviously, no longer the case when crane interdependencies come into play.

The tower crane selection and positioning problem in a grid, TCSP-GRID for short, researched in the current chapter will be concisely defined, its computational complexity will be settled and the contribution to the academic literature will be stated in Chapter 4.1. Afterwards, two different solution approaches will be presented (Chapter 4.2). The first one employs a standard solver which is used based on four different mixed-integer programming formulations (Chapter 4.2.3), the second one is a branch and bound procedure (Chapter 4.2.4). Both approaches require a data pre-processing step transforming the mainly geometric input of an instance into data suited for applying standard OR techniques. This pre-processing will, thus, be described before the approaches themselves are presented in Chapter 4.2.1 and the resulting notation to be used when presenting the solution approaches will be summarized in Chapter 4.2.2. Both approaches are, afterwards, tested regarding their computational performance in Chapter 4.3. The evaluation starts with a comparison of the standard solver's performances using the four MIP formulations (Chapter 4.3.2). In doing so, drivers of computational effort are identified. Once this evaluation has been finished, the B & B approach is tested against the best-performing MIP formulation (Chapter 4.3.3).

4.1 Problem Definition, Computational Complexity and Contribution

In Chapter 4.1.1, a concise problem definition is given. The construction site with the structures contained therein and the different tower crane types are basically identical to the setting described in

Chapter 3.1.1 with one enhancement: forbidden areas have heights now in order to represent existing on-site structures such as fully constructed buildings. As mentioned above, the former setting is enriched by peculiar interrelations of cranes among each other and cranes with on-site structures, but the continuous perspective on space from Chapter 3 is, therefore, altered to a discrete one which is achieved by employing a grid. In order to keep the chapter at hand self-contained, the problem will be fully introduced in Chapter 4.1.1 and, afterwards, a proof regarding the problem's computational complexity is presented (Chapter 4.1.2). Finally, a brief overview on the scientific contribution will be given in Chapter 4.1.3.

4.1.1 Problem Definition: TCSP-GRID

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

Like in Chapter 3, “the construction site is given by a (not necessarily convex) simple polygon. We address each point on the construction site by its Cartesian coordinates within a plane the construction site is embedded in. Both the supply and demand areas are represented by simple polygons, as well. Additionally, there are areas where a crane may not be located (e.g. due to ground conditions or prescribed minimum distances between cranes and existing structures such as demand and supply areas) that are also represented by simple polygons. We address the set of demand polygons as D and the set of supply polygons as S . Each demand site d has a given height $h_d \in \mathbb{N}$ and a given maximum load weight $w_d \in \mathbb{N}$. As we consider a set T of different crane types with different specifications, there is a crane type-dependent set of forbidden polygons F_t . Each forbidden area $f \in F$ has a given height $h_f \in \mathbb{N}^0$, as well. Each $s \in S$, $d \in D$ and $f \in \bigcup_{t \in T} F_t$ is contained completely in the construction site polygon.

With each crane type $t \in T$ we associate a given fixed cost π_t (representing, e.g., rental cost over a fixed period, set-up cost, etc.), a maximum operating radius r_t^{max} , a maximum operating radius $r_{t,w}$ for a given weight w and a maximum operating height h_t . We assume that an infinite number of cranes is available for each crane type. Cranes have to be located on-site (including the construction site polygon's edges), but may not be located within any polygon in D , S and F (including the polygons' edges) and have to keep at least a given type-dependent minimum distance $D_{t,t'}^{min}$ between their centers (i.e. the cranes' locations).” Like in Chapter 3, “each demand site $d \in D$ is supplied by exactly one supply site $s_d \in S$ and we refer to such an assignment of a supply area and a demand area as pair (d, s_d) . A supply site may supply multiple demand sites. Note that we can easily incorporate demand sites that receive material from multiple supply sites without altering the structure of what follows. For ease of notation, however, we restrict ourselves to the case with exactly one supply site per demand site. Each pair has to be covered completely by at least one single crane, i.e. one crane has to be able to move from each point in the supply area to each point in the demand area of the respective pair. This implies that there are no material handovers between cranes. In fact, in practice handovers are prevented whenever possible since they cause additional handling effort and give rise to additional interdependencies between cranes. In the following, we will detail under which conditions a crane can cover a pair.

A pair (d, s_d) with h_d and w_d is said to be in reach of a crane of type t located at location i if

- all points of the corresponding polygons are within the crane’s operating radius for weight w_d , r_{t,w_d} , i. e. they have Euclidean distance less than or equal to r_{t,w_d} to the crane’s center at location i and
- the crane has a sufficient operating height, i. e. $h_t \geq h_d$.”

However, as we want to account for interferences between both cranes and cranes and other on-site structures it is not sufficient for a pair to be covered to be in reach of a crane. We will, first, have a look at inter-crane interferences and, afterwards, transfer these considerations to interferences of cranes with other blocking structures. In order to analyze such interferences we rely on the concept of polar coordinate systems. For a given crane, we interpret its “position i as the pole of a polar coordinate system and define a horizontal half-line l starting at this pole as the polar axis. The polar coordinate of any point p in the polar coordinate system of the crane in position i is then well defined as (ρ_p^i, Θ_p^i) with ρ_p^i being the radial coordinate and Θ_p^i being the angular coordinate of point p .”

For two cranes c and c' of type t_c and $t_{c'}$ with operating heights $h_{t_c} \leq h_{t_{c'}}$ located in positions i_c and $i_{c'}$, c' limits the operating range of c if the Euclidean distance between i_c and $i_{c'}$ is not larger than $r_{t_c}^{max}$. That means crane c cannot reach any point p with $(\rho_p^{i_c}, \Theta_p^{i_c} = \Theta_{i_{c'}}^{i_c})$. As c may not move past the blocking crane c' the jib of c may be trapped between several blocking cranes within its operating range, thus, limiting the effective operating range of c to one of the circular sectors formed by blocking cranes.

As mentioned above, a crane’s operating range may not only be limited by other cranes, but by fixed on-site structures, as well. Such an on-site structure may be a demand area $d \in D$ or an infeasible area $f \in F$ of sufficient height to block the jib of crane c , i. e. with $h_d > h_{t_c}$ or $h_f > h_{t_c}$, respectively. If any point p' of such an on-site structure is in Euclidean distance of no more than $r_{t_c}^{max}$ from the position i_c of crane c of type t_c , then c cannot reach any point p with $(\rho_p^{i_c}, \Theta_p^{i_c} = \Theta_{p'}^{i_c})$. Like for blocking cranes, there may be several blocking on-site structures or combinations of blocking cranes and on-site structures that trap the jib of c and limit its effective operating range to one of the circular sectors formed thereby.

“Figure 4.1 shows how a crane can be blocked by either” an on-site structure (Figure 4.1a) “or another crane (Figure 4.1b) of sufficient height or a combination of both (Figure 4.1c).

In Figure 4.1a there is a single demand site with a height that does not allow crane c to move its jib over the demand area. Consequently, those points that do not lie within the demand area, but require crane c to move its jib over it cannot be reached. This results in the gray circular sector that cannot be reached by crane c . In Figure 4.1b there is a single crane c' with a height that prevents crane c from reaching any point in the circle that is located on the dashed line. In Figure 4.1c we have a combination of” on-site structures (one demand site and two forbidden areas) “and other cranes limiting the operating range of crane c . Again, points in gray circular sectors and on dashed lines cannot be reached. We, furthermore, can see that crane c cannot reach all of the remaining points simultaneously. Instead, it can serve points in a single white circular sector only. The circular sector it serves can be chosen by setting up crane c appropriately, but cannot be changed afterwards.

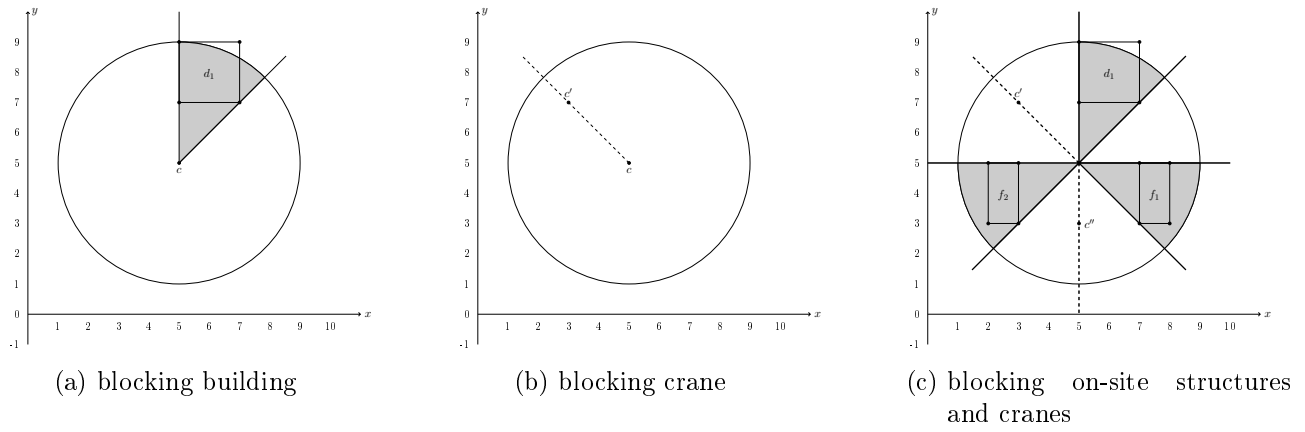


Figure 4.1: Blocking of a crane

A grid is laid over the site and only the grid's intersection points are considered as potential locations for setting up a crane. This is a simplifying assumption as it artificially boils down the infinite number of potential crane locations for a crane of type t to a finite set G_t . Set G_t contains all intersection points that do not lie in any supply site (or on its boundaries), in any demand site (or on its boundaries), or in any forbidden area in F_t (or on the corresponding boundaries). Note that while the grid restricts the set of locations it is up to the decision maker to finetune the granularity of the grid.

A solution is

- a number $k_t \in \mathbb{N}^0$ for each crane type $t \in T$ and a set $G_t^* \subseteq G_t$ of k_t different intersection points for each $t \in T$ so that $G_t^* \cap G_{t'}^* = \emptyset$ for $t \neq t'$ and
- an assignment of each pair (d, s_d) to exactly one intersection point $g(d) \in \bigcup_{t \in T} G_t^*$.

Such a solution implies that for each crane type $t \in T$ we have k_t cranes positioned with their centers at the respective intersection points G_t^* (one crane on each point) and pair (d, s_d) is served by the crane located in $g(d)$. We say that (d, s_d) is assigned to crane c if c is located in $g(d)$.

For a solution to be feasible, the latter point implies each pair (d, s_d) to be accessible with respect to the angular coordinate by crane c located at $g(d)$ the pair is assigned to. This is the case if

- no crane limiting c 's operating range has the same angular coordinate as any point of (d, s_d) ,
- no point of a demand area or a forbidden area limiting c 's operating range has the same angular coordinate as any point of (d, s_d) and
- if there are multiple cranes, demand areas or forbidden areas limiting c 's operating range then each point of (d, s_d) lies in the same circular sector formed by limiting cranes, demand areas or forbidden areas.

“Finally, we say a solution is feasible if

- each pair of cranes located at $g \in G_t^*$ and $g' \in G_{t'}^*$ has Euclidean distance of at least $D_{t,t'}^{min}$,
- each pair is assigned to a located crane c so that the pair is in reach of c and the pair is accessible

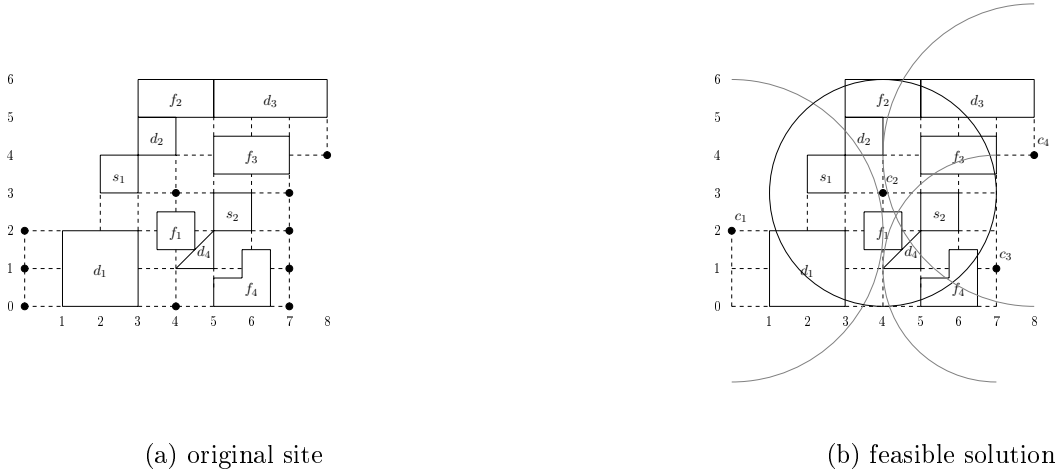


Figure 4.2: Illustrative site and feasible solution

by c with respect to the angular coordinate, and

- all pairs assigned to the same crane lie in the same circular sector.

Figure 4.2 depicts an illustrative construction site on the left and a feasible solution on the right. There are four forbidden areas for placing any crane and four pairs, namely (d_1, s_1) , (d_2, s_1) , (d_3, s_2) , (d_4, s_2) . For covering these pairs, there are two different crane types available, t_1 and t_2 , with $r_{t_1, w} < r_{t_2, w}$ for any weight w and $h_{t_1} < h_{t_2}$. Buildings d_1 and d_3 each have a height of h_{t_2} – i. e. they cannot be covered by cranes of type t_1 , but may block such cranes –, buildings d_2 and d_4 have a height of h_{t_1} . For the sake of simplicity, let $w_{d_1} = w_{d_2} = w_{d_3} = w_{d_4} = 0$ and $h_{f_1} = h_{f_2} = h_{f_3} = h_{f_4} = 0$. “With the given grid granularity, the highlighted intersection points in Figure 4.2a remain as feasible positions for locating a crane. In Figure 4.2b, a feasible solution is pictured that contains two candidates with cranes of the smaller type t_1 (c_2 and c_3) and two candidates with cranes of the larger type t_2 (c_1 and c_4). In the given solution, c_1 covers (d_1, s_1) , c_2 covers (d_2, s_1) , c_3 covers (d_4, s_2) and, finally, c_4 covers (d_3, s_2) . Note that c_2 cannot cover pairs (d_2, s_1) and (d_4, s_2) simultaneously since its jib is blocked by buildings d_1 and d_3 . Also note that there are no cranes blocking each other in the given solution. But, e. g. locating a crane of type t_2 at point $(7, 3)$ would block c_4 and prevent it from serving pair (d_3, s_2) .”

A total cost of $\sum_{t \in T} k_t \pi_t$ is associated with a solution and the discrete tower crane selection and positioning problem in a grid with minimum distances and slewing ranges (TCSPP-GRID) is to find a feasible solution with minimum total cost among all feasible solutions.”

4.1.2 Computational Complexity of TCSPP-GRID

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

Theorem 4. *TCSPP-GRID is strongly NP-hard.*

The theorem is proven by reduction from a variant of the problem to find a minimum cardinality

independent dominating set in a grid graph, namely IND-DOM-GRID, which has been proven to be strongly NP-hard by Clark et al. [22]. In a grid graph each node corresponds to a circle with radius $1/2$ with its center at integer Cartesian coordinates in a plane. Two nodes are connected by an undirected edge if and only if the corresponding circles intersect. “We see an example of such a graph in Figure 4.3a depicted by nodes (in gray) and edges (black lines) embedded in circles (black) placed in a plane that imply the graph.” Our variant, namely IND-DOM-GRID-BIG, is the problem to find a minimum cardinality independent dominating set in a grid graph with more than four nodes. Note that NP-hardness of IND-DOM-GRID-BIG follows trivially from NP-hardness of IND-DOM-GRID since only instances that can be solved in constant time are excluded.

“A dominating set of a graph is a subset of nodes so that each node of the graph is either in the subset or has a neighbour in the subset. An independent set is a subset of nodes so that for each node in the subset no neighbour is in the subset. An independent dominating set is a subset of nodes that is both a dominating set and an independent set. A minimum cardinality independent dominating set is an independent dominating set so that no independent dominating set with fewer nodes exists.

Proof. We consider an instance I of the decision version of IND-DOM-GRID-BIG which asks whether an independent dominating set of a certain cardinality exists. Instance I is specified by n circles and their coordinates. In the following, we construct an instance I' of TCSPP-GRID. We “restrict ourselves to connected graphs since otherwise I decomposes.

Let x_{\min} , x_{\max} , y_{\min} , and y_{\max} be the minimum and maximum first and second, respectively, coordinate of centers among all circles. For simplicity we reduce each first coordinate by x_{\min} and each second coordinate by y_{\min} . Note that in the following we will use x_{\min} , x_{\max} , y_{\min} , and y_{\max} with respect to the modified coordinates. Obviously, this does not change the structure of the graph, but gives us $x_{\min} = y_{\min} = 0$. Since we assume the graph to be connected the number n of nodes is at least $x_{\max} + y_{\max} + 1$.

We start by defining the grid in instance I' of TCSPP-GRID as the points in the integer coordinate set $\{(x, y) \mid 0 \leq x \leq x_{\max}, 0 \leq y \leq y_{\max}\}$. Note that the number of grid points is in $\mathcal{O}(x_{\max} \cdot y_{\max})$ and, thus, polynomial in n . The construction site is then the polygon containing exactly the convex hull of all grid points. Thus, all grid points lie within the construction site. We refer to the grid points with coordinates corresponding to the position of a circle in I as node grid points and to the remaining grid points as dummy grid points in the following. Let q be the number of dummy grid points.

We consider two types of cranes, that is $T = \{t_1, t_2\}$, which we will specify rather informally in the following. Type t_2 cranes can lift heavy weights and have a small height and a maximum operating radius of ϵ , $0 < \epsilon < 0.1$. Cranes of type t_2 can lift the heavy weights at their maximum operating radius. Finally, the cost of type t_2 is $\pi_{t_2} = (x_{\max} + 1) \cdot (y_{\max} + 1)$. Type t_1 cranes can lift light weights only and have a large height and a maximum operating radius of $1 + \epsilon$. The cost of type t_1 is $\pi_{t_1} = 1$. Minimum distances are not an issue, that is we can assume w.l.o.g. $D_{t_1, t_1}^{\min} = D_{t_1, t_2}^{\min} = D_{t_2, t_1}^{\min} = D_{t_2, t_2}^{\min} = 0$. Note that cranes of type t_2 cannot interfere with any other crane due to their maximum operating radii and their heights. Therefore, the only type of interference which has to be taken into account is between

two cranes of type t_1 that are positioned in Euclidean distance of 1 to each other.

It remains to define the pairs, that consist of single points only. We have two types of such pairs. The second type requires a heavy weight to be lifted, and, therefore, can be served by cranes of type t_2 only. The first type requires a light weight to be lifted and, therefore, can be served by both types of cranes. The pairs are specified as follows.

- For each pair of adjacent nodes in the graph, that is for each pair of node grid points (x, y) and (x', y') with Euclidean distance of 1 to each other, we have two pairs of the light type. The first one has Euclidean distance of $1 - \epsilon$ and ϵ to (x, y) and (x', y') , respectively. The second one has Euclidean distance of $1 - \epsilon$ and ϵ to (x', y') and (x, y) , respectively. Note that both pairs lie on the line between both points. Note, furthermore, that there is at least one pair in Euclidean distance of ϵ to each node grid point since we assume the graph to be connected.
- For each dummy grid point (x, y) we have a pair of the heavy type at an arbitrary point within Euclidean distance of ϵ to (x, y) and within the construction site.

Note the scheme described above constructs $q + 2|E|$ pairs where $|E| \in \mathcal{O}(n^2)$ is the number of edges in the grid graph induced by I . Since $q \in \mathcal{O}(n^2)$ the number of pairs constructed is in $\mathcal{O}(n^2)$ and, thus, polynomial in the size of I .

Finally, we have no forbidden areas, that is $F_{t_1} = F_{t_2} = \emptyset$. This completes the construction.

We shall verify that the reduction is indeed in polynomial time. Hence, we summarize the construction as follows.

- The number of grid points is in $\mathcal{O}(n^2)$. The construction site is defined by the four coordinates $(0, 0)$, $(x_{\max}, 0)$, $(0, y_{\max})$, and (x_{\max}, y_{\max}) and, thus, can be constructed in constant time.
- The number of crane types is constant and, thus, in $\mathcal{O}(1)$.
- The number of pairs is in $\mathcal{O}(n^2)$. Since each pair covers a single point only it can be constructed in constant time (we set coordinates of the single point, maximum load weight, and height).
- The number of forbidden areas is zero and, thus, constant.

We conclude that the reduction can be done in polynomial time and claim that there is a feasible solution to I' with cost of $q\pi_{t_2} + p$ if and only if there is an independent dominating set of cardinality p in the grid graph.”

Consequently, we need to show that a feasible solution to I' with cost of $q\pi_{t_2} + p$ implies an independent dominating set of cardinality p in the grid graph and that an independent dominating set of cardinality p implies a feasible solution with cost $q\pi_{t_2} + p$.

We start by showing the former point:

- On “each dummy grid point a crane of type t_2 is placed since there is a pair of the heavy type close by that can be served only by a crane of type t_2 placed on the corresponding dummy grid point. Thus, we need at least q cranes of type t_2 . But we cannot use more than q of them since $\pi_{t_2} = (x_{\max} + 1) \cdot (y_{\max} + 1)$ and, thus, $(q + 1)\pi_{t_2} > q\pi_{t_2} + p$. Hence, all other cranes used in

the solution are of type t_1 and there are p of them. No crane of type t_2 can serve any other pair since they are too distant. Hence, the p cranes of type t_1 serve all pairs of the light type.”

- A “pair positioned in Euclidean distance of ϵ to (x, y) can only be served by a crane of type t_2 positioned at (x, y) or by a crane of type t_1 positioned at (x', y') with $|x - x'| + |y - y'| \leq 1$ since all other node grid points have distance of more than $\sqrt{2} - \epsilon > 1.3$. Since all pairs of the light type are served by cranes of type t_1 , the placement of cranes of type t_1 on node grid points constitutes a dominating set of the graph.”
- The placement of cranes of type t_1 on node grid points constitutes an independent set of the graph, as well, see Proposition 1 after the proof of Theorem 4.

“Concluding, the placement of cranes of type t_1 on node grid points constitutes an independent dominating set of the graph with cardinality p .”

It remains to show that an independent dominating set of cardinality p implies a feasible solution with cost $q\pi_{t_2} + p$. We construct such a solution as follows. We place cranes of type t_1 on node grid points as implied by the independent dominating set and place a crane of type t_2 on each dummy grid point. Note that pairs of the heavy type are covered since each such pair gets a dedicated crane of type t_2 . No crane of type t_2 can interfere with any other crane due to the granularity of the grid. Since we position cranes of type t_1 according to an independent set they have minimum pairwise distance of $\sqrt{2}$ and, therefore, do not interfere neither. A crane of type t_1 positioned at (x, y) can cover all pairs of the light type in Euclidean distance of ϵ to (x', y') with $|x - x'| + |y - y'| \leq 1$. Since we position cranes of type t_1 according to a dominating set each pair of the light type is covered. Hence, the solution is feasible. Clearly, the cost of the constructed solution is $q\pi_{t_2} + p$. \square

The reduction is illustrated in Figure 4.3. “In Figure 4.3a, there is a grid graph of a given instance I of IND-DOM-GRID. Figure 4.3b depicts the constructed instance I' of TCSPP-GRID. We have node grid points represented by large black dots and dummy grid points represented by small black dots. The construction site is, thus, given by the convex hull of the set of black dots. The small gray dots represent the pairs of the light type to be covered by cranes of type t_1 . There are two of them on the line between each pair of node grid points with Euclidean distance of 1. The small white dots represent the pairs of the heavy type to be covered by cranes of type t_2 . They are positioned close to the dummy grid points within the construction site.

Figure 4.4 illustrates solutions to the instances depicted in Figure 4.3. In Figure 4.4a the set of nodes highlighted in black constitutes an independent dominating set with 3 nodes. In fact, it is a minimum cardinality independent dominating set (not the only one). Figure 4.4b depicts the corresponding solution to I' of TCSPP-GRID. There is a crane of type t_1 positioned on each node grid point corresponding to a node in the independent dominating set. Additionally, there is a crane of type t_2 positioned on each dummy grid point.

In Figure 4.4 operating areas of cranes of type t_1 do not overlap at all. However, this is not necessarily the case, as we can see from the solutions of the same problem instances depicted in Figure 4.5. Both solutions are not optimal. As we can see, here operating ranges overlap. However, no crane’s operating

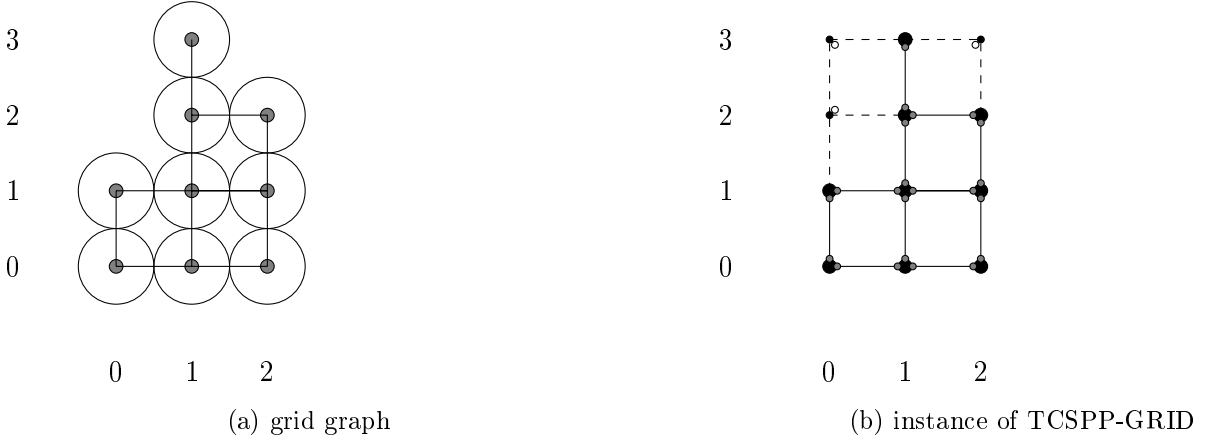
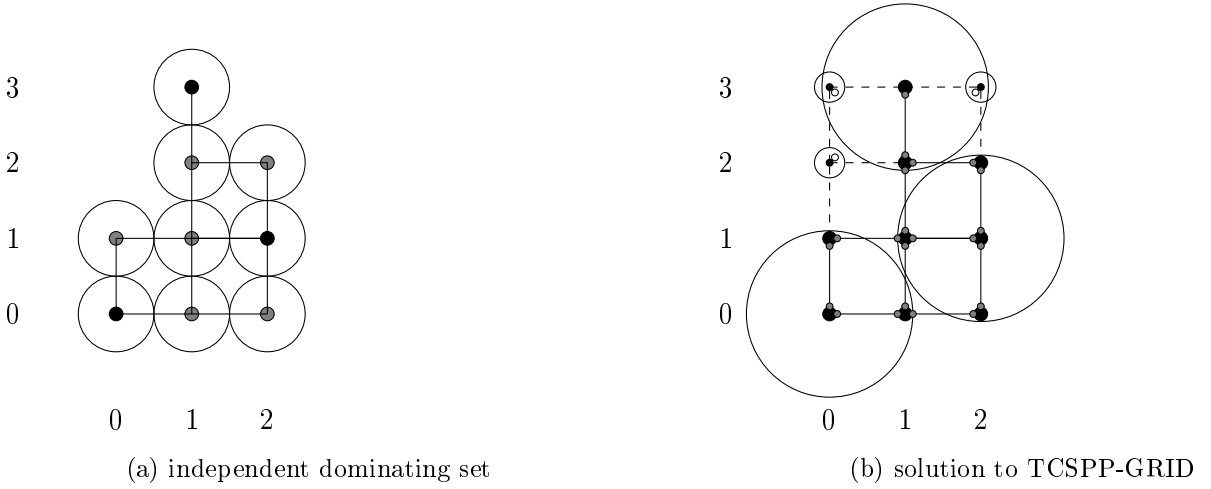


Figure 4.3: Grid graph and an instance of TCSPP-GRID


 Figure 4.4: Independent dominating set of cardinality 3 and a solution with cost $3\pi_{t_2} + 3$

area encloses the position of another crane of type t_1 and, therefore, no crane prevents another crane from reaching its full operating range.

The proof of Theorem 4 shows that TCSPP-GRID is strongly NP-hard even if $|T| = 2$, $F = \emptyset$, and each demand site consists of a single point only and coincides with its supply site. It is not hard to see that we can modify the proof to show that it is also strongly NP-hard even if $|T| = 1$ and each demand site consists of a single point only and coincides with its supply site (by using forbidden areas in order to prevent cranes of type t_1 from being placed on dummy grid points)."

In the proof of Theorem 4, the following proposition was used.

Proposition 1. *The placement of cranes of type t_1 in a feasible solution to I' with cost of $q\pi_{t_2} + p$ constitutes an independent set of the graph.*

We will prove Proposition 1 in the following.

Proof. We consider the subgraph G' constituted by the nodes of the grid which have a crane of type

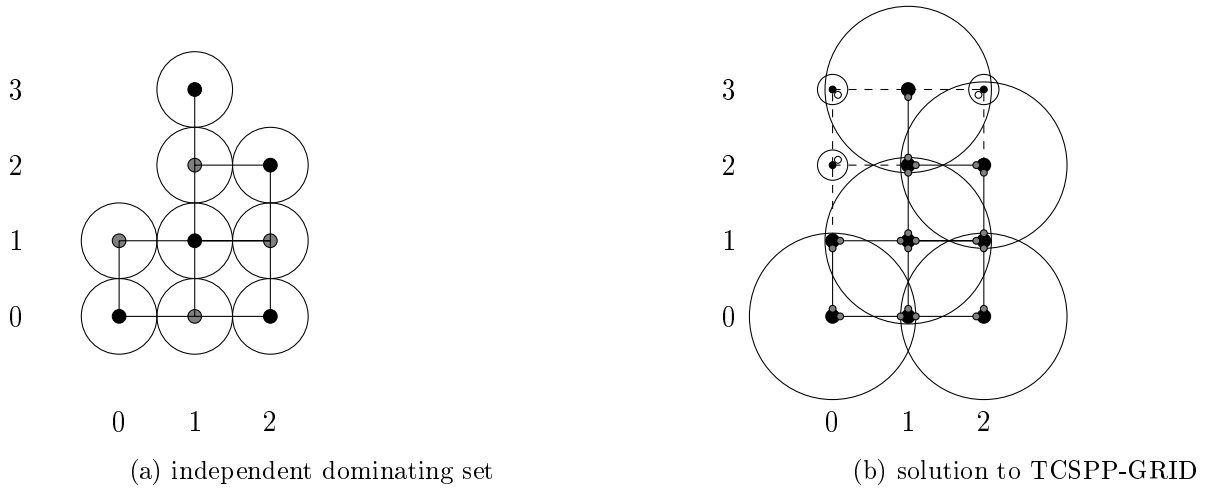


Figure 4.5: Independent dominating set of cardinality 5 and a solution with cost $3\pi t_2 + 5$

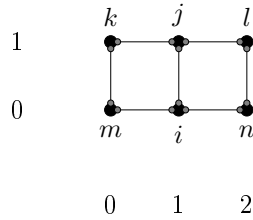
t_1 assigned. In order to allow a reasoning as intuitive as possible we will imagine the nodes of the grid graph G to be embedded in a plane according to the coordinates of the circles' centers. This enables us to speak about horizontal and vertical edges and pairs of edges being orthogonal to each other. As we consider cranes of type t_1 only in the following we will talk of a node covering a pair when a crane of type t_1 located at this node covers the respective pair.

If G' has no edges, the positions of cranes of type t_1 constitute an independent set which is in line with Proposition 1. Now, in contrast, assume G' has an edge (i, j) , i. e. two adjacent nodes i and j are selected for locating a crane of type t_1 on each of them. The pairs on edge (i, j) cannot be covered by these nodes as they obstruct each other. Thus, only other nodes adjacent to i (j) can potentially cover the pair on edge (i, j) close by node i (j) due to the crane type's operating radius. From these adjacent nodes, however, only those lying in orthogonal orientation with respect to edge (i, j) can potentially cover the respective pair (any other node is obstructed by i (j)). Consequently, the selection of two adjacent nodes i and j requires the selection of at least one more adjacent node for each of them in order to potentially cover the pairs on edge (i, j) . This, in turn, requires G' to have additional edges connected to edge (i, j) .

Resulting from the previous considerations, we can exclude that any node in G' has a degree of one or larger than four. Additionally, we can rule out nodes with degree three according to the following Lemma 1.

Lemma 1. *In G' , no node may have a degree of three.*

Proof. Assume there is a node i having a degree of three. Then there are three nodes j , m and n adjacent to i (cf. Figure 4.6). Then, two of these nodes, m and n , have the same y -coordinate (x -coordinate) as i , and the third node, j , has the same x -coordinate (y -coordinate) as i . Node j has to have at least a degree of two with at least one edge (j, k) being orthogonally oriented with respect to edge (i, j) for the pair on edge (i, j) close by j to be covered. Then, the jib of the crane located at j is blocked by both the crane located at i and k and, hence, cannot simultaneously reach both the pair on edge (i, m) and on edge (i, n) close by i . Thus, one of these pairs remains uncovered. \square


 Figure 4.6: Node i in G' with a degree of three resulting in infeasibility

Now, we are going to prove that if G' has an edge, then each node in G' has a degree of exactly two and G consists of a single cycle of four nodes.

In order to see this, first note that there has to be at least one top horizontal edge (i, j) in G' since any adjacent vertical edge would require another horizontal edge for covering both pairs lying on this vertical edge. Then, neither i nor j do have a degree of four and – considering the above – have a degree of two. Furthermore, there has to be one vertical edge adjacent to i , (i, m) , and one vertical edge adjacent to j , (j, n) . Both such edges must be below (i, j) due to the assumption that (i, j) is top-most. Then, nodes i, j, m and n constitute a cycle. From the previous considerations, it can be concluded that the jibs of the cranes located at these four nodes are trapped within this cycle, i. e. they can only rotate within the cycle, but cannot leave it as this would require slewing through another crane's mast: m and n have to cover the pairs on (i, j) , i has to cover the pair on (j, n) close by j and j has to cover the pair on (i, m) close by i .

Now assume that G has at least one more node. Then there is a node a which is connected to the cycle as G is connected. Then, in G there are two pairs on the edge connecting a to the cycle. One of these pairs is close to a node of the cycle and, thus, can only potentially be covered by a node in the cycle or adjacent to the same node in the cycle like a . Since i, j, m and n are trapped in the cycle they cannot cover this pair and, thus, there has to be at least one additional node in G' . However, according to Lemma 2, this cannot be the case.

Lemma 2. *In G' , m and n have a degree of two.*

Proof. We restrict ourselves to consideration of node m since node n having a degree of two can be established in the same way. Since degrees zero, one and three have already been excluded above it remains to exclude a degree of four.

Assume that m has a degree of four. In this case (cf. Figure 4.7), there are two nodes k and o with coordinates $(x_k = x_m - 1, y_k = y_m)$ and $(x_o = x_m, y_o = y_m - 1)$, respectively, in G' . This means that there is a pair on edge (k, m) close by k . As i may not have a degree of three there may not be a node p with coordinates $(x_p = x_i - 1, y_p = y_i)$ so that there has to be node q with coordinates $(x_q = x_k, y_q = y_k - 1)$ to cover the pair close by k . This, in turn, means that there is a pair on edge (k, q) close by k . This pair cannot be covered by m as m serves the pair on edge (i, j) close by i and is trapped between i and n . Thus, there needs to be node r with coordinates $(x_r = x_k - 1, y_r = y_k)$ in order to cover the pair. But then node k has a degree of three which is not possible due to Lemma 1.

Consequently, neither m nor n may have a degree different from two. \square

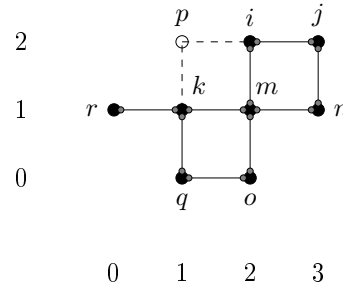


Figure 4.7: Node m in G' with a degree of four resulting in infeasibility

Hence, with Lemma 2 holding, it can be concluded that G has only the four nodes i , j , m , and n if G' is not an independent set. However, G has more than four nodes due to the definition of IND-DOM-GRID-BIG and, hence, we have a contradiction. \square

4.1.3 Research Gap

Note: this chapter is based on Briskorn and Dienstknecht [12].

As already mentioned when reviewing related literature in Chapter 2.2.2, single facettes of the problem described in Chapter 4.1.1 have been studied to-date. However, the combination of these facettes has not been considered so far. The following features are added to the scientific literature:

- Facilities with limited operating areas have been introduced by Toregas et al. [115]. In the current work, facilities (i.e. cranes) have limited operating areas, as well, but these are additionally affected by interferences with on-site objects. Thus, the actual operating area of a facility is not known in advance and, furthermore, depends on both a facility's location and other facilities' locations.
- Minimum distances between facilities to be located have been studied in both the discrete p -dispersion problem (selection of p facility locations from a given finite set of facility locations so that the minimum distance between any two of them is maximized; Erkut et al. [33]) and the discrete anti-covering location problem (selection of a maximum subset from a given finite set of facility locations so that any two of the selected points keep at least a given minimum distance; Murray and Church [75]). In the current work, the selection of facilities and their locations respects minimum distances, as well, but, additionally, the coverage of pairs of polygons has to be respected while simultaneously not knowing which and how many facilities to select and where to locate them.
- Additionally, analogously to the contributions mentioned in Chapter 3.1.3, the common assumptions of point-based demand and homogeneous facilities do not hold in the problem studied since pairs of polygons have to be covered by cranes with different specifications.

4.2 Solution Approaches

In this chapter, two different approaches for solving TCSP-GRID as described in Chapter 4.1.1 will be presented. The first approach relies on employing a standard solver based on a representation of the problem as mixed-integer program. Four different MIP formulations motivated by i) different perspectives on representing coverage and ii) a trade-off between the number of variables and constraints are presented in Chapter 4.2.3. The second approach is a branch and bound procedure which will be detailed in Chapter 4.2.4. Both approaches require processing the mainly geometric input data of an instance of TCSP-GRID in order to allow for the application of the respective techniques. This data pre-processing will be described in Chapter 4.2.1. Chapter 4.2.2 summarizes the resulting sets along with the remainder of the notation used when detailing the solution approaches.

4.2.1 Data Pre-Processing

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

We, now, detail the pre-processing of an instance's geometric input data. In this pre-processing step, sets are derived from the geometric information which allows for the linear models and the B & B approach presented in Chapter 4.2.3 and Chapter 4.2.4, respectively. After a general description of the information to be processed and the resulting sets, a more detailed example for the derivation of one such set is given.

“An instance of TCSP-GRID is specified by the data introduced in Chapter 4.1.1. In order to keep the part at hand self-contained, we shortly recapitulate the notation:

- the set T of crane types with the single types' specifications as mentioned in Chapter 4.1.1,
- the site, i.e. the site polygon with the sets of supply areas S , demand areas D (including the assignment of supply areas to demand areas, i.e. the set P containing all pairs (d, s_d)) and forbidden areas $\bigcup_{t \in T} F_t$,
- the prescribed minimum distances between cranes of type t and t' , $D_{t,t'}^{min}$,
- the granularity of the grid, i.e. the horizontal and vertical distance between two intersection points of the grid.”

In the following, we point out how the inputs for the MIPs and the B & B approach are derived from the geometric information.

“The idea of using a grid aims at discretizing the by nature continuous site space and, thus, generating a finite set of potential crane locations. The intersection points of the grid are regarded as potential locations for erecting a crane. Grid points are infeasible for any crane type if they are located outside the site's boundaries or within any $s \in S$ or $d \in D$ (including edges). Additionally, a grid point is infeasible for a specific crane type t if the grid point is located within any $f \in F_t$ (including edges). Thus, in a first step, each grid point that is not infeasible in general is assigned a crane of each type

t that may be feasibly located in that point. Each assignment of location and crane type is called a candidate and added to the candidate set C . Selecting candidate c comes at a cost of π_c which corresponds to the fix cost of crane type t_c associated with candidate c , π_{t_c} .

Once the candidate set is determined, information on minimum distances among the single candidates can be processed. Minimum distance constraints require two cranes of types t_c and $t_{c'}$ to keep at least a Euclidean distance of $D_{t_c, t_{c'}}^{min}$ between the cranes' centers. Thus, cranes may not be positioned independently: we cannot simultaneously locate cranes of types t_c and $t_{c'}$ at intersection points of the grid that have a distance of less than $D_{t_c, t_{c'}}^{min}$, i.e. the corresponding candidates c and c' may not be selected simultaneously. Consequently, all candidates that violate the minimum distance constraint with respect to candidate c constitute the set $N_c = \{c' \in C \mid dist_{c,c'} < D_{t_c, t_{c'}}^{min}\}$ with $|N_c| = n_c$ for a specific candidate c (with $dist_{c,c'}$ being the Euclidean distance between the crane locations associated with candidates c and c').

The crucial part is to derive coverage information for the single candidates. Here, interferences of cranes and of cranes with "on-site structures (i.e. buildings and forbidden areas) "have to be respected. In order to identify such interference effects, we first have a look at the prerequisites for a crane to cover a pair and then focus on interference by any object (i.e. cranes and "on-site structures).

"In the following, we focus on a candidate c , i.e. a position given by Cartesian coordinates and an assigned crane of type t_c , and a pair (d, s_d) . The basic prerequisites for c to cover the pair are that the pair is in reach of c (as defined in Chapter 4.1.1) and c can establish an uninterrupted path between d and s_d . If a pair is in reach of c , then all points of both the supply area and the demand area are within the crane's operating radius. The supply and the demand area are each located in a sector of the circle representing the crane's operating area – we call these sectors the supply sector and the demand sector. More precisely, these sectors are the smallest sectors (in terms of the angle between the respective sector's boundaries) that contain all points of the supply and demand polygon, respectively. We distinguish between two cases.

1. The sectors are disjoint, cf. pair (d_1, s_1) in Figure 4.8a.
2. The sectors overlap (including the case of one sector being completely contained within the other sector), cf. pair (d_2, s_2) in Figure 4.8a.

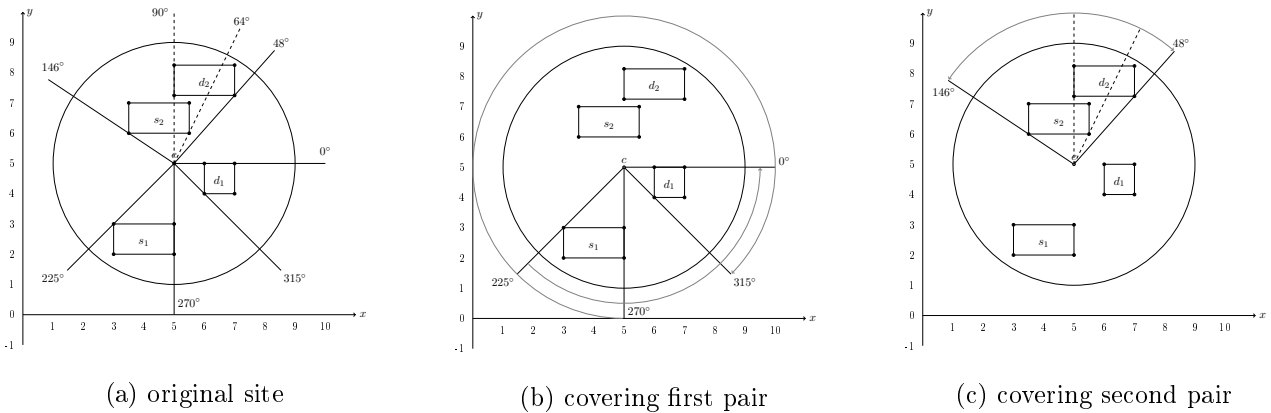


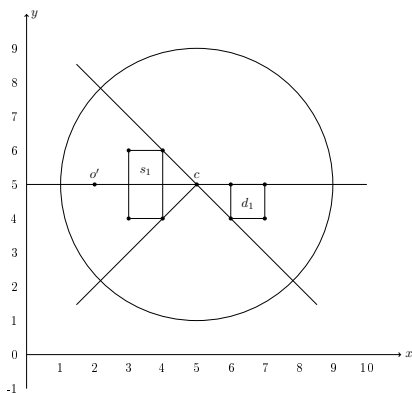
Figure 4.8: Slewing directions and coverage

In the first case, the pair divides the crane's operating area into four sectors, namely the supply sector, the demand sector and two transport sectors. We distinguish between those sectors with regard to the lifting operation of cranes (oriented from the supply site to the demand site) during which the crane moves either clockwise or counter-clockwise. Consequently, we refer to the transport sector in which the crane moves clockwise (counter-clockwise) from supply site to demand site as clockwise (counter-clockwise) transport sector. In Figure 4.8a the clockwise transport sector of pair (d_1, s_1) contains all points in reach having an angular coordinate larger than 0 and lower than 225 and the counter-clockwise transport sector contains all points in reach having an angular coordinate larger than 270 and lower than 315. In order to cover such a pair, the crane has to cover the supply and the demand sectors and at least one of the transport sectors. This can be considered as clockwise or counter-clockwise load-carrying rotation of the crane reaching each point in three of the four sectors as depicted in Figure 4.8b where the direction of the load-carrying moves is indicated by the arrow heads. In the second case, it suffices to cover the union of supply and demand sector which can be performed by either a clockwise or counter-clockwise rotation of the crane as depicted in Figure 4.8c (the borders of sectors lying properly within the union of sectors are depicted using dashed lines).

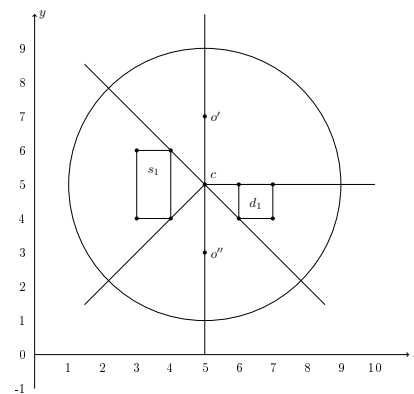
Once this is established, we can focus on the conditions for objects of sufficient height – cranes or on-site structures – “to prevent a crane from covering a pair. Again, we distinguish the two cases 1. and 2. specified above.

In the first case (supply sector and demand sector are disjoint), a pair cannot be covered by the candidate under consideration if

- (a) at least one blocking object is (partially) positioned within the supply sector,
- (b) at least one blocking object is (partially) positioned within the demand sector, or
- (c) at least one blocking object is (partially) positioned in each transport sector.



(a) one object preventing coverage



(b) two objects preventing coverage

Figure 4.9: Objects of sufficient height preventing a crane from covering a pair

Case (a) is illustrated in Figure 4.9a where a single object o' prevents c from covering pair (d_1, s_1) since some points within s_1 cannot be reached; for case (b) we can imagine a single object being positioned within the demand sector instead. Case (c) is depicted in Figure 4.9b where two objects o' and o'' prevent c from covering the pair. Each point in both supply sector and demand sector can be reached

but the crane cannot move from the supply sector to the demand sector (and vice versa) since in both transport sectors some points cannot be reached.

In the second case (supply sector and demand sector overlap), a crane can only be prevented from covering a pair if an object is located within the union of the supply and the demand sector (that is only cases (a) and (b) are relevant).

Summarizing, depending on the very position of the blocking objects a single object may suffice to prevent a candidate from covering a pair or two objects can only jointly do so.

For the MIP formulations” and the B & B approach to be developed in Chapters 4.2.3 and 4.2.4 “it is important to note that in turn if a candidate is prevented from covering a pair (that is in reach) we can identify a single object or a pair of objects that is sufficient for prevention. This motivates to consider single objects or pairs of objects that have potential to prevent coverage when designing the MIP formulations and when exploiting structural knowledge in the B & B approach. When doing so we have to distinguish between blocking” on-site structures and blocking cranes.

Blocking on-site structures are easy to check with regard to their impact since they are given and, naturally, have a static position. “So, for each candidate c and each pair (d, s_d) we can predetermine whether a blocking” on-site structure “(partially) lies in the supply sector or the demand sector or lies in one of the transport sectors. In the former case candidate c cannot cover pair (d, s_d) independent from other” on-site structures “or crane locations. In the latter case, if there are blocking” on-site structures “in exactly one transport sector a single crane in the other transport sector suffices to prevent c from covering (d, s_d) . If there are blocking” on-site structures “in both transport sectors c cannot cover (d, s_d) independent from other” on-site structures or crane locations.

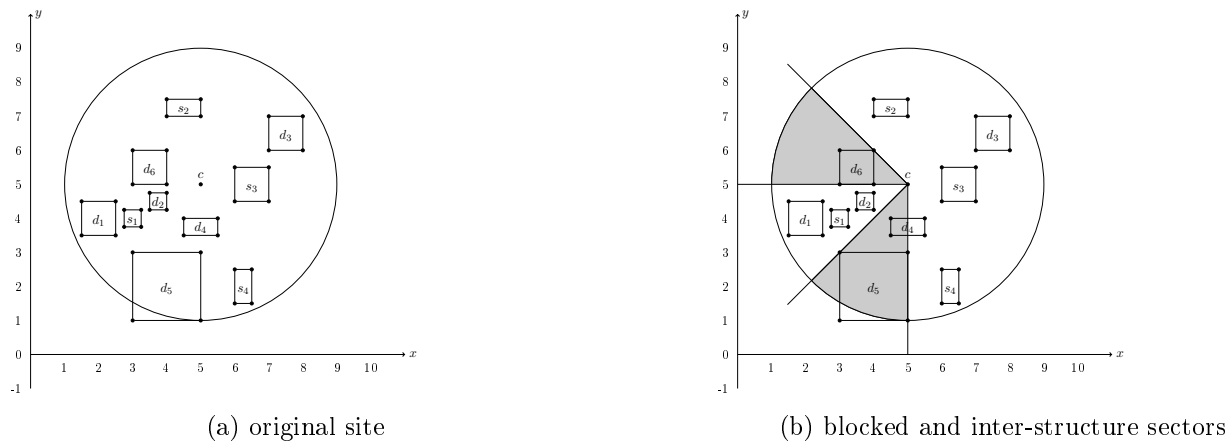


Figure 4.10: Inter-structure sectors and coverage

The concept of blocking on-site structures “is illustrated in Figure 4.10 (with blocking demand areas). Demand areas d_5 and d_6 block the crane associated with candidate c and create two blocked sectors (gray sectors) that divide the crane’s operating area into two inter-structure sectors, one containing pair (d_1, s_1) and the other one containing pair (d_3, s_3) . It can be seen that neither pair (d_2, s_2) nor pair (d_4, s_4) can be covered by c : for (d_2, s_2) , its supply and demand site are in different inter-structure sectors, for (d_4, s_4) , the demand site is partially in a blocked sector.

For a given instance of TCSP-GRID, the set $B_c = \{B_c^1, \dots, B_c^{|B_c|}\}$ is the set of inter-structure sectors specified by on-site structures blocking c ’s jib. We consider only inter-structure sectors that

fully contain at least one pair (d, s_d) in reach of c . If there is no blocking on-site structure for candidate c , then $B_c = \emptyset$. We say that a candidate c can cover a pair (d, s_d) with respect to structures if there are further cranes necessary for c not being able to cover (d, s_d) . For each pair $(d, s_d) \in P$ we can define a set C_d containing all candidates that can cover (d, s_d) with respect to structures. A candidate c is in C_d if and only if (d, s_d) is in reach of c and either $B_c = \emptyset$ or (d, s_d) is fully contained in one of c 's inter-structure sectors. In turn, for each candidate c , we can define a set P_c containing all pairs that can be covered by c with respect to structures, i. e. pair (d, s_d) is in P_c if and only if (d, s_d) can be covered by c with respect to structures.”

As stated in the introductory part of Chapter 4.2, the MIPs to be developed will differ with respect to their representation of coverage. More specifically, two MIPs will rely on the slewing direction of a candidate c covering a pair (d, s_d) . If there are blocking on-site structures “for c it may well be that c can cover (d, s_d) with respect to structures, but it cannot move its jib through the counter-clockwise (clockwise) transport sector. We, thus, define set P_c^{nco} (P_c^{ncl}) as the set of pairs in P_c where demand sector and supply sector do not overlap with respect to c and that cannot be covered by candidate c using the counter-clockwise (clockwise) transport sector with respect to structures.”

In contrast to blocking on-site structures, “cranes and their locations are not given in advance. However, taking into account locations of on-site structures for each candidate c and pair (d, s_d) we can derive all single candidates and pairs of candidates that prevent c from covering (d, s_d) . Blocking by a single crane or two cranes without” on-site structures “being involved are depicted in Figure 4.9. In Figure 4.9a, a crane located at d' prevents c from covering s_1 and, thus, the pair while in Figure 4.9b both transport sectors are blocked by cranes located at d' and d'' . Finally, blocking both transport sectors can be jointly achieved by a single crane and” an on-site structure “as depicted in Figure 4.11 where crane c' and a sector blocked by a demand area (gray sector) prevent coverage.

Consequently, for each candidate c , we need to identify the sets of a single other candidate c' and two other candidates c' and c'' , respectively, that block candidate c 's jib, i. e. that have sufficient height and proximity. We call these sets T_c and T'_c . More formally, T_c is the set of tuples (c, c') where candidate c' blocks candidate c 's jib and T'_c is the set of triples (c, c', c'') where candidates c' and c'' block candidate c 's jib. These tuples and triples, respectively, are to be taken into account in order to consider mutual interference of cranes which might result in c being prevented from covering a pair which it can cover with respect to structures.

For a given candidate c , set $P_{(c,c')} \subseteq P_c$ is the set of pairs that can be covered by c with respect to structures, but cannot be covered by c anymore if c' is selected simultaneously. Sets $P_{(c,c')}^{co}$ and $P_{(c,c')}^{cl}$ reflect the sectors formed by on-site structures and cranes. Consider candidate c and one of its inter-structure sectors B_c^i (if existent). A candidate c' located in B_c^i divides it into two sectors (that may be divided further by other cranes). A pair $(d, s_d) \in B_c^i$ cannot be covered if c' is chosen and the pair does not fully lie in one of these sectors. If it fully lies in one of these sectors each point of pair (d, s_d) can be reached by moving c 's jib from c' either counter-clockwise or clockwise. Depending on which direction has to be chosen, (d, s_d) is in $P_{(c,c')}^{co}$ or $P_{(c,c')}^{cl}$. Set $P_{(c,c')}^{nco}$ and $P_{(c,c')}^{ncl}$ is the complementary set of $P_{(c,c')}^{co}$ and $P_{(c,c')}^{cl}$ with respect to the set of pairs in the inter-structure sector where c' is located in. Analogously, sets $P_{(c,c',c'')}^{co}$, $P_{(c,c',c'')}^{cl}$, and $P_{(c,c',c'')}^{nco}$ reflect sectors formed by two candidates. Set $P_{(c,c',c'')}$ is the set of pairs that can be covered by c with respect to structures, but cannot be covered by c if

both c' and c'' are chosen. This is the case if c' and c'' are located in both transport sectors of the pair and, consequently, neither can be used for moving c 's jib. Sets $P_{(c,c',c'')}^{co}$ and $P_{(c,c',c'')}^{cl}$ reflect the pairs that can be fully reached by candidate c by moving its jib from c' to c'' in counter-clockwise direction and clockwise direction, respectively.

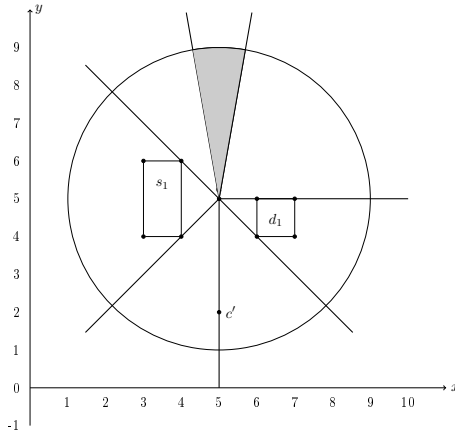
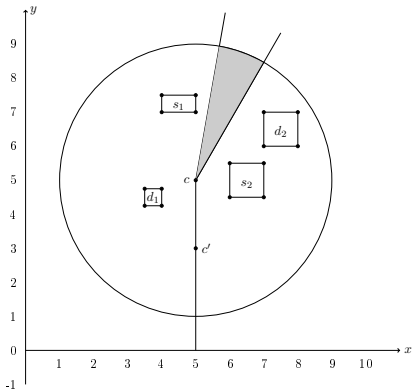
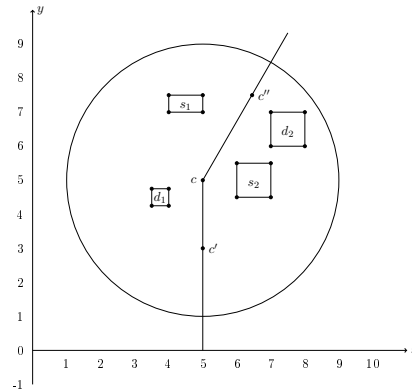


Figure 4.11: Demand area and crane jointly preventing coverage

The above clarifies the conditions preventing a candidate from covering a pair. However, there is one additional issue to be considered. Although two pairs can be covered by a candidate c it may be infeasible to cover them by c simultaneously. For example, in Figure 4.10b (d_1, s_1) and (d_3, s_3) can be covered by c . However, c cannot cover both pairs simultaneously since they do not lie in the same inter-structure sector and the crane cannot move its jib from one inter-structure sector to the other. If blocking cranes are involved, similar concepts may be applied which is illustrated in Figure 4.12. In Figure 4.12a, a blocking on-site structure and a blocking crane c' divide c 's operating area into two sub-sectors of which c may at most cover one. Similarly, two cranes c' and c'' divide c 's operating area into two sub-sectors in Figure 4.12b.



(a) single crane creating sub-sectors



(b) two cranes jointly creating sub-sectors

Figure 4.12: Creation of sub-sectors by cranes

Summarizing, for a set of pairs to be covered by a candidate simultaneously these pairs need to be fully located in the same sub-sector formed by on-site structures and/or other candidates.

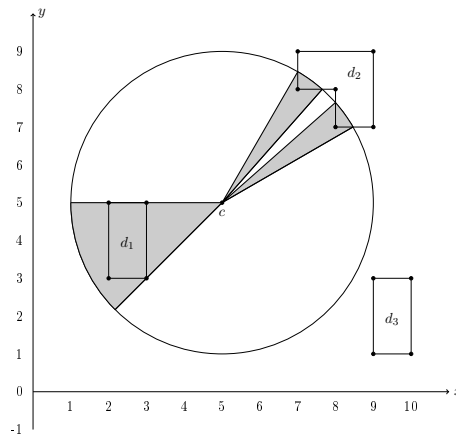


Figure 4.13: Blocking on-site structures and inter-structure sectors

All these sets described in this chapter can be determined by implementing geometric checking routines that determine the situation and orientation of objects in the plane in relation to each other. “We will exemplify the generation of sets containing coverage information now by deriving the set of inter-structure sectors B_c for a given candidate c . The ideas presented can be transferred to all other coverage-related sets. As mentioned earlier in this chapter, B_c is determined by on-site structures of sufficient height to block the jib of the crane associated with candidate c . In general, a circular sector is defined by the circle’s center (i. e. the crane location in our case) and two (polar) angles defining the sector’s start and end. This also holds for sectors in the paper at hand. Once these angles are known for the single inter-structure sectors, it can be derived which pairs are completely contained in them. This, finally, gives us sets B_c^i that, in turn, constitute set B_c . Note that any $B_c^i = \emptyset$ can be eliminated as an empty sector does not need to be covered.

We start by identifying the angles defining the single inter-structure sectors. For each on-site structure of sufficient height to block c ’s jib, there are three potential situations: it is fully, partially or not at all located within c ’s operating radius (cf. Figure 4.13). In the latter case, the structure does not block c . In the first case, all of its nodes and edges are located in c ’s operating radius, so the structure blocks a sector of c ’s operating area. This blocked sector can be identified as the smallest circular sector containing all nodes and edges of the structure. This circular sector is specified by the two outermost nodes of the polygon representing the structure with respect to their angular coordinates in the polar coordinate system of c ’s position. In the second case, the parts of the structure lying in c ’s operating area can be identified by traversing along the polygon’s edges in either clockwise or counter-clockwise orientation. Once, an edge intersects the circle representing c ’s operating area, this edge and, consequently, the structure is partially situated within c ’s operating radius. These parts of the structure reflect blocked sectors of c ’s operating area for which, in turn, the defining angles can be derived from the structure’s nodes and the intersection points of edges and the circle representing c ’s operating area. After identifying the sectors blocked by the single structures, these sectors have to be checked for pairwise overlaps. Whenever two such sectors overlap, we merge them.

Once we determined all sectors not blocked by on-site structures (white sectors in Figure 4.13) we still for each such sector have to identify those pairs that completely lie within the sector. A pair is said to

be completely contained in a certain inter-structure sector if all of its nodes and edges are contained in the sector, i. e. between the angles defining the sector. If an inter-structure sector contains at least one pair completely the sector is added to the set of inter-structure sectors B_c of candidate c .

Sectors determined by cranes or by cranes and blocking on-site structures can be derived from the angular coordinates of the polar coordinates of the blocking cranes and on-site structures, analogously, by treating blocking cranes as on-site structures of sufficient height with punctate groundplan.”

4.2.2 Notation

Note: this chapter is based on Briskorn and Dienstknecht [12].

In this chapter, the notation introduced in Chapter 4.2.1 is summarized. Furthermore, the remaining notation to be used in the MIP formulations (Chapter 4.2.3) and the B & B approach (Chapter 4.2.4) is introduced in order to improve readability. This summary is presented in Table 4.1. More detailed explanations regarding the notation will be given at the respective point of occurrence.

Notation	Description
Sets	
$B_c = \{B_c^1, \dots, B_c^{ B_c }\}$	set of inter-structure sectors of candidate c
C	set of candidates
C_d	set of candidates that can cover pair (d, s_d) with respect to on-site structures
C^{ex}	in the B & B approach: set of candidates that are excluded from being selected due to branching decisions
C^{sel}	in the B & B approach: set of candidates that have been selected by branching decisions
C^*	in the B & B approach: set of candidates that have been selected in a lower bound solution
N_c	set of candidates that cannot be chosen due to minimum distances if candidate c is selected
N_c^{UB}	in the B & B approach: set of candidates that cannot be chosen due to minimum distances if candidate c is selected in the upper bound computation
P	set of pairs
P_c	set of pairs that can be covered by candidate c with respect to buildings
P^{cov}	in the B & B approach: set of pairs that are definitely covered by selected candidates due to branching decisions
P_c^{nco}, P_c^{ncl}	set of pairs in P_c that cannot be covered by candidate c using the counter-clockwise or clockwise, respectively, transport sector
$P_{(c,c')}$	set of pairs in P_c that candidate c cannot cover if candidate c' is chosen
$P_{(c,c')}^{co}, P_{(c,c')}^{cl}$	set of pairs that can be fully reached by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c'

$P_{(c,c')}^{nco}, P_{(c,c')}^{ncl}$	set of pairs that cannot be covered by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c'
$P_{(c,c',c'')}$	set of pairs that cannot be covered by candidate c when candidates c' and c'' are selected
$P_{(c,c',c'')}^{co}, P_{(c,c',c'')}^{cl}$	set of pairs that can be fully reached by candidate c by slewing counter-clockwise or clockwise, respectively, from candidate c' to candidate c''
\mathcal{S}_c	in the B & B approach: set of sectors of candidate c 's operating area formed by blocking cranes and / or blocking on-site structures
T_c	set of tuples (c, c') with candidate c' blocking candidate c 's jib
T'_c	set of triples (c, c', c'') with candidates c' and c'' blocking candidate c 's jib
Parameters	
π_c	cost of crane type t associated with candidate c
n_c	sufficiently large number, e. g. $n_c = N_c $
Variables	
α_c	binary variable; equals 1, if candidate c is selected; 0 otherwise
$\beta_{c,d}$	binary variable; equals 1, if candidate c covers pair (d, s_d) ; 0 otherwise
γ_{c,B_c^i}	binary variable; equals 1, if candidate c covers inter-structure sector $B_c^i \in B_c$; 0 otherwise
$\delta_{(c,c')}^{co}, \delta_{(c,c')}^{cl}$	binary variable; equals 1, if candidate c slews counter-clockwise or clockwise, respectively, with respect to selected candidate c' ; 0 otherwise
$\epsilon_{(c,c',c'')}^{co}, \epsilon_{(c,c',c'')}^{cl}$	binary variable; equals 1, if candidate c slews counter-clockwise or clockwise, respectively, from selected candidate c' to selected candidate c'' ; 0 otherwise
τ_c^j	in the B & B approach: binary variable; equals 1, if candidate c covers its sector $j \in \mathcal{S}_c$; 0 otherwise
$\omega_{c,d}^{co}, \omega_{c,d}^{cl}$	binary variable; equals 1, if candidate c covers pair (d, s_d) by slewing counter-clockwise or clockwise, respectively; 0 otherwise

Table 4.1: Notation for the MIP formulations and the B & B approach

4.2.3 Mixed-Integer Programming Formulations

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

Once the pre-processing of the geometric inputs has been finished, the MIP formulations representing TCSP-GRID can be introduced. As already indicated at the beginning of the current chapter these formulations “are motivated by i) different perspectives on representing coverage and ii) a trade-off between the number of variables and constraints.

The MIP formulations in Chapters 4.2.3.1 and 4.2.3.2 take the same perspective on modelling the coverage of pairs by asking whether a pair is covered by a candidate or not. Contrastingly, the MIPs

presented in Chapters 4.2.3.3 and 4.2.3.4 represent coverage by asking whether a candidate covers a pair by slewing in a certain direction (clockwise or counter-clockwise) from the supply site to the demand site and, thus, use more variables for modelling coverage.

For each perspective on coverage, we examine two ways of accounting for objects limiting the cranes' slewing ranges. One way implies more variables (Chapters 4.2.3.1 and 4.2.3.3, respectively), the other way usually leads to a higher number of constraints (Chapters 4.2.3.2 and 4.2.3.4, respectively)."

4.2.3.1 Undirected Coverage Variables and Explicit Choice of Sectors

"The following model takes the simple perspective on coverage, i. e. asks whether a candidate covers a pair or not. Consequently, variables α_c and $\beta_{c,d}$ reflecting choice of candidates and coverage assignments of candidates and pairs, respectively, are employed. Whenever the working range of a crane is limited to one of multiple sectors (between pairs of" on-site structures, cranes and on-site structures, "or pairs of cranes) we represent the choice of the sector explicitly by employing different variables: γ_{c,B_c^i} indicates which inter-structure sector $B_c^i \in B_c$ is covered by candidate c ; when c 's working range is limited by cranes and on-site structures, $\delta_{(c,c')}^{co}$ and $\delta_{(c,c')}^{cl}$ signal whether c 's jib moves counter-clockwise or clockwise from c' in the respective inter-structure sector; for c being blocked by pairs of cranes, $\epsilon_{(c,c',c'')}^{co}$ and $\epsilon_{(c,c',c'')}^{cl}$ represent whether c 's jib moves counter-clockwise or clockwise from c' to c'' (identifying one of the two sectors formed by c' and c''). Thus, the chosen sector formed by two on-site structures, two cranes, or one on-site structure and one crane is then represented as the intersection of sectors chosen according to variables γ_{c,B_c^i} , $\delta_{(c,c')}^{co}$, $\delta_{(c,c')}^{cl}$, $\epsilon_{(c,c',c'')}^{co}$ and $\epsilon_{(c,c',c'')}^{cl}$.

MIP UN-EX

$$\text{Min } Z = \sum_{c \in C} \alpha_c \cdot \pi_c \quad (4.1)$$

$$\text{s. t. } \quad n_c \cdot \alpha_c + \sum_{c' \in N_c} \alpha_{c'} \leq n_c \quad \forall c \in C \quad (4.2)$$

$$\sum_{c \in C_d} \beta_{c,d} \geq 1 \quad \forall (d, s_d) \in P \quad (4.3)$$

$$\beta_{c,d} \leq \alpha_c \quad \forall c \in C; (d, s_d) \in P_c; |B_c| < 2 \quad (4.4)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c,B_c^i} \leq \alpha_c \quad \forall c \in C; |B_c| \geq 2 \quad (4.5)$$

$$\beta_{c,d} \leq \gamma_{c,B_c^i} \quad \forall c \in C; i = 1, \dots, |B_c|; (d, s_d) \in B_c^i; |B_c| \geq 2 \quad (4.6)$$

$$\sum_{(d,s_d) \in P_{(c,c')}} \beta_{c,d} \leq (1 - \alpha_{c'}) \cdot |P_{(c,c')}| \quad \forall c \in C; (c, c') \in T_c \quad (4.7)$$

$$\sum_{(d,s_d) \in P_{(c,c',c'')}} \beta_{c,d} \leq (2 - \alpha_{c'} - \alpha_{c''}) \cdot |P_{(c,c',c'')}}| \quad \forall c \in C; (c, c', c'') \in T'_c \quad (4.8)$$

$$2 - \alpha_{c'} \geq \delta_{(c,c')}^{co} + \delta_{(c,c')}^{cl} \quad \forall c \in C; (c, c') \in T_c; |B_c| \geq 1 \quad (4.9)$$

$$\beta_{c,d} \leq \delta_{(c,c')}^i \quad \forall c \in C; (c, c') \in T_c; i \in \{co; cl\}; (d, s_d) \in P_{(c,c')}^i; |B_c| \geq 1 \quad (4.10)$$

$$3 - (\alpha_{c'} + \alpha_{c''}) \geq \epsilon_{(c,c',c'')}^{co} + \epsilon_{(c,c',c'')}^{cl} \quad \forall c \in C; (c, c', c'') \in T'_c \quad (4.11)$$

$$\beta_{c,d} \leq \epsilon_{(c,c',c'')}^i \quad \forall c \in C; (c, c', c'') \in T'_c; i \in \{co; cl\}; (d, s_d) \in P_{(c,c',c'')}^i \quad (4.12)$$

$$\alpha_c \in \{0, 1\} \quad \forall c \in C \quad (4.13)$$

$$\beta_{c,d} \in \{0, 1\} \quad \forall c \in C; (d, s_d) \in P_c \quad (4.14)$$

$$\gamma_{c,B_c^i} \in \{0, 1\} \quad \forall c \in C; i = 1, \dots, |B_c|; |B_c| \geq 2 \quad (4.15)$$

$$\delta_{(c,c')}^{co}, \delta_{(c,c')}^{cl} \in \{0, 1\} \quad \forall c \in C; (c, c') \in T_c; |B_c| \geq 1 \quad (4.16)$$

$$\epsilon_{(c,c',c'')}^{co}, \epsilon_{(c,c',c'')}^{cl} \in \{0, 1\} \quad \forall c \in C; (c, c', c'') \in T'_c \quad (4.17)$$

The objective is to minimize total cost for selected candidates and is reflected by objective function (4.1).

Constraints (4.2) ensure that, if candidate c is selected, no other candidate violating the prescribed minimum distance can be selected. The set of constraints given by (4.3) ensures that each pair is covered by at least one selected candidate.

Constraints (4.4) to (4.6) link candidate selection variables and pair coverage variables and ensure that each candidate can serve at most one inter-structure sector. If there are less than two inter-structure sectors, (4.4) states that a pair may only be covered by candidate c if c is selected. If there are at least two inter-structure sectors, (4.5) ensure that at most one of them may be selected for coverage by candidate c and if one is selected, then candidate c is chosen. Then, constraints (4.6) ensure that

only pairs in the selected inter-structure sector may be covered by candidate c . Thus, constraints (4.4) to (4.6) make sure that limitations of the operating ranges by on-site structures alone are chosen and respected.

Constraints (4.7) to (4.12) reflect restrictions of coverage caused by other candidates (possibly jointly with on-site structures, see Chapter 4.2.1). Constraints (4.7) and (4.8) make sure that candidate c may not cover pairs that are blocked by a single selected candidate c' and two selected candidates c' and c'' , respectively. Constraints (4.9) and (4.10) handle the case where there is at least one on-site structure that blocks the moves of a candidate c and another candidate c' is selected that blocks c , as well. Constraints (4.9) require the selection of one sub-sector formed by on-site structures and c' and (4.10) ensure that pairs are covered by candidate c only in accordance with the chosen sub-sector. Similarly, constraints (4.11) and (4.12) handle two candidates c' and c'' that block c and divide c 's (full circle) operating area into two sub-sectors. If both c' and c'' are chosen, at most one of these sub-sectors can be covered by c .

Finally, constraints (4.13) to (4.17) define the domains of the decision variables.

While correctness of the model should be settled for all decision variables being binary from the above we shall shortly justify the possible relaxation of α_c , γ_{c,B_c^i} , $\delta_{(c,c')}^{co}$, $\delta_{(c,c')}^{cl}$, $\epsilon_{(c,c',c')}^{co}$ and $\epsilon_{(c,c',c')}^{cl}$ to continuous variables on the interval $[0, 1]$. We start with α_c which will be chosen as low as feasible due to the objective function. Lower bounds on α_c are imposed only by constraints (4.4) to (4.6). The imposed lower bounds are integer and, therefore, α_c takes integer values in optimum solutions. Variables γ_{c,B_c^i} are involved in constraints (4.5) and (4.6) only. It is easy to see that for any feasible solution with non-integer values of γ_{c,B_c^i} we can round these values down to zero and obtain another feasible solution with the same objective value (since the left hand side of (4.6) is binary). The very same argument holds for $\delta_{(c,c')}^{co}$ and $\delta_{(c,c')}^{cl}$ ($\epsilon_{(c,c',c')}^{co}$ and $\epsilon_{(c,c',c')}^{cl}$) which appear in (4.9) and (4.10) ((4.11) and (4.12)) only and can be rounded down to the next integer since the only lower bound imposed by (4.10) ((4.12)) is binary. This allows us to replace constraints (4.13), (4.15), (4.16) and (4.17) by the following constraints.

$$0 \leq \alpha_c \leq 1 \quad \forall c \in C \tag{4.18}$$

$$0 \leq \gamma_{c,B_c^i} \leq 1 \quad \forall c \in C; i = 1, \dots, |B_c|; |B_c| \geq 2 \tag{4.19}$$

$$0 \leq \delta_{(c,c')}^{co}, \delta_{(c,c')}^{cl} \leq 1 \quad \forall c \in C; (c, c') \in T_c; |B_c| \geq 1 \tag{4.20}$$

$$0 \leq \epsilon_{(c,c',c')}^{co}, \epsilon_{(c,c',c')}^{cl} \leq 1 \quad \forall c \in C; (c, c', c'') \in T'_c \tag{4.21}$$

MIP UN-EX employs $\mathcal{O}(|C| \cdot |D|)$ binary variables, $\mathcal{O}(|C| \cdot |D| + |C|^3)$ continuous variables and $\mathcal{O}(|C|^3 \cdot |D|)$ linear constraints.

4.2.3.2 Undirected Coverage Variables and Implicit Choice of Sectors

The following model takes, again, the simple perspective on coverage. However, the choice of the sector a candidate operates on is represented implicitly reducing the set of variables employed to α_c and $\beta_{c,d}$. It does not come as a surprise then that the objective function and those constraints where only α_c and $\beta_{c,d}$ are used are identical to the MIP formulation in Chapter 4.2.3.1.

MIP UN-IM

$$\text{Min } Z = \sum_{c \in C} \alpha_c \cdot \pi_c \quad (4.1)$$

s. t. (4.2), (4.3), (4.4), (4.7), (4.8), (4.14), (4.18)

$$\beta_{c,d} + \beta_{c,d'} \leq \alpha_c \quad \forall c \in C; (d, s_d) \in B_c^i; (d', s_{d'}) \in B_c^j; j > i \in \{1, \dots, |B_c|\}; |B_c| \geq 2 \quad (4.22)$$

$$2 - \alpha_{c'} \geq \beta_{c,d} + \beta_{c,d'} \quad \forall c \in C; (c, c') \in T_c; (d, s_d) \in P_{(c,c')}^{co}; (d', s_{d'}) \in P_{(c,c')}^{cl}; |B_c| \geq 1 \quad (4.23)$$

$$3 - (\alpha_{c'} + \alpha_{c''}) \geq \beta_{c,d} + \beta_{c,d'} \quad \forall c \in C; (c, c', c'') \in T_c'; (d, s_d) \in P_{(c,c',c'')}^{co}; (d', s_{d'}) \in P_{(c,c',c'')}^{cl} \quad (4.24)$$

Constraints (4.22) replace constraints (4.5) and (4.6), constraints (4.23) replace constraints (4.9) and (4.10) and, finally, constraints (4.24) replace constraints (4.11) and (4.12). The potential choices to be made for a candidate c of one inter-structure sector, of one sub-sector formed by "on-site structures" and a candidate, and of one sub-sector formed by two candidates are represented by enforcing compatible coverage decisions. Constraints (4.22) prevent pairs in different inter-structure sectors from being covered simultaneously. Similarly, constraints (4.23) prevent pairs in different sub-sectors formed by on-site structures and other candidates from being covered simultaneously. Finally, constraints (4.24) prevent pairs in different sub-sectors formed by other candidates from being covered simultaneously.

MIP UN-IM employs $\mathcal{O}(|C| \cdot |D|)$ binary variables, $\mathcal{O}(|C|)$ continuous variables and $\mathcal{O}(|C|^3 \cdot |D|^2)$ linear constraints.

4.2.3.3 Directed Coverage Variables and Explicit Choice of Sectors

The following model takes the more involved perspective on coverage, i. e. asks whether a candidate covers a pair by counter-clockwise or clockwise load-carrying moves. Consequently, besides variables α_c , $\omega_{c,d}^{co}$ and $\omega_{c,d}^{cl}$ representing this decision are employed. Note that the direction refers to load-carrying moves in case supply site and demand site do not overlap with respect to candidate c . In case they do the choice is arbitrary. Whenever the working range of a crane is limited to one of multiple sectors

(between pairs of” on-site structures, cranes and on-site structures, “or pairs of cranes) we represent the choice of the sector explicitly by employing variables γ_{c,B_c^i} , $\delta_{(c,c')}^{co}$, $\delta_{(c,c')}^{cl}$, $\epsilon_{(c,c',c'')}^{co}$ and $\epsilon_{(c,c',c'')}^{cl}$ (as in Chapter 4.2.3.1).

MIP D-EX

$$\text{Min } Z = \sum_{c \in C} \alpha_c \cdot \pi_c \quad (4.1)$$

s. t. (4.2), (4.5), (4.9), (4.11), (4.18), (4.19), (4.20), (4.21)

$$\sum_{c \in C_d} (\omega_{c,d}^{cl} + \omega_{c,d}^{co}) \geq 1 \quad \forall (d, s_d) \in P \quad (4.25)$$

$$\omega_{c,d}^{cl} + \omega_{c,d}^{co} \leq \alpha_c \quad \forall c \in C; (d, s_d) \in P_c; |B_c| < 2 \quad (4.26)$$

$$\omega_{c,d}^{cl} + \omega_{c,d}^{co} \leq \gamma_{c,B_c^i} \quad \forall c \in C; i = 1, \dots, |B_c|; (d, s_d) \in B_c^i; |B_c| \geq 2 \quad (4.27)$$

$$1 - \alpha_{c'} \geq \omega_{c,d}^{co} \quad \forall c \in C; (c, c') \in T_c; (d, s_d) \in P_{(c,c')}^{nco} \quad (4.28)$$

$$1 - \alpha_{c'} \geq \omega_{c,d}^{cl} \quad \forall c \in C; (c, c') \in T_c; (d, s_d) \in P_{(c,c')}^{ncl} \quad (4.29)$$

$$\omega_{c,d}^{co} + \omega_{c,d}^{cl} \leq \delta_{(c,c')}^i \quad \forall c \in C; (c, c') \in T_c; i \in \{co; cl\}; (d, s_d) \in P_{(c,c')}^i; |B_c| \geq 1 \quad (4.30)$$

$$\omega_{c,d}^{co} + \omega_{c,d}^{cl} \leq \epsilon_{(c,c',c'')}^i \quad \forall c \in C; (c, c', c'') \in T'_c; i \in \{co; cl\}; (d, s_d) \in P_{(c,c',c'')}^i \quad (4.31)$$

$$\omega_{c,d}^{co} = 0 \quad \forall c \in C; (d, s_d) \in P_c^{nco} \quad (4.32)$$

$$\omega_{c,d}^{cl} = 0 \quad \forall c \in C; (d, s_d) \in P_c^{ncl} \quad (4.33)$$

$$\omega_{c,d}^{co}, \omega_{c,d}^{cl} \in \{0, 1\} \quad \forall c \in C; (d, s_d) \in P_c \quad (4.34)$$

We detail only those constraints that are not obvious from what we have discussed above. Roughly speaking, variable $\beta_{c,d}$ is replaced by $\omega_{c,d}^{co} + \omega_{c,d}^{cl}$. Note that (4.26) implies $\omega_{c,d}^{co} + \omega_{c,d}^{cl} \leq 1$. Constraints (4.25), (4.26) and (4.27) then immediately correspond to constraints” (4.3), (4.4) and (4.6).

“Constraints (4.28) to (4.29) force $\omega_{c,d}^{co}$ ($\omega_{c,d}^{cl}$) to zero in case candidate c' is chosen and prevents c from serving (d, s_d) with a counter-clockwise (clockwise) load-carrying move. Constraints (4.30) as well as (4.31) ensure consistency between coverage variables and choice of sub-sectors formed by” on-site structures “and single candidates as well as by pairs of candidates, respectively. Finally, constraints (4.32) and (4.33) prohibit infeasible slewing directions for certain pairs.

MIP D-EX employs $\mathcal{O}(|C| \cdot |D|)$ binary variables, $\mathcal{O}(|C| \cdot |D| + |C|^3)$ continuous variables and $\mathcal{O}(|C|^3 \cdot |D|)$ linear constraints.

4.2.3.4 Directed Coverage Variables and Implicit Choice of Sectors

The following model takes, again, the more involved perspective on coverage by employing variables α_c , $\omega_{c,d}^{co}$, and $\omega_{c,d}^{cl}$. The choice of the sector a candidate operates on is represented implicitly and, thus, allows us to drop variables γ_{c,B_c^i} , $\delta_{(c,c')}^{co}$, $\delta_{(c,c')}^{cl}$, $\epsilon_{(c,c',c'')}^{co}$ and $\epsilon_{(c,c',c'')}^{cl}$ (as in Chapter 4.2.3.2).

MIP D-IM

$$\text{Min } Z = \sum_{c \in C} \alpha_c \cdot \pi_c \quad (4.1)$$

s. t. (4.2), (4.18), (4.25), (4.26), (4.28), (4.29), (4.32), (4.33), (4.34)

$$\begin{aligned} \left(\omega_{c,d}^{cl} + \omega_{c,d}^{co} \right) + \left(\omega_{c,d'}^{cl} + \omega_{c,d'}^{co} \right) &\leq \alpha_c \\ \forall c \in C; (d, s_d) \in B_c^i; (d', s_{d'}) \in B_c^j; j > i \in \{1, \dots, |B_c|\}; |B_c| \geq 2 \end{aligned} \quad (4.35)$$

$$\begin{aligned} 5 - 4 \cdot \alpha_{c'} &\geq \left(\omega_{c,d}^{cl} + \omega_{c,d}^{co} \right) + \left(\omega_{c,d'}^{cl} + \omega_{c,d'}^{co} \right) \\ \forall c \in C; (c, c') \in T_c; (d, s_d) \in P_{(c,c')}^{co}; (d', s_{d'}) \in P_{(c,c')}^{cl}; |B_c| \geq 1 \end{aligned} \quad (4.36)$$

$$\begin{aligned} 5 - 2 \cdot (\alpha_{c'} + \alpha_{c''}) &\geq \left(\omega_{c,d}^{cl} + \omega_{c,d}^{co} \right) + \left(\omega_{c,d'}^{cl} + \omega_{c,d'}^{co} \right) \\ \forall c \in C; (c, c', c'') \in T_c'; (d, s_d) \in P_{(c,c',c'')}^{co}; (d', s_{d'}) \in P_{(c,c',c'')}^{cl} \end{aligned} \quad (4.37)$$

The structures of constraints resemble those of the constraints used in Chapters 4.2.3.3 (where variables α_c , $\omega_{c,d}^{co}$ and $\omega_{c,d}^{cl}$ have been employed, too) and 4.2.3.2 (where choice of (sub-)sectors for candidates is implied by consistent choice of candidates to cover pairs).

MIP D-IM employs $\mathcal{O}(|C| \cdot |D|)$ binary variables, $\mathcal{O}(|C|)$ continuous variables and $\mathcal{O}(|C|^3 \cdot |D|^2)$ linear constraints.”

4.2.4 Branch and Bound Procedure

Now, a simple exact solution procedure for TCSPP-GRID relying on the inputs generated as described in Chapter 4.2.1 is proposed. The key components of this branch and bound approach with best-first search, i. e. the branching scheme, the computation of upper and lower bounds and, finally, the fathoming of nodes in the B & B tree, will be described in Chapters 4.2.4.1 to 4.2.4.3.

4.2.4.1 Branching Scheme

In this part, the branching scheme is detailed. Branching is performed with regard to the coverage assignment of candidates and pairs as this allows us to make use of structural knowledge in order to exclude certain coverage assignments or even candidates from a solution. Thus, the branching scheme is based on variables $\beta_{c,d}$ in model UN-EX and UN-IM presented in Chapters 4.2.3.2 and 4.2.3.1.

The basic idea can be sketched as follows: A pair (d, s_d) can be covered by a set of candidates C_d , but it is sufficient for pair (d, s_d) to be covered by only one $c \in C_d$. Thus, it needs to be decided which $c \in C_d$ covers the pair. Once such an assignment has been fixed, pair (d, s_d) is guaranteed to be covered in the respective branch of the B & B tree and can be added to the set of covered pairs P^{cov} . Such an assignment for guaranteeing a pair's coverage may very well require excluding certain candidates or combinations of candidates from a solution as their selection would render the coverage assignment non-viable. We will detail the considerations for excluding candidates or combinations of them below. Note that the proposed branching scheme gives us at most $|P| + 1 = |D| + 1$ levels in the the B & B tree.

The next pair to be assigned to a covering candidate is a pair that has not been assigned to a candidate, i. e. it is selected from the set $P \setminus P^{cov}$. Out of this set $P \setminus P^{cov}$ we select the pair (d^*, s_{d^*}) with the fewest potential covering candidates which have not been excluded from the solution. With C^{ex} being the set of candidates being definitely excluded from the solution, we determine $(d^*, s_{d^*}) = \arg \min_{(d,s_d) \in P \setminus P^{cov}} \{|C_d \setminus C^{ex}|\}$.

The exclusion of candidates and of coverage assignments as mentioned above utilizes structural information and is a significant part of the branching step as it allows to boil down the solution space considerably. We will address the related concepts now based on a node n in the B & B tree which is generated from a parent node by assigning the still unassigned pair (d^*, s_{d^*}) selected as described above to a candidate $c \in C_d \setminus C^{ex}$. This, obviously, gives us $|C_d \setminus C^{ex}|$ children for a parent node.

Assigning a certain pair (d, s_d) to a certain candidate c for being covered has several implications which can be classified into two types, namely

1. the exclusion of candidates

- Once candidate c is selected, any candidate $c' \in N_c$ may not be selected.
- For any tuple $(c, c') \in T_c$, c' has to be excluded if $(d, s_d) \in P_{(c,c')}$.
- For any triple $(c, c', c'') \in T'_c$, the combined selection of c' and c'' has to be prevented if

$(d, s_d) \in P_{(c,c',c'')}$. Consequently, if c' (c'') has already been selected c'' (c') may not be selected anymore.

2. the exclusion of coverage assignments between candidates and pairs

- By assigning pair (d, s_d) to a candidate $c \in C_d$ all assignments of pair (d, s_d) to any other candidate $c' \neq c \in C_d$ can be excluded since multiple coverage of a pair does not add any value.
- If pair $(d, s_d) \in B_c^i$, any pair $(d', s_{d'}) \notin B_c^i$ cannot be assigned to candidate c .
- Selecting candidate c may affect the covering potential of another candidate c' , as well, as c may block c' and, thus, prevent c' from covering certain pairs due to limiting these pairs' accessibility by c' . This holds for any pair $(d', s_{d'}) \in P_{(c',c)}$ and – if at least one pair has been assigned to candidate c'' – for any pair $(d', s_{d'}) \in P_{(c',c,c'')}$. Furthermore, if any pair $(d', s_{d'})$ has been assigned to candidate c' and if $B_{c'} \neq \emptyset$ and if $(d', s_{d'}) \in P_{(c',c)}^{co}$ ($(d', s_{d'}) \in P_{(c',c)}^{cl}$), any pair $(d'', s_{d''}) \notin P_{(c',c)}^{co}$ ($(d'', s_{d''}) \notin P_{(c',c)}^{cl}$) may not be assigned to c' . Similarly, if any pair $(d', s_{d'})$ has been assigned to candidate c' and if at least one pair has been assigned to candidate c'' and if $(d', s_{d'}) \in P_{(c',c,c'')}^{co}$ ($(d', s_{d'}) \in P_{(c',c,c'')}^{cl}$), any pair $(d'', s_{d''}) \notin P_{(c',c,c'')}^{co}$ ($(d'', s_{d''}) \notin P_{(c',c,c'')}^{cl}$) may not be assigned to c' .

4.2.4.2 Lower and Upper Bounds

Besides the branching scheme, a crucial part in B&B procedures is the determination of preferably tight upper and lower bounds. In this part, we start by defining our lower bound (LB) and, afterwards, present an upper bound (UB).

For determining a lower bound, we neglect crane interferences, i. e. we only consider given on-site structures (demand areas and forbidden areas) obstructing cranes. We, then, seek a cost-optimal selection C^* of candidates that have not been excluded so far, i. e. $C^* \subseteq (C \setminus C^{ex})$, which can cover the whole set of pairs P while respecting the single candidates' inter-structure sectors B_c . The latter means that for each $c \in C^*$ at most one $B_c^i \in B_c$ may be serviced by c . Additionally, prescribed minimum distances between candidates have to be respected, i. e. $dist_{c,c'} \geq D_{t_c,t_{c'}}^{min}$ has to be fulfilled for all $c \neq c' \in C^*$ with $dist_{c,c'}$ being the Euclidean distance between the crane locations associated with candidates c and c' .

With C^{sel} being the set of candidates already selected by assigning at least one pair $(d, s_d) \in P$ to each candidate $c \in C^{sel}$ in the branching step and γ_{c,B_c^i} being the binary variable indicating whether the i -th inter-structure sector $B_c^i \in B_c$ of candidate c is selected or not (as already introduced for the MIP formulations in Chapters 4.2.3.1 and 4.2.3.3), we can formulate the following MIP for determining a lower bound.

MIP LB

$$\text{Min } Z = \sum_{c \in C \setminus C^{ex}} \sum_{i=1}^{|B_c|} \gamma_{c, B_c^i} \cdot \pi_{t_c} \quad (4.38)$$

$$\sum_{c \in C \setminus C^{ex}} \sum_{\substack{i=1 \\ (d, s_d) \in B_c^i}}^{|B_c|} \gamma_{c, B_c^i} \geq 1 \quad \forall (d, s_d) \in P \quad (4.39)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c, B_c^i} + \sum_{j=1}^{|B_{c'}|} \gamma_{c', B_{c'}^j} \leq 1 \quad \forall c \in C \setminus C^{ex}; c' \in (C \setminus C^{ex}) \cap N_c \quad (4.40)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c, B_c^i} = 1 \quad \forall c \in C^{sel} \quad (4.41)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c, B_c^i} \leq 1 \quad \forall c \in C \setminus (C^{ex} \cup C^{sel}) \quad (4.42)$$

$$\gamma_{c, B_c^i} \in \{0, 1\} \quad \forall c \in C \setminus C^{ex}; i = 1, \dots, |B_c| \quad (4.43)$$

The objective (4.38) is to minimize total cost for selected inter-structure sectors of candidates not having been excluded while branching. In this selection, each pair $(d, s_d) \in P$ has to be contained in at least one sector (constraints (4.39)). When selecting the sectors, it has to be taken into account that sectors of candidates violating the minimum distance constraint are not selected simultaneously (constraints (4.40)). Additionally, for a candidate having been selected during branching, there is exactly one sector selected (constraints (4.41)). Similarly, for any other candidate neither having been excluded nor selected so far, at most one sector may be selected (constraints (4.42)).

For determining an upper bound, we choose an approach similar to the one for getting a lower bound. The basic idea is that the lower bound may be infeasible with regard to the original problem as it neglects crane interferences. An easy way to exclude solutions with crane interferences is to require cranes to be located with sufficient distance from each other, i. e. for two candidates c and c' (and their associated cranes of types t_c and $t_{c'}$), minimum distance requirements have to be set to a value that prevents the smaller crane (regarding height) from being blocked by the larger one (or, for cranes of identical types, from blocking each other). Thus, for $h_{t_c} \leq h_{t_{c'}}$, we can set $D_{t_c, t_{c'}}^{min, UB} = r_{t_c}^{max}$ and define set $N_c^{UB} = \left\{ c' \in C \mid dist_{c, c'} \leq D_{t_c, t_{c'}}^{min, UB} \right\}$, accordingly, with $dist_{c, c'}$ being the Euclidean distance between the crane locations associated with candidates c and c' . Then, using notation as introduced so far, an optimal solution to the following MIP UB is a feasible solution to the original problem.

MIP UB

$$\text{Min } Z = \sum_{c \in C} \sum_{i=1}^{|B_c|} \gamma_{c,B_c^i} \cdot \pi_{t_c} \quad (4.44)$$

$$\sum_{c \in C} \sum_{\substack{i=1 \\ (d,s_d) \in B_c^i}}^{|B_c|} \gamma_{c,B_c^i} \geq 1 \quad \forall (d, s_d) \in P \quad (4.45)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c,B_c^i} + \sum_{j=1}^{|B_{c'}|} \gamma_{c',B_{c'}^j} \leq 1 \quad \forall c \in C; c' \in N_c^{UB} \quad (4.46)$$

$$\sum_{i=1}^{|B_c|} \gamma_{c,B_c^i} \leq 1 \quad \forall c \in C \quad (4.47)$$

$$\gamma_{c,B_c^i} \in \{0, 1\} \quad \forall c \in C; i = 1, \dots, |B_c| \quad (4.48)$$

Note that, due to the modified minimum distances, MIP UB cannot guarantee to find a feasible solution even if there is a feasible solution to the original problem. Then, an upper bound can be given by requiring each pair to be covered by its own crane of the most expensive type.

4.2.4.3 Fathoming of Sub-Problems

Finally, we have a look at fathoming sub-problems in order to control the B & B tree's size. Once a feasible solution with objective value Z^{UB} has been obtained, there are three criteria which can be applied individually to check whether the current B & B node n can be fathomed:

1. n 's lower bound Z_n^{LB} is not better than the best feasible solution, i. e. $Z_n^{LB} \geq Z^{UB}$,
2. n 's lower bound Z_n^{LB} is better than the best feasible solution, i. e. $Z_n^{LB} < Z^{UB}$, and the corresponding solution of n is feasible (which gives a new best feasible solution),
3. for node n , there is no feasible lower bound (there cannot be a feasible solution to the original problem in this branch of the B & B tree).

As can be seen, criterion 2. requires to check the feasibility of the lower-bound solution at node n , P_n^{LB} . Remember that our lower bound accounts for minimum distances between selected candidates and interferences of cranes with on-site structures, but not for inter-crane interferences. Thus, we have to incorporate these in a respective feasibility check in order to verify (or falsify) compliance of P_n^{LB} with the feasibility criteria given at the end of Chapter 4.1.1. With the instance data processed

as described in Chapter 4.2.1 and the information contained in P_n^{LB} we can formulate the following constraint satisfaction problem CSP FC reflecting the feasibility check.

CSP FC

$$\sum_{c \in C^*} \sum_{\substack{j \in \mathcal{S}_c \\ (d, s_d) \in j}} \tau_c^j \geq 1 \quad \forall (d, s_d) \in P \quad (4.49)$$

$$\sum_{j \in \mathcal{S}_c} \tau_c^j = 1 \quad \forall c \in C^* \quad (4.50)$$

$$\tau_c^j \in \{0, 1\} \quad \forall c \in C^*; j \in \mathcal{S}_c \quad (4.51)$$

With the set of selected candidates C^* and information on blocking on-site structures, for each $c \in C^*$, we can derive the set \mathcal{S}_c of sectors formed by blocking cranes and / or blocking on-site structures of which c may at most cover one. Note that only those elements of \mathcal{S}_c which contain at least one pair $(d, s_d) \in P_c$ are relevant. Binary variable τ_c^j equals one if the crane associated with candidate c operates on sector j and zero otherwise. A solution is feasible if there is a selection of exactly one sector $j \in \mathcal{S}_c$ for each candidate (constraints (4.50)) so that each pair $(d, s_d) \in P$ is in at least one of the selected sectors (constraints (4.49)).

Note that, here, we do not respect the coverage assignments from the branching step. This provides some flexibility as it allows a re-assignment of pairs to candidates, thus, creating a feasible solution for a given selection of candidates that may have been infeasible due to the original assignments.

4.3 Computational Evaluation

In the chapter at hand, the computational performance of standard solver CPLEX based on the MIP formulations presented in Chapter 4.2.3 and of the B & B approach developed in Chapter 4.2.4 is evaluated. When comparing the different MIP formulations in Chapter 4.3.2 drivers of computational effort (measured in computing time) are identified. Once the MIPs have been evaluated the performance capabilities of the B & B approach are analyzed and compared to the best performing MIP formulation (Chapter 4.3.3). All evaluations are based on a test set whose generation will be described first in Chapter 4.3.1. The implementation of both the instance generator and the solution approaches has been done in Java 8 using the Eclipse development environment. All computational studies have been performed on a computer with 32GB RAM and an i7-4790 CPU @ 3.6GHz employing CPLEX 12.6.3 as solver.

4.3.1 Test Set Generation

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

With the studies to be conducted, we aim at investigating how various parameters influence the computational performance and at analyzing the performance capabilities of the approaches developed. “In our study we vary

- the number of pairs to be covered,
- the number of crane types available,
- the site’s size, and
- the grid’s granularity.

The number of pairs drives the number of binary coverage variables and the remaining parameters drive the number of candidates and, thus, also the number of binary variables. Note that we cannot expect the number of variables to exactly scale with the parameters above. For example, for a given size of the site and a given grid granularity, more pairs will leave fewer feasible grid points (i. e. candidates) as compared to fewer pairs. Still, we expect to control the computational effort of instances by varying these parameters when generating the test set.

Throughout our computational study we consider the same four crane types. These types differ in their maximum operating height and radius as well as the cost for selecting one crane of the respective type. Furthermore, we have given required minimum distances of cranes which depend on the respective types. The specific values are given in Table 4.2. We restrict the computational study to instances with unique maximum weights to be lifted associated with demand areas. Therefore, we do not have to consider a maximum operating radius with respect to a certain weight to be lifted. Note, furthermore, that the maximum operating height is not given by a number, but related to a crane type, i. e. a larger crane type corresponds to a larger maximum operating height. We vary the number of available crane

Crane type	Maximum operating radius	Cost per crane	Minimum distances			
			1	2	3	4
1	10	500	5	6	8	12
2	20	1,500	6	10	12	14
3	30	4,000	8	12	15	17
4	40	10,000	12	14	17	20

Table 4.2: Maximum operating radii, crane cost and minimum inter-crane distances

types in instances by parameter κ specifying that κ arbitrarily chosen types are available.

For the sake of comparability, we restrict our study to square-shaped sites and choose the side length with respect to the maximum operating radius of the smallest crane type $t = 1$. The side length equals ζ times the maximum operating radius of the smallest crane type $t = 1$ where ζ is an integer parameter

controlling the site’s size. We restrict ourselves to supply areas and demand areas [...] that are either rectangles or L-shaped areas. Their edges are parallel to those of the construction site. There are two sizes of rectangles and one size of L-shaped area, the latter being rotated randomly by 0, 90, 180, or 270 degrees when being placed on-site. Demand and supply areas are located randomly, but non-overlappingly on-site employing the same procedure as described in Chapter 3.3.1. Hence, there are at most two demand sites per supply site. The number of demand sites to be placed is given by parameter η . A demand site’s height is given in terms of the smallest crane type that is capable of serving the demand site.” When comparing the single MIP formulations no forbidden areas are considered since these only reduce the number of candidates being available and – as stated above – we can control for this by varying the parameters listed above. However, when comparing the B & B approach to the MIP-based approach we add forbidden areas with specific heights as an additional source of limiting the cranes’ operating areas. Forbidden areas are located with a fixed ratio with respect to the site’s size, i. e. an instance’s number of obstacles is given by $\lfloor \frac{\zeta^2}{4} \rfloor$. We assume forbidden areas to be square-shaped and to be located non-overlappingly with respect to supply, demand and other forbidden areas. A forbidden area’s height is given by the largest crane type being blocked by it. “The grid’s granularity is given by parameter θ as the distance of consecutive (horizontally or vertically) intersection points.

We encode a class of instances using the scheme $\kappa - \eta - \zeta - \theta$ with the notation introduced above, i. e. there are κ crane types available for a square-shaped site of size $\zeta \times \zeta$ (with the operating radius of the smallest crane as unit) containing η pairs and a grid with horizontal and vertical distances of θ between the grid’s intersection points. In our study, we employ the following values:

- $\kappa \in \{1, 2, 3, 4\}$
- $\eta \in \{20, 30, 40\}$
- $\zeta \in \{5, 8, 10\}$
- $\theta = \{5, 10\}$ ”

For the comparison of the MIP formulations, we “generate a class of instances for each combination of these values which gives us 72 different classes of instances. Our test set includes 15 instances per class which results in a total of 1,080 instances” (as mentioned above without forbidden areas). Similarly, for the B & B evaluation, we generate a class of instances for each combination of these values which gives us 72 different classes of instances with, again, 15 instances per class. This gives us another 1,080 instances (this time, with forbidden areas).

4.3.2 Evaluation of the MIP-Based Approach

Note: all contents presented and all passages quoted in this chapter are taken from Briskorn and Dienstknecht [12].

Now, we report the results of the study concerning the MIP formulations. Table 4.3 “summarizes computing times (average and maximum time in seconds) per instance class with $\theta = 10$ for the single MIPs formulated in Chapter 4.2.3 (results for $\theta = 5$ are not reported as we observed out-of-

memory issues even for the small instance classes with respect to κ , η and ζ). Note that the maximum allowed computing time for each MIP has been limited to one hour (3,600 seconds). We refrain from giving separate times for input generation for the MIPs and solving the respective MIP since the input generation time is marginal (less than three seconds in the worst case).

instance class	UN-EX		UN-IM		D-EX		D-IM	
	avg.	max.	avg.	max.	avg.	max.	avg.	max.
1-20-5-10	4.32	14.15	1.98	6.22	17.68	72.99	2.91	9.89
1-20-8-10	16.25	73.24	3.67	10.99	8.77	30.88	4.86	19.61
1-20-10-10	20.78	123.13	2.15	8.79	27.93	162.78	3.51	12.04
1-30-5-10	36.33	103.31	5.38	17.19	24.69	128.84	7.99	20.19
1-30-8-10	273.27	1,101.36	14.24	47.98	266.27	1,029.58	17.74	59.77
1-30-10-10	197.29	925.13	7.93	24.42	181.34	809.85	10.44	34.30
1-40-5-10	66.86	196.93	8.27	32.88	5.44	16.75	15.06	44.99
1-40-8-10	570.17	1,920.03	30.08	86.03	597.01	2,398.22	35.12	95.77
1-40-10-10	632.05	3,137.45	29.90	169.64	697.53	2,666.51	38.34	216.68
2-20-5-10	16.75	72.21	4.59	10.09	37.30	184.65	5.90	12.04
2-20-8-10	103.32	360.10	7.03	18.21	54.93	546.05	9.79	22.22
2-20-10-10	76.87	235.88	5.52	13.81	42.29	296.26	7.93	18.46
2-30-5-10	74.11	277.87	10.20	25.97	85.10	282.62	14.97	40.26
2-30-8-10	446.40	1,547.94	25.38	73.10	484.02	1,447.63	24.53	78.19
2-30-10-10	372.98	1,163.49	12.64	31.16	85.66	656.70	19.01	49.11
2-40-5-10	143.99	452.20	19.26	49.10	55.09	330.56	28.63	56.26
2-40-8-10	1,053.69	3,600.00	54.63	175.29	1,021.60	3,600.00	82.30	544.70
2-40-10-10	1,117.99	3,600.00	37.25	124.87	1,247.12	3,600.00	50.37	176.17
3-20-5-10	41.91	110.95	6.28	9.88	53.51	175.72	8.80	12.70
3-20-8-10	220.64	643.74	12.39	29.06	46.51	165.52	18.42	33.98
3-20-10-10	194.14	572.51	9.43	16.60	113.05	377.56	15.22	27.37
3-30-5-10	167.79	581.78	15.72	42.03	167.10	379.84	21.68	47.06
3-30-8-10	842.75	2,116.53	35.19	88.82	779.63	2,125.52	49.14	115.66
3-30-10-10	571.41	1,671.44	24.77	52.94	171.10	829.66	29.70	50.07
3-40-5-10	323.74	722.29	27.88	52.71	315.35	713.16	41.62	69.15
3-40-8-10	1,762.36	3,600.00	85.97	231.16	1,811.82	3,600.00	104.61	321.45
3-40-10-10	1,914.82	3,600.00	59.05	127.99	1,832.33	3,600.00	77.62	176.00
4-20-5-10	75.45	146.11	8.87	18.48	88.02	176.42	11.08	14.40
4-20-8-10	294.84	567.23	14.81	24.55	54.49	75.96	21.94	28.93
4-20-10-10	221.76	371.31	12.21	17.22	173.29	383.98	19.60	32.08
4-30-5-10	254.77	823.09	19.14	37.68	263.17	555.27	26.97	44.09
4-30-8-10	1,011.01	2,103.95	38.41	86.45	1,100.77	2,319.06	52.51	105.00
4-30-10-10	760.68	1,403.44	28.12	41.06	228.53	983.59	40.93	52.47
4-40-5-10	391.29	615.23	33.75	51.03	384.46	572.15	53.28	79.61
4-40-8-10	2,174.98	3,600.00	99.18	230.48	2,137.67	3,600.00	124.90	193.74
4-40-10-10	2,520.14	3,600.00	87.38	157.11	2,366.81	3,600.00	123.83	214.82

Table 4.3: Average and maximum computing times (in seconds) per MIP for grid granularity $\theta = 10$

With the study, we aim at three goals.

1. Verify drivers of computational effort (measured in computing times).
2. Compare the MIP models developed in Chapter 4.2.3 with regard to their computational performance and – if possible – give recommendations on which one to employ.
3. Determine the performance capabilities of the MIP models developed in Chapter 4.2.3.

We start by identifying drivers of computational effort. Here, we obtained the same insights for each model formulation. For a given κ , ζ and θ the computational effort increases with η – an expected effect since more pairs result in more variables and more constraints. As mentioned before more pairs lead to less candidates which reduces the number of variables and constraints. Nevertheless, not surprisingly the former effect turns out to be stronger than the latter. Similarly, computing times increase with κ for given values of ζ , η and θ . Again, this does not come as a surprise since more crane types result in more candidates which, in turn, lead to more variables and constraints. With respect to the available crane types, it can be stated (from the single instances) that the larger the types available are, the higher the computing times are. This may be explained by a higher potential for crane interferences, i. e. more tuples and triples of candidates to be considered leading to more variables and constraints. We observe a similar effect when decreasing θ since the grid's granularity affects the number of candidates. Note that reducing θ by 50% for a given site results in the quadruple number of grid points. Hence, in a study with $\theta = 5$ instead of $\theta = 10$ we observed both increasing computing times and, more importantly, out-of-memory issues even for small instances (with regard to κ , η and ζ). An interesting fact is that for a given κ , η and θ , average computing times are maximal for $\zeta = 8$, i. e. for mid-sized sites. We can only speculate that this is attributed to the instance classes' geometric structures. On large sites, pairs are more dispersed in comparison to small sites. Thus, on large sites crane decisions tend to decompose whereas on small sites, few cranes suffice to cover all pairs. On medium-size sites we need a considerable number of cranes and the instances tend to not decompose. Hence, interference of a considerable number of cranes is to be handled which might complicate decisions.

Now, we compare the different MIP formulations developed in Chapter 4.2.3. With one exception (instance class 1-40-5-10), the models with implicit choice of sectors, i. e. UN-IM and D-IM, outperform the corresponding ones with explicit choice of sectors, i. e. UN-EX and D-EX, with respect to both, average and maximum computing times. Additionally, UN-IM dominates D-IM regarding average and maximum computing times with one (2-30-8-10) and three exceptions (3-30-10-10, 4-20-5-10 and 4-40-8-10), respectively. Such a consistent pattern cannot be observed for models UN-EX and D-EX.

As we enforced a time limit of one hour, optimal solutions could not be found with every MIP for every single instance although feasible solutions (if existent) were found by each MIP for all instances. Whereas UN-IM and D-IM never reached the limit and found optimal solutions for all instances, UN-EX and D-EX failed to find an optimal solution for 14 and 19 instances, respectively. In cases where only a feasible solution could be obtained, objective values are between 400% and 800% of the optimal objective value.

Concluding the analysis of the models, model UN-IM usually dominates all other variants. Note that the MIP models employing directed coverage variables seem to provide more information than the others (by prescribing the direction of load-carrying moves). However, we can easily check which direction is feasible once we obtained a solution to a MIP model employing undirected coverage variables

and, therefore, can derive this seemingly additional piece of information.

The previous study shows that the best-performing MIP UN-IM is capable of solving the most challenging instances with $\theta = 10$ within a few minutes. As there are up to 40 demand areas on-site this size of instances is relevant for real-world applications. One may even argue that this is too large a number of buildings for real-world construction sites. However, even if there are usually less than 40 buildings on real-world construction sites, the granularity of representing such buildings could be increased, i. e. a building could be partitioned into several lots each constituting a demand site. Nevertheless, with regard to the granularity of the grid the applicability of the MIP models seems to be restricted.”

4.3.3 Evaluation of the Branch and Bound Approach

In the previous part, MIP UN-IM has been identified as the best-performing representation of TCSP-GRID. We, now, report the results of the comparison between MIP UN-IM and the B & B approach developed in Chapter 4.2.4. Since, in Chapter 4.3.2, it was found that the MIP-based approach could handle instances with a grid granularity of $\theta = 10$ quite well, but struggled with the finer granularity of $\theta = 5$ we will focus the following analysis on $\theta = 5$ and just briefly summarize that the B & B approach clearly outperforms the MIP-based approach with regard to computing times with only very few exceptions for $\theta = 10$. The results for $\theta = 5$ are summarized in Table 4.4. For both approaches, average and maximum computing times are given as well as the number of instances having been aborted due to reaching the time limit of half an hour (1,800 seconds) or running out of memory. The last two columns report the B & B approach’s average and maximum percentage savings with regard to computing times for those instances which have not been aborted and with regard to costs (objective value) for those instances for which a feasible solution was found when aborted.

Since drivers of computational effort have already been identified in Chapter 4.3.2 the focus in our analysis is on comparing the approaches’ performances, now.

A comparison of computing times reveals that the B & B approach significantly outperforms standard solver CPLEX in the MIP-based approach. In total, there have been only six instances where CPLEX was faster than B & B with CPLEX achieving an average percentage time saving of 46.59% compared to B & B. On the whole set of instances, B & B has only been aborted in five cases due to reaching the time limit, finding a feasible solution within several seconds in each case with an average percentage gap compared to the lower bound of 4.75%. CPLEX, in contrast, reached the time limit for 88 instances, failing to provide a feasible solution for 32 of them. As can be seen in the last column of Table 4.4 the objective function-effect of poor feasible solutions found by CPLEX until aborted is significant, as well, which, in practical applications, can easily be worth several thousand dollars.

Furthermore, the B & B approach resolves the memory issues observed employing the MIP-based approach since the former has never been aborted due to memory issues whereas the latter stopped 42 times.

With regard to further enhancing the computational performance of the B & B approach we will now have a closer look at the lower and upper bounds found during the computations. As already mentioned above, for the instances aborted, B & B finds feasible solutions with a marginal gap with respect to

instance class	MIP UN-IM				B & B				$\left(\frac{\text{B \& B} - \text{MIP}}{\text{MIP}}\right) \cdot 100\%$			
	computing time		instances aborted		computing time		instances aborted		computing time		objective value	
	avg.	max.	time limit	memory	avg.	max.	time limit	memory	avg.	max.	avg.	max.
1-20-5-5	43.42	93.47	0	0	1.80	3.36	0	0	-84.12	-96.49	—	—
1-20-8-5	170.34	659.33	0	0	21.62	192.34	0	0	-76.01	-97.49	—	—
1-20-10-5	173.53	656.20	0	0	10.98	26.14	0	0	-61.91	-96.47	—	—
1-30-5-5	214.47	523.45	0	0	3.44	8.93	0	0	-89.97	-99.06	—	—
1-30-8-5	529.89	1,800.00	3	0	8.01	21.70	0	0	-77.05	-98.77	-25.00	-25.00
1-30-10-5	279.41	1,800.00	2	0	14.26	37.45	0	0	-74.95	-94.92	-50.00	-50.00
1-40-5-5	627.58	1,374.35	0	0	2.98	6.11	0	0	-84.45	-99.56	—	—
1-40-8-5	495.88	1,800.00	2	0	13.00	55.69	0	0	-79.79	-99.19	-25.00	-25.00
1-40-10-5	468.71	1,800.00	2	0	20.22	93.49	0	0	-82.15	-99.91	-37.50	-37.50
2-20-5-5	68.59	264.11	0	0	2.62	8.50	0	0	-92.01	-98.64	—	—
2-20-8-5	213.45	968.20	0	0	10.45	30.27	0	0	-85.25	-97.98	—	—
2-20-10-5	164.13	735.48	0	0	13.85	52.79	0	0	-71.91	-96.41	—	—
2-30-5-5	402.47	1,800.00	1	0	8.60	55.89	0	0	-93.77	-99.49	—	—
2-30-8-5	798.80	1,800.00	5	0	13.84	35.36	0	0	-89.97	-98.19	-49.96	-62.73
2-30-10-5	578.18	1,800.00	3	0	22.69	65.52	0	0	-72.46	-98.43	-46.53	-57.58
2-40-5-5	762.21	1,800.00	4	0	4.25	9.11	0	0	-98.32	-99.59	-40.36	-52.38
2-40-8-5	388.30	1,800.00	5	0	16.47	41.10	0	0	-90.65	-97.68	-1.22	-1.22
2-40-10-5	799.34	1,800.00	4	3	163.17	1,800.00	1	0	-90.92	-98.22	-46.91	-50.91
3-20-5-5	100.85	235.94	0	0	5.03	9.33	0	0	-93.76	-98.30	—	—
3-20-8-5	557.39	1,709.20	0	0	21.19	42.11	0	0	-92.56	-98.40	—	—
3-20-10-5	326.47	1,546.19	0	0	31.90	61.87	0	0	-82.30	-96.26	—	—
3-30-5-5	547.99	1,800.00	2	0	13.03	91.43	0	0	-97.91	-98.96	-15.00	-15.00
3-30-8-5	1,207.03	1,800.00	7	0	26.37	48.72	0	0	-95.05	-98.36	-53.35	-68.00
3-30-10-5	945.54	1,800.00	5	0	162.36	1,800.00	1	0	-92.38	-96.91	-49.43	-50.00
3-40-5-5	1,152.24	1,800.00	2	3	8.15	15.12	0	0	-99.15	-99.64	-56.38	-71.74
3-40-8-5	1,015.52	1,800.00	4	6	27.99	56.05	0	0	-94.02	-95.71	-60.14	-64.48
3-40-10-5	1,277.11	1,800.00	5	7	324.92	1,800.00	2	0	-88.90	-88.90	-45.37	-46.00
4-20-5-5	128.10	204.41	0	0	12.89	81.83	0	0	-79.34	-96.63	—	—
4-20-8-5	811.35	1,800.00	1	0	34.58	42.52	0	0	-91.87	-97.51	-69.57	-69.57
4-20-10-5	636.99	1,800.00	1	0	57.69	66.18	0	0	-84.81	-95.24	-71.88	-71.88
4-30-5-5	758.76	1,800.00	1	0	36.1	215.09	0	0	-95.10	-99.18	—	—
4-30-8-5	1,783.27	1,800.00	9	4	41.71	55.11	0	0	-95.18	-95.18	-58.29	-70.49
4-30-10-5	1,209.70	1,800.00	7	0	75.83	135.28	0	0	-92.39	-95.12	-56.39	-65.03
4-40-5-5	1,680.53	1,800.00	6	0	11.13	15.70	0	0	-99.31	-99.50	-45.08	-66.67
4-40-8-5	1,800.00	1,800.00	3	9	49.94	57.19	0	0	—	—	-59.38	-59.38
4-40-10-5	1,800.00	1,800.00	4	9	220.91	1,800.00	1	0	—	—	-69.03	-69.03

Table 4.4: Computational results of MIP UN-IM and B&B for grid granularity $\theta = 5$

the lower bound. For the remaining instances, the optimal solution was obtained within a few seconds (with one exception where it took about 200 seconds to find the optimal solution). As, then, can be concluded from the summary in Table 4.4 proving optimality consumed quite an amount of time. One may suspect that this is attributed to poor lower bounds. Indeed, there are instances with lower bounds of 20% to 25% gap with regard to the optimal solution, but this only holds for very few instances. In general, this gap is rather small amounting to less than 5%. However, despite of these tight bounds, many nodes in the B & B tree are evaluated.

5 Concluding Remarks and Outlook

Finally, the thesis' content and contribution are summarized (Chapter 5.1) and ideas for future research are presented (Chapter 5.2).

5.1 Conclusion

This thesis was concerned with the selection and on-site location of fixed-base tower cranes on construction sites. Although tower cranes are a key factor in construction projects from both an economic and operational perspective an extensive literature review revealed that research on the topic has been rather limited to date. The current work presented two new concisely defined problem settings capturing relevant aspects of tower crane selection and location in real-world applications. With these problems defined and approaches developed to solve the respective problems, this research adds to both the application-oriented and the methodological branch of literature.

The first problem, TCSPP, was concerned with cost-optimally selecting tower cranes from a given set of types with different specifications, i. e. cost, operating height and load weight-dependent operating radii, and locating them on a polygonal construction site in order to establish transport relations for given pairs of polygonal supply and demand areas. Such a transport relation was considered to be established if each point of both the supply and the demand area was within one crane's operating radius for the maximum weight to be lifted and the crane had sufficient height for serving the demand area with its given height. Crane locations could be chosen arbitrarily within the site polygon as long as no crane was positioned within an infeasible area. The problem was proven to be strongly NP-complete. The approach developed proved that the by nature infinite set of position candidates within the site polygon can be boiled down to a finite set without loss of optimality which, then, allows to represent the problem as classical set cover problem and to apply corresponding solution techniques developed over decades. This set cover problem was shown to be solvable by standard solver CPLEX within several minutes even for rather challenging instance sizes.

The second problem, TCSPP-GRID, considered basically the same construction site setting like the first one, but added interdependencies between cranes and between cranes and on-site structures to the problem. These interdependencies included minimum distance requirements between single cranes and interferences of cranes and of cranes with on-site objects of sufficient height. Consequently, in contrast to the first problem, cranes could not be located independently which required a new approach to solving the problem. Thus, the problem introduced a grid laid over the site polygon with only the grid's intersection points being potential crane locations, i. e. a discretization of space with potential

loss of optimality took place. The problem was proven to be strongly NP-hard. Two approaches were developed for solving the problem: the first one was a MIP-based approach employing a standard solver (CPLEX in this thesis), the second one was a branch and bound approach. For the MIP-based approach four MIP formulations differing in the number of constraints and variables were proposed and computationally tested. A computational study revealed that this approach particularly struggled with fine grid granularities, i. e. small horizontal and vertical distances between the grid's intersection points. The branch and bound approach exploiting structural problem knowledge relieved the issue and competed favorable against the MIP-based approach.

5.2 Ideas for Future Research

Note: this chapter is based on Briskorn and Dienstknecht [10] and Briskorn and Dienstknecht [12].

One way to extend TCSPP and TCSPP-GRID is to include new features.

This may be a time dimension and, related to that, more complex cost structures. In Chapter 3.4, a rather simple approach to time dynamics has been presented. However, this may be enhanced, e. g. as the site changes over time certain supply or demand or forbidden areas may be valid for certain intervals and cranes may have to be re-located in order to still comply with the current site requirements. Then cranes need to be erected, dismantled and re-located with each of these processes being charged with a certain cost. Additionally, operating cost may be considered as it might be cost-optimal to dismantle and re-erect a crane instead of keeping it in-place and operating it all the time.

Crane capacities, i. e. maximum work loads for each crane, can be added as already indicated in Chapter 3.4. Along with this, it may be allowed to split servicing a pair of demand and supply area between multiple cranes. In addition, not only the single cranes' capacities may be limited, but the number of cranes of each type being available.

From both a practical and a theoretical point of view, enhancements like material hand-overs, i. e. supply chains of multiple cranes, and lift path planning / checking are interesting. Clearly, there are downsides of material hand-overs such as increased crane conflict potential and increased material handling effort. However, it may still be cost-optimal due to smaller (and cheaper) cranes being capable of establishing transport relations or it may simply be inevitable as large cranes cannot be located on suggested sites. In the work at hand, transport relations were considered established if there was an uninterrupted path between every point of the supply site and every point of the corresponding demand site. In practice, there may be requirements regarding such a lift path, e. g. the path has to have a certain width in order to really make materials transportable.

Ultimately, it may be desired to integratedly plan the whole site layout, not limited to cranes. Steps for reaching that goal can be the integrated location of cranes and supply sites (which, in turn, may require planning the sites' situation, orientation or dimensions) or the integrated planning of several crane types, i. e. not limited to fixed-base tower cranes, but considering other types of tower cranes or even mobile cranes. Depending on the type of crane and its operating characteristics, more complex geometric considerations are required.

Closely related to the last point mentioned above is the matter of how objects are represented. In

the work at hand, all structures were represented by simple polygons and cranes were represented by points. This may be inappropriate – particularly the point-based representation of cranes as cranes in reality have a two-dimensional footprint.

With regard to the approaches proposed, the development of new more efficient procedures may be a promising, but challenging task. One way to achieve this goal could be the exploitation of geometric problem properties. Regarding the branch and bound approach presented in Chapter 4.2.4, although the computational study revealed the lower and the upper bounds to be quite tight in general, the lower bound had a significant gap with respect to the optimal solution for some instances and, furthermore, there was no guarantee of finding a feasible initial solution as an upper bound.

The ultimate goal, however, will be to combine the continuous representation of space from TCSPP with the operational restrictions from TCSPP-GRID. Then, again, solution procedures may be developed, but it would be even more interesting to identify structural properties that, e. g., allow to boil down the solution space without loss of optimality as provided for TCSPP in this thesis.

All these aspects mentioned above – in combination with construction cranes' high relevance from both an operational and economic perspective – make research regarding cranes an interesting subject as it offers the opportunity to develop theoretical insights and to provide decision support for real-world problem settings.

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