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On deformations of the direct sum of a regular and another indecomposable module over a tame quiver algebra

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Dissertation

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1 Introduction

Given an associative algebra A of dimension r over an algebraically closed field k, we can identify the finite dimensional module structures to a fixed dimension d in a very natural way with the points of an affine variety Mod_A^d, by considering r-tuples of $d \times d$-matrices $m = (m_1, \ldots, m_r)$ representing the multiplication with the elements of a fixed basis of A. The general linear group $\mathrm{Gl}_d(k)$ acts on Mod_A^d by conjugation and every orbit $\mathcal{O}(m)$ for $m \in \mathrm{Mod}_A^d$ corresponds to an isomorphism class of a module structure M on k^d given by m. In this situation we do not strictly differ the isomorphism class of M and the module M itself in the sense, that we use for both the same notation. On account of the above identification we write $\mathcal{O}(M)$ for $\mathcal{O}(m)$ too. Since $\mathrm{Gl}_d(k)$ is an algebraic group, we know that the closure of an orbit $\mathcal{O}(M)$ is a union of the orbit itself together with orbits of strictly lower dimension. Our aim is to give a characterization of the isomorphism classes, which belong to a given orbit closure. This problem is called THE DEGENERATION PROBLEM and we say, that a module N is a DEGENERATION of a module M, respectively M is a DEFORMATION of N, if $\mathcal{O}(N)$ belongs in the closure of $\mathcal{O}(M)$ and denote this fact by $M \leq_{\deg} N$. This gives a partial order on the isomorphism classes of modules of a fixed dimension.
For arbitrary algebras the degeneration problem is very hard even though Zwara has characterized $M \leq_{\deg} N$ by the existence of an exact sequence $0 \to Z \to Z \oplus M \to N \to 0$. But for certain algebras one can describe the degenerations explicitly because the degeneration order is equivalent to two other partial orders defined on the set of isomorphism classes of modules of fixed dimension. We define

- $M \leq_{ext} N$ $:\Leftrightarrow$ there are modules M_i, U_i, V_i and short exact sequences $0 \to U_i \to M_i \to V_i \to 0$ such that $M = M_1$, $M_{i+1} = U_i \oplus V_i$, $1 \leq i \leq s$, and $N = M_{s+1}$ for some natural number s.

- $M \leq N$ $:\Leftrightarrow$ $\dim_k \mathrm{Hom}_{\mathrm{A}}(M, X) \leq \dim_k \mathrm{Hom}_{\mathrm{A}}(N, X)$ holds for all modules X. As from now on we denote $\dim_k \mathrm{Hom}_{\mathrm{A}}(M, X)$ shorter by $[M, X]$ and $\dim_k \mathrm{Ext}^1_{\mathrm{A}}(M, X)$ by $[M, X]^1$.

The implications $M \leq_{ext} N \Rightarrow M \leq_{\deg} N \Rightarrow M \leq N$ hold for all algebras. By [1] and [12] the reverse implications are also true for representation finite and tame quiver algebras. So all three partial orders coincide in that case. Moreover, the partial order $\leq$ can really be computed, because all indecomposables and all homomorphism spaces are known. In particular, one can study the minimal degenerations $M < N$, where no module L satisfies $M < L < N$. Here one can by [5] reduce to the case where M and N are disjoint, i.e. have no common direct summand. Then there is a short exact sequence $0 \to U \to M \to V \to 0$ with indecomposable modules U and V whose direct sum is N. So we are led to study more general deformations M of the direct sum $U \oplus V$ of two indecomposables. The following questions are natural:

1. What is the codimension of the minimal deformations?
2. What deformations are extensions?
3. What singularities occur at $U \oplus V$ for minimal deformations?

For representation-finite quivers all three questions are answered in [9]: The codimension is always one, all deformations are extensions and there are no singularities. For tame quivers there are several cases depending on the nature of U and V. If both are preprojective, the codimension is bounded by two and all deformations are extensions as shown in [5]. So by [11] the singularities are known.

This article deals with the case where at least one of the modules say V is regular. In case U is regular, the codimensions are bounded by two and the singularities are known. This is already contained in Kempken (see [7]). We give in addition a precise criterion when a deformation is an extension.

Up to duality, we can assume know that U is preprojective. We show by theoretical means that we can restrict to the case where the dimension of V is less or equal to the null-root without affecting the codimension or the type of singularity. Thus we are left with finitely many cases that we have analyzed completely with the help of a computer. The codimensions are again bounded by two, so that the minimal singularities are known again by [11]. Thanks to Lemma 4.2 and Theorem 4.1, it is enough to look at the minimal deformations of $U \oplus V$, with $\underline{\dim}\, V$ is the null-root, to get the result of the bounded codimensions. The degeneration diagrams for this cases are given in the appendix.

In subsection 4.4 Theorem 4.9 we give an answer for question 2 for this case, but again it is not only done with theoretical arguments. In most of the cases (namely for all U, with defect$(U) \geq -4$) we marked the deformations, which are not extensions of $U \oplus V$, in the degeneration diagrams (see the appendix).

Finally we describe a tranversal slice that allows us (in principle) to give equations for the singularities of all deformations, but we have performed the necessary tedious calculation only in a view cases. Unfortunately, we did not find a new interesting singularity.

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2 Basics about tame path algebras

2.1 Quivers and their representations

A QUIVER $Q = (Q_0, Q_1)$ consists a set of vertices Q_0 and a set of arrows Q_1. For $\alpha \in Q_1$ we denote by $s(\alpha) \in Q_0$ its starting point and by $n(\alpha) \in Q_0$ its end. A nonempty PATH of length r from x to y is a sequence $\alpha_r \dots \alpha_1$ of arrows with $s(\alpha_1) = x$, $n(\alpha_r) = y$ and $s(\alpha_{i+1}) = n(\alpha_i)$ for $i < r$. Additionally there is the empty path e_i (of lentgh 0) for each vertex i.

For a field k the PATH ALGEBRA kQ has as a basis the set of paths and the product of two paths w_1 and w_2 is the composed path, if possible, otherwise 0. Now we have the following statement:

The path algebra kQ is finite dimensional if and only if

1.) the quiver Q is finite that is Q_0 and Q_1 are finite sets and

2.) there are no oriented cycles.

If Q is a quiver and k is a field, a finite dimensional REPRESENTATION $V = \{V_i, f_\alpha | i \in Q_0,\ \alpha \in Q_1\}$ of Q consists of finite dimensional vector spaces V_i for each $i \in Q_0$ and $f_\alpha \in \mathrm{Hom}(V_{s(\alpha)}, V_{b(\alpha)})$ for each $\alpha \in Q_1$. We get a morphism between two representations by the obvious commutativity conditions. This defines a category Rep_Q of representations of Q. An interesting aspect is that this category is equivalent to the category Mod_{kQ} of finite dimensional kQ-modules.

For a quiver $Q = (Q_0, Q_1)$, where $Q_0 = \{1, \dots, n\}$, the EULER FORM or HOMOLOGICAL BILINEAR FORM

$$\langle -, - \rangle : \mathbb{Z}^n \times \mathbb{Z}^n \to \mathbb{Z}$$

is defined by $\langle x, y \rangle = \sum_{i \in Q_0} x_i y_i - \sum_{\alpha \in Q_1} x_{s(\alpha)} y_{n(\alpha)}$. This is a (non symmetric) bilinear form.

Thus we get the TITS FORM $q_Q = q$ by $q(z) = \langle z, z \rangle$, which is an integral quadratic form with corresponding symmetric bilinear form (-,-) defined by $(x, y) = \langle x, y \rangle + \langle y, x \rangle$.

The radical rad$q$ of q is the subspace $\{y \in \mathbb{Z}^n | (y, -) = 0\}$.

The DIMENSION VECTOR of a module X is defined as $\underline{dim} X = (\dim X(i))_{i \in Q_0}$.

Theorem 2.1:

Let Q be a connected quiver with path algebra $A = kQ$ for some field k.

- *A is representation finite if and only if q_Q is positive definite. The underlying graph of Q is a Dynkin diagram $A_n (n \geq 1), D_n (n \geq 4), E_6, E_7, E_8$ in this case.*

- q_Q *is positive semidefinite but not positive definite if and only if the underlaying graph of* Q *is an Euclidean (or extended Dynkin) graph* $\tilde{A}_n (n \geq 1)$, $\tilde{D}_n (n \geq 4), \tilde{E}_6, \tilde{E}_7, \tilde{E}_8$. *In this case* A *and* Q *are called tame and the radical of* q *is one dimensional with a unique strictly positive generator* $\underline{\delta}$, *having smallest component 1.*

- *In all other cases* q_Q *is indefinite. Then* Q *is called a wild quiver.*

For two kQ-modules X and Y we have

- $\langle \underline{\dim}\, X, \underline{\dim}\, Y \rangle = [X, Y] - [X, Y]^1$

- $\langle \underline{\dim}\, P(i), x \rangle = \langle x, \underline{\dim}\, I(i) \rangle = x_i$ for $x \in \mathbb{Z}^n$, where $P(i)$, respectively $I(i)$ denotes the projective indecomposable to the vertex $i \in Q_0$, respectively the injective.

2.2 Auslander-Reiten Translation

For an $A-$module U the Auslander-Reiten translation is given by $\tau U = \mathrm{DTr} U$, where D means the duality and Tr is defined by the exact sequence $\mathrm{Hom_A}(P_0, A) \to \mathrm{Hom_A}(P_1, A) \to \mathrm{Tr}U \to 0$ induced by a minimal projective solution $P_1 \to P_0 \to U \to 0$. We get τ^- by TrD. It holds, that τ gives a bijection from the non-projective indecomposables to the non-injective indecomposables with inverse τ^-.
There are two nice and important formulas due to Auslander-Reiten (see [8]):
Namely for two modules X and Y with the same dimension vector $\underline{\dim}\, X = \underline{\dim}\, Y$ we get for all indecomposables T

$$[X, T] - [Y, T] = [\tau^- T, X] - [\tau^- T, Y].$$

Hence, $X \leq Y$ is also equivalent to $[T, X] \leq [T, Y]$ for all indecomposables T. Furthermore we get for a path algebra A always

$$\mathrm{D}\,\mathrm{Hom_A}(X, Y) \simeq \mathrm{Ext}^1_{\mathrm{A}}(Y, \tau X).$$

2.3 Types of indecomposables

We have three types of indecomposable modules X for a path algebra.

- X is preprojective $\Leftrightarrow \tau^i X = 0$ for $i >> 0$
- X is preinjective $\Leftrightarrow \tau^{-i} X = 0$ for $i >> 0$
- X is regular $\Leftrightarrow \tau^i X \neq 0$ for all $i \in \mathbb{Z}$

and we say, that a module is preprojective, regular or preinjective if each indecomposable summand is. Thus every module X decomposes in its preprojective part $X_{\mathcal{P}}$, regular part $X_{\mathcal{R}}$ and preinjective part $X_{\mathcal{Q}}$, i.e. we have $X = X_{\mathcal{P}} \oplus X_{\mathcal{R}} \oplus X_{\mathcal{Q}}$. Furthermore it holds the following result.

Lemma 2.2:

Let X, Y be indecomposable.

- *If Y is preprojective and X is not, then* $\mathrm{Hom}_{\mathrm{A}}(X, Y) = 0$ *and* $\mathrm{Ext}^1_{\mathrm{A}}(Y, X) = 0$
- *If Y is preinjective and X is not, then* $\mathrm{Hom}_{\mathrm{A}}(Y, X) = 0$ *and* $\mathrm{Ext}^1_{\mathrm{A}}(X, Y) = 0$

There is a nice characterization for modules being preprojective, regular or injective over tame quivers.

So from now on A denotes a tame path algebra.

With $\underline{\delta}$ from Theorem 2.1 we get the definition of the defect.

Definition 2.3:
The defect $\partial(X)$ of a module X is $\langle \underline{\delta}, \underline{\dim}\, X \rangle = - \langle \underline{\dim}\, X, \underline{\delta} \rangle$.

Now it holds that

- If X is preprojective, then $\partial(X) < 0$,
- If X is regular, then $\partial(X) = 0$,
- If X is preinjective, then $\partial(X) > 0$.

Finally, we collect some properties of regular modules

- The category $\mathcal{R}$ of regular modules is the product of uniserial categories $\mathcal{T}_\mu$, $\mu \in \mathbb{P}^1$, called TUBES.

- Each tube $\mathcal{T}_\mu$ contains only finitely many, say p_μ, simples that are of the form $\tau^i X$, $1 \leq i \leq p_\mu$, for some simple X. We have $\sum_{i=1}^{p_\mu} \underline{\dim}\, \tau^i X = \underline{\delta}$ and $\tau^{p_\mu} Y \simeq Y$ for all Y in $\mathcal{T}_\mu$. p_μ is called the PERIOD of $\mathcal{T}_\mu$.
- There are at most 3 tubes with $p_\mu > 1$. These tubes are called EXCEPTIONAL, the others HOMOGENEOUS.

3 Deformations of $U \oplus V$ for regular modules U and V

3.1 First observations and notations

The category $\mathcal{R}$ of regular modules is the product of uniserial categories $\mathcal{T}_\mu$. So non-trivial deformations can only occur if U and V belong to the same uniserial category $\mathcal{T}_\mu$, where the Auslander-Reiten quiver is a stable tube of rank p. To describe an indecomposable module $X \in \mathcal{T}_\mu$ you only have to know the regular socle and length of X. Namely for any simple module $S \in \mathcal{T}_\mu$ there is a unique infinite path of monomorphisms

$$S = S(1) \hookrightarrow S(2) \hookrightarrow S(3) \hookrightarrow \dots .$$

So every indecomposable module $X \in \mathcal{T}_\mu$ has up to isomorphism a unique form $X = S(l)$ for a simple regular module $S \in \mathcal{T}_\mu$ and we call $l = l(X)$ the regular length of X and S the regular socle of X. It is convenient to define $S(0) = 0$.
It is well-known that $\mathcal{T}_\mu$ is equivalent to the category $\mathcal{N}_p$ of nilpotent representations of an oriented cycle with p points and arrows. The equivalence preserves the order $\leq$, the existence of short exact sequences and the codimensions of the orbits. So we can work with the category $\mathcal{N}_p$ instead of $\mathcal{T}_\mu$ and reversely the same. This makes the notations easier replacing the regular socle by the socle, the regular top by the top etc. Furthermore the category $\mathcal{N}_p$ is obviously self-dual.
Now the degenerations inside $\mathcal{N}_p$ are already given in [7], but for the convenience of the reader we include here a complete description of all deformations M of $U \oplus V$ and we determine, which of them are in the middle of an exact sequence with end terms U and V.
In the first instance we declare for a module $X \in \mathcal{T}_\mu$ the following: Let S be any simple regular module in $\mathcal{T}_\mu$, then we denote by $l_S(X)$ the multiplicity of S as a composition factor in the composition series of X in the category $\mathcal{T}_\mu$. With this we get a nice description for the dimension of $\mathrm{Hom}_\mathrm{A}(X, Y)$ for indecomposable modules $X,\ Y \in \mathcal{T}_\mu$ (see for it Lemma 5.1 in [10]).

Lemma 3.1:

Using the above notations, if $l(Y) \geq l(X)$, then we get

1. $[X, Y] = l_{\mathrm{Soc}(Y)}(X)$,

2. $[Y,X] = l_{\mathrm{Top}(Y)}(X)$.

The next statement gives a precise criterion when a deformation is an extension of U and V and shows that the codimensions are bounded by two.

Lemma 3.2:

Let $U = S_i(k)$ and $V = S_j(l)$ be indecomposables in $\mathcal{T}_\mu$ and let r be the minimal length of an indecomposable module W with $\mathrm{Top}(W) = \mathrm{Top}(V)$ and $\mathrm{Soc}(W) = \tau^- \mathrm{Top}(U)$. Then the partially ordered sets $\mathcal{S}(V,U) = \{m \mid m \in \mathbb{N}\ ,\ l \geq r + mp > l - k\}$ and $\mathcal{E}(V,U) = \{M \mid M < U \oplus V\ ,\ \exists$ exact sequence $0 \to U \to M \to V \to 0\}$ are in bijection under the order-preserving map $m \mapsto S_i(k+r+mp) \oplus S_j(l-r-mp)$.
For the unique minimal element M in $\mathcal{E}(V,U)$ we have

$$\mathit{codim}(U \oplus V, M) = \begin{cases} 2 & \text{, for } l(U) \geq l(V) \text{ and } \mathrm{Top}(U) = \mathrm{Top}(V) \\ 2 & \text{, for } l(U) < l(V) \text{ and } \mathrm{Soc}(U) = \mathrm{Soc}(V) \\ 1 & \text{, otherwise.} \end{cases}$$

Proof. First we show that $S_i(k+r+mp) \oplus S_j(l-r-mp)$ belongs to $\mathcal{E}(V,U)$ for $m \in \mathcal{S}(V,U)$. Set $M_1 = S_i(k+r+mp)$, then we have $\mathrm{Soc}(M_1) = U$ and $l(M_1) > l(U)$. So there is a proper monomorphism $\varepsilon_1 : U \to M_1$. Similarly we have $\mathrm{Top}(M_1) = \mathrm{Top}(V)$, $l(M_1) > l(V)$ and a proper epimorphism $\pi_1 : M_1 \to V$. For $l = r + mp$ there is an obvious exact sequence $0 \to U \to M_1 \to V \to 0$. In the other case $K = \ker \pi_1$ is a proper submodule of U with canonical projection $\varepsilon_2 : U \to U/K$. Set $M_2 = U/K$ and $\varepsilon = \binom{\varepsilon_1}{\varepsilon_2}$ and look at the exact sequence $0 \to U \xrightarrow{\varepsilon} M_1 \oplus M_2 \xrightarrow{\pi = (\pi_1\ \pi_2)} C \to 0$. By construction, $\mathrm{Top}(\varepsilon_2)\ :\ \mathrm{Top}(U) \to \mathrm{Top}(M_2)$ is an isomorphism, whence $\mathrm{Top}(U) \to \mathrm{Top}(M_1) \oplus \mathrm{Top}(M_2) \to \mathrm{Top}(C) \to 0$ is exact and $\mathrm{Top}(\pi_1)$ is an isomorphism. Counting lengths we see that $C = S_j(l)$ and $M_2 = S_j(l-r-mp)$.
To see that the map is surjective we take a non-split exact sequence $0 \to U \xrightarrow{\varepsilon} M \xrightarrow{\pi} V \to 0$. The induced exact sequence $0 \to \mathrm{Soc}(U) \to \mathrm{Soc}(M) \to \mathrm{Soc}(V)$ shows $l(\mathrm{Soc}(M)) \leq l(\mathrm{Soc}(U)) + l(\mathrm{Soc}(V)) = 2$. So M is indecomposable or the direct sum $M_1 \oplus M_2$ of two indecomposables. In the first case we have $M = S_i(k+l) = S_i(k+r+mp)$ with $r + mp = l$. So M is in the image of the map. For $M = M_1 \oplus M_2$ we assume $l(M_1) \geq l(M_2)$ and decompose ε and π.Now, $\ker \varepsilon_i \neq 0$ for both i implies directly

$\mathrm{Soc}(U) \subseteq \ker \varepsilon_1 \cap \ker \varepsilon_2 = \ker \varepsilon = 0$, a contradiction. So one ε_i is a monomorphism. Similarly, if $\varepsilon(U)$ is contained in $\mathrm{rad}M$, then $\mathrm{Top}(\varepsilon) = 0$ implies $\mathrm{Top}(M) \simeq \mathrm{Top}(V)$, a contradiction again. So one of the $\mathrm{Top}(\varepsilon_j)$ is an epimorphism, whence one ε_j. The case $i = j$ leads to ε_i is an isomorphism and the sequence splits. Thus we have $i \neq j$ and ε_i is a proper mono, ε_j is a proper epi. Finally the assumption $l(M_1) \geq l(M_2)$ implies $i = 1, j = 2$.
Dually, one π_k is a proper monomorphism, the other is a proper epimorphism. The case π_1 monomorphism and π_2 epimorphism leads to the inequalities $l(U) < l(M_1) \leq l(V) \leq l(M_2) \leq l(U)$, a contradiction.
So we have $M_1 = S_i(k + r + mp)$ and $M_2 = S_j(l - r - mp)$ for some m with $l > r + mp > l - k$. The injectivity of the map is obvious.
Now we take $m < m + 1$ in $\mathcal{S}$ and show that
$X = S_i(k+r+(m+1)p) \oplus S_j(l-r-(m+1)p) < Y = S_i(k+r+mp) \oplus S_j(l-r-mp)$.
Take any indecomposable Z with $l(Z) \leq k+r+mp$. Then the image of any $f : Z \to S_i(k+r+(m+1)p)$ has length $\leq k+r+mp$. So it factorizes through $S_i(k+r+mp) \hookrightarrow S_i(k+r+(m+1)p)$ and we have $[Z, S_i(k+r+mp)] = [Z, S_i(k+r+(m+1)p)]$. Because of $[Z, S_j(l-r-mp)] \geq [Z, S_j(l-r-(m+1)p)]$ the inequality $[Z, X] \leq [Z, Y]$ holds. If $l(Z) > k+r+mp$ we have $[Z, Y] = l_{\mathrm{Top}(Z)}(Y) = l_{\mathrm{Top}(Z)}(X) \geq [Z, X]$ by Lemma 3.1.
Finally, to derive the codimension formula we can assume that $k \geq l$ up to duality. The minimal element in $\mathcal{E}(V, U)$ is then given by $S_i(k+r) \oplus S_j(k-r) = M_1 \oplus M_2$. We calculate the codimension

$$\begin{aligned} c &= [U \oplus V, U \oplus V] - [M, M] \\ &= ([U \oplus V, U \oplus V] - [U \oplus V, M]) + ([U \oplus V, M] - [M, M]) \,. \end{aligned}$$

Since M_1 has maximal length and $0 \to U \to M \to V \to 0$ is exact we have
$[U \oplus V, M_1] = [M, M_1] = l_{\mathrm{Soc}(M_1)}(M)$. The surjection $M_1 \xrightarrow{\pi_1} V$ induces an isomorphism $\mathrm{Hom}_{\mathrm{A}}(V, M_2) \simeq \mathrm{Hom}_{\mathrm{A}}(M_1, M_2)$ because any $f : M_1 \to M_2$ has $\ker \pi_1$ in its kernel because of $l(\ker f) > l(\ker \pi_1)$. The surjection $U \xrightarrow{\varepsilon_2} M_2$ also induces an isomorphism $\mathrm{Hom}_{\mathrm{A}}(U, M_2) \simeq \mathrm{Hom}_{\mathrm{A}}(M_2, M_2)$. So we get $[U \oplus V, M] - [M, M] = 0$.
The inclusion $U \hookrightarrow M_1$ gives $\mathrm{Hom}_{\mathrm{A}}(U, U) \simeq \mathrm{Hom}_{\mathrm{A}}(U, M_1)$ and $\mathrm{Hom}_{\mathrm{A}}(V, U) \simeq \mathrm{Hom}_{\mathrm{A}}(V, M_1)$. We always have $[V, V] - [V, M_2] = l_{\mathrm{Top}(V)}(V) - l_{\mathrm{Top}(V)}(M_2) \leq 1$ and $[V, V] - [V, M_2] = 1$ because the identity does not factor through the inclusion $M_2 \hookrightarrow V$.

So we get

$$\begin{aligned} c &= 1 + [U, V] - [U, M_2] \\ &= 1 + l_{\mathrm{Top}(U)}(V) - l_{\mathrm{Top}(U)}(M_2) = 1 + l_{\mathrm{Top}(U)}(V/M_2) \\ &= 1 + l_{\mathrm{Top}(U)}(M_1/U). \end{aligned}$$

The wanted formula is now obvious. □

3.2 Construction of a complete list of deformations of $U \oplus V$

The proof of Lemma 3.2 has shown that every deformation M of $U \oplus V$ has at most two direct indecomposable summands, more precisely $M = M_1 \oplus M_2$ and for $M_i \neq 0$ it holds:

$$\mathrm{Soc}(M_i) = \mathrm{Soc}(U) \text{ or } \mathrm{Soc}(M_i) = \mathrm{Soc}(V). \tag{1}$$

Dual to this we get for $M_i \neq 0$:

$$\mathrm{Top}(M_i) = \mathrm{Top}(U) \text{ or } \mathrm{Top}(M_i) = \mathrm{Top}(V). \tag{2}$$

By means of the map of Lemma 3.2 and the top-socle-conditions (1) and (2), we are now going to describe, how we get a complete list of deformations of $U \oplus V$:

In the 1st step we construct all modules $M^{(t)} = M_1^{(t)} \oplus M_2^{(t)}$ which comply the conditions of Lemma 3.2. For this we assume that $\mathcal{S}(V, U)$ is not empty and we take the unique minimal element m_{min} of $\mathcal{S}(V, U)$. Then we set

$$M_1^{(1)} \oplus M_2^{(1)} = S_i(k + r + m_{min}p) \oplus S_j(l - r - m_{min}p).$$

To complete the list of the $M_1^{(t)} \oplus M_2^{(t)}$ we only have to decrease the length of $M_2^{(1)}$ by the period p as long as it is possible, which means that it has to be greater or equal to zero. Then clearly $M_1^{(t)}$ has to be a module with length $l(U) + l(V) - l(M_2^{(t)})$ and the same socle as U. Dual to the 1st step we construct in the 2nd step modules $\tilde{M_1}^{(\tilde{t})}$, $\tilde{M_2}^{(\tilde{t})}$ by using the minimal element of $\mathcal{S}(U, V)$.
In the 3rd step we adapt the same procedure from the 1st step to $M_1^{(1)} \oplus M_2^{(1)}$. This yields lists of modules $N^{(s)} = N_1^{(s)} \oplus N_2^{(s)}$ and $\tilde{N_1}^{(\tilde{s})} \oplus \tilde{N_2}^{(\tilde{s})}$, similar in the 4th step with $\tilde{M_1}^{(1)} \oplus \tilde{M_2}^{(1)}$. But now we get again the same modules as in the 3rd step.
Hence we covered all possibilities to get deformations of $U \oplus V$ from supermodules of

U and submodules of V by the $M_1 \oplus M_2$'s and the $\tilde{N}_1 \oplus \tilde{N}_2$'s. Dual the $\tilde{M}_1 \oplus \tilde{M}_2$'s and the $N_1 \oplus N_2$ yields a complete list of deformations of $U \oplus V$, which arose as a result out of submodules of U with supermodules of V. Due to (1) and (2) there can be no further deformations. The comparison of the lists among and with one another leads to the following degeneration diagram (fig. 3.1), at this there are four cases to distinguish.

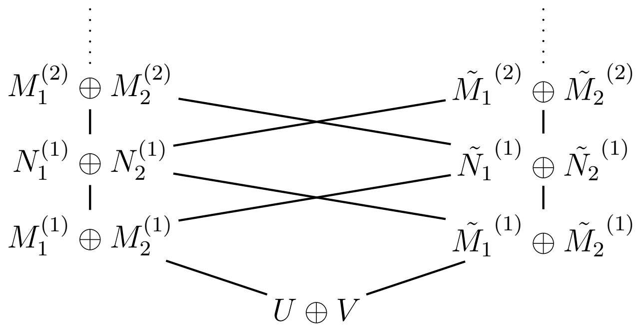


fig. 3.1: Degeneration-Diagram of $U \oplus V$

Case 1: $\mathrm{Soc}(U) \neq \mathrm{Soc}(V)$ and $\mathrm{Top}(U) \neq \mathrm{Top}(V)$ obtain. Then all $M_1^{(t)} \oplus M_2^{(t)}$ and $\tilde{M}_1^{(\tilde{t})} \oplus \tilde{M}_2^{(\tilde{t})}$ exist as middle terms of a short exact sequence between U and V. Following Lemma 3.2 $N_1^{(s)} \oplus N_2^{(s)}$ and $\tilde{N}_1^{(\tilde{s})} \oplus \tilde{N}_2^{(\tilde{s})}$ cannot accomplish that.

Case 2: $\mathrm{Soc}(U) \neq \mathrm{Soc}(V)$ and $\mathrm{Top}(U) = \mathrm{Top}(V)$ obtain. In this case $M_i^{(t)} = N_i^{(t)}$ and $\tilde{M}_i^{(\tilde{t})} = \tilde{N}_i^{(\tilde{t})}$ holds for all t, $\tilde{t}$ and $i \in \{1, 2\}$. Again following Lemma 3.2 we have that $M_1^{(t)} \oplus M_2^{(t)}$ and $\tilde{M}_1^{(\tilde{t})} \oplus \tilde{M}_2^{(\tilde{t})}$ are middle terms of short exact sequences between U and V.

Case 3: $\mathrm{Soc}(U) = \mathrm{Soc}(V)$ and $\mathrm{Top}(U) \neq \mathrm{Top}(V)$ obtain. This time it holds that $M_i^{(t)} = \tilde{N}_j^{(\tilde{t})}$ and $\tilde{M}_i^{(\tilde{t})} = N_j^{(t)}$ for all t, $\tilde{t}$ and $i \neq j$. The residual is analogous to case 2.

Case 4: $\mathrm{Soc}(U) = \mathrm{Soc}(V)$ and $\mathrm{Top}(U) = \mathrm{Top}(V)$ obtain. It now holds that $M_1^{(t)} = \tilde{N}_2^{(\tilde{t})} = \tilde{M}_2^{(\tilde{t})} = N_1^{(t)}$ and $\tilde{M}_1^{(\tilde{t})} = N_2^{(t)} = M_2^{(t)} = \tilde{N}_1^{(\tilde{t})}$. All $M_1^{(t)} \oplus M_2^{(t)}$ are middle terms of a short exact sequence between U and V.

A detailed proof of this needs an elaborate notation and is unpleasantly technical (but not really difficult). So we only want to discuss an example for the case 1 to get the idea of the proof.

3.3 Example

We consider a tube of period 4. Thus we have four simple modules S_1, S_2, S_3, S_4 and get the following figure for the Auslander-Reiten quiver:

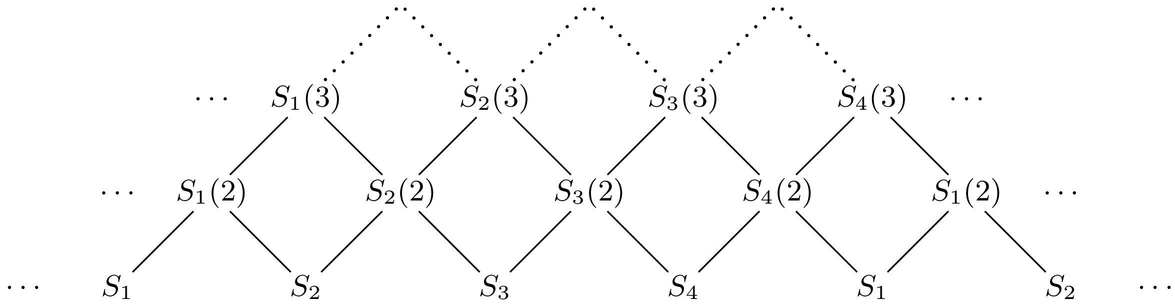


fig. 3.2: Tube of period 4

Now we set

$$U = S_1(10) \text{ and } V = S_3(15) \ ,$$

thus we are in case 1 with $\mathrm{Top}(U) = S_2 = \tau\,\mathrm{Soc}(V)$ and $\mathrm{Top}(V) = S_1$. Figure 3.3 shows all direct summands of the deformations of $U \oplus V$ in the Auslander-Reiten quiver.

In the notations of Lemma 3.2 we have $i = 1, j = 3, k = 10$ and $l = 15$. Thus we get

$$\mathcal{S}(V,U) = \{1,2,3\} \text{ (with } r = 3) \text{ and}$$
$$\mathcal{S}(U,V) = \{0,1,2\} \text{ (with } r = 1).$$

1st step: List of the $M_1^{(t)} \oplus M_2^{(t)}$'s.
The minimal element of $\mathcal{S}(V,U)$ is $m_{min} = 1$, thus we set

$$M_1^{(1)} := S_1(17) \text{ and } M_2^{(1)} := S_3(8).$$

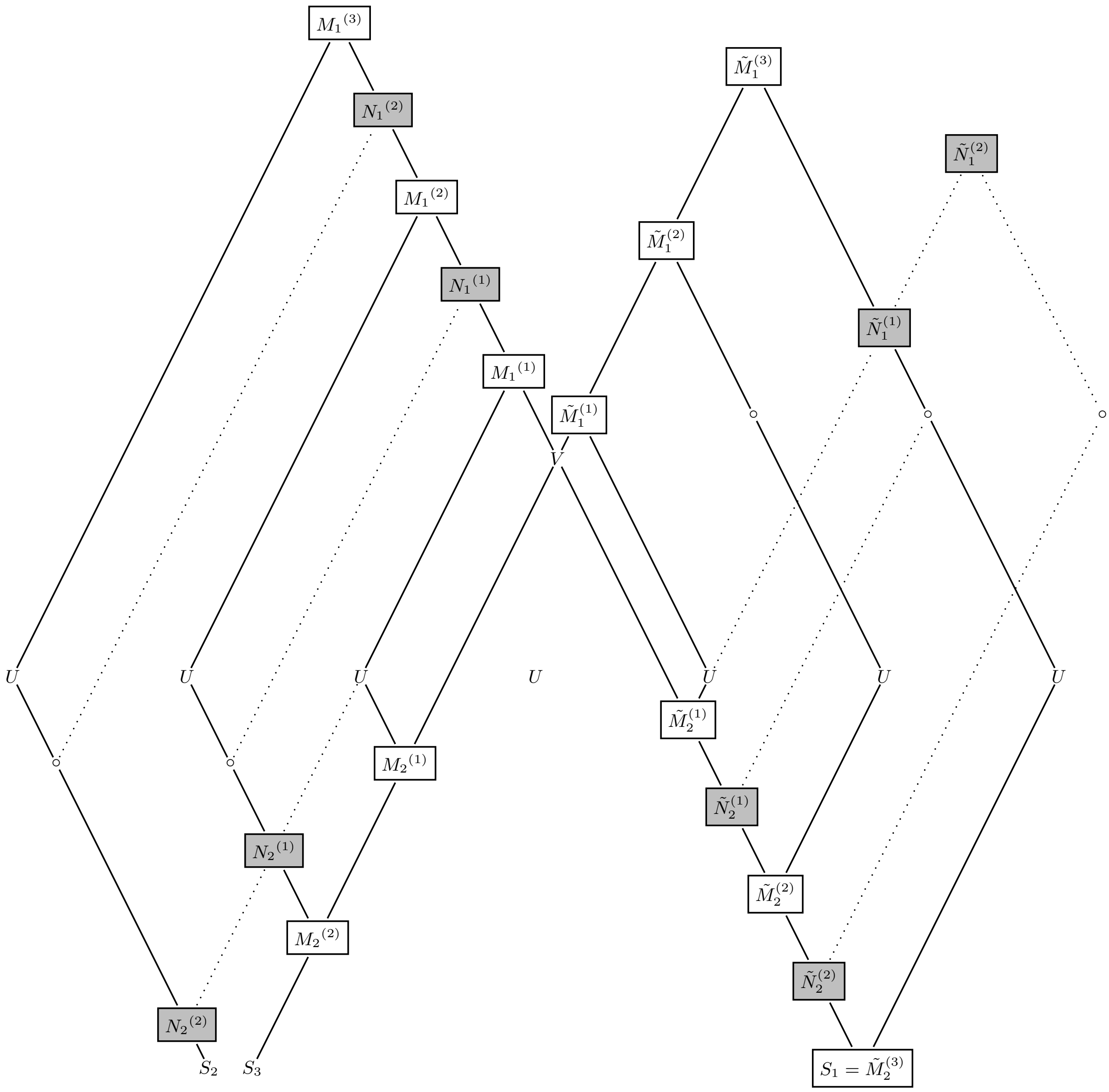


fig. 3.3: Deformations of $U \oplus V$

By decreasing the length of $M_2^{(1)}$ by the period as long as it is possible, we get

$$M_1^{(2)} := S_1(21) \text{ and } M_2^{(2)} := S_3(4) \ ,$$
$$M_1^{(3)} := S_1(25) \text{ and } M_2^{(3)} := 0 \ .$$

Thanks to Lemma 3.2 and the construction we get that there are a short exact sequences $0 \to U \to M_1^{(t)} \oplus M_2^{(t)} \to V \to 0$ for all $k = 1, 2, 3$. For this take a look on the left side of figure 3.3 (marked by lines).

2nd step: List of the $\tilde{M_1}^{(\tilde{t})} \oplus \tilde{M_2}^{(\tilde{t})}$'s.

The minimal element of $\mathcal{S}(U, V)$ is $m_{min} = 0$, thus we set

$$\tilde{M_1}^{(1)} := S_3(16) \text{ and } \tilde{M_2}^{(1)} := S_1(9) \ ,$$
$$\tilde{M_1}^{(2)} := S_3(20) \text{ and } \tilde{M_2}^{(2)} := S_1(5) \ ,$$
$$\tilde{M_1}^{(3)} := S_3(24) \text{ and } \tilde{M_2}^{(3)} := S_1 \ ,$$

The successive application of the result to the minimal steps yields then figure 3.4:

$$\begin{tikzcd}
M_1^{(3)} \oplus M_2^{(3)} & & \tilde{M_1}^{(3)} \oplus \tilde{M_2}^{(3)} \\
M_1^{(2)} \oplus M_2^{(2)} \arrow[u, dash] & & \tilde{M_1}^{(2)} \oplus \tilde{M_2}^{(2)} \arrow[u, dash] \\
M_1^{(1)} \oplus M_2^{(1)} \arrow[u, dash] & & \tilde{M_1}^{(1)} \oplus \tilde{M_2}^{(1)} \arrow[u, dash] \\
& U \oplus V \arrow[ul, dash] \arrow[ur, dash] &
\end{tikzcd}$$

fig. 3.4: Degeneration-Diagram of $U \oplus V$ after 1st and 2nd step

3rd step: List of the $N_1^{(t)} \oplus N_2^{(t)}$'s and $\tilde{N_1}^{(\tilde{t})} \oplus \tilde{N_2}^{(\tilde{t})}$'s.

Now we have $\mathcal{S}(M_1^{(1)}, M_2^{(1)}) = \{2, 3\}$ (with $r = 3$) and $\mathcal{S}(M_2^{(1)}, M_1^{(1)}) = \{0, 1\}$ (with

$r = 1$). Thus we get

$$
\begin{aligned}
N_1^{(1)} &:= S_3(19) \text{ and } N_2^{(1)} := S_1(6)\ , \\
N_1^{(2)} &:= S_3(23) \text{ and } N_2^{(2)} := S_1(2)\ , \\
\tilde{N_1}^{(1)} &:= S_3(19) \text{ and } \tilde{N_2}^{(1)} := S_1(6)\ , \\
\tilde{N_1}^{(2)} &:= S_3(23) \text{ and } \tilde{N_2}^{(2)} := S_1(2)\ .
\end{aligned}
$$

Again the successive application of the result to the minimal steps yields then figure 3.5:

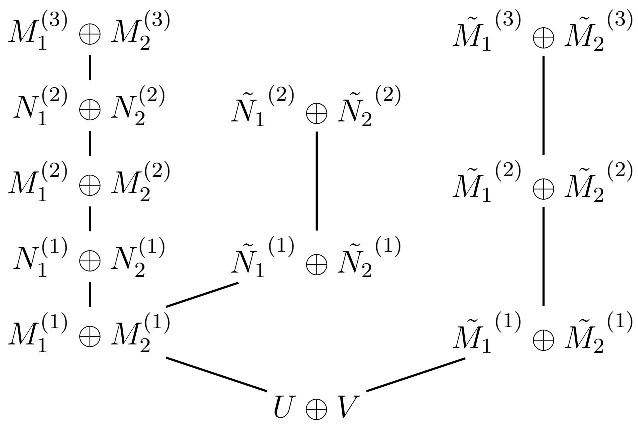


fig. 3.5: Degeneration-Diagram of $U \oplus V$ after 1st till 3rd step

4th step:

We have $\mathcal{S}(\tilde{M_1}^{(1)}, \tilde{M_2}^{(1)}) = \{2, 3\}$ (with $r = 1$) and $\mathcal{S}(\tilde{M_2}^{(1)}, \tilde{M_1}^{(1)}) = \{0, 1\}$ (with $r = 3$). Thus we get again the modules as constructed before and the following figure 3.6

If you go on step-by-step with this procedure, you get the last cross connections as in figure 3.1.

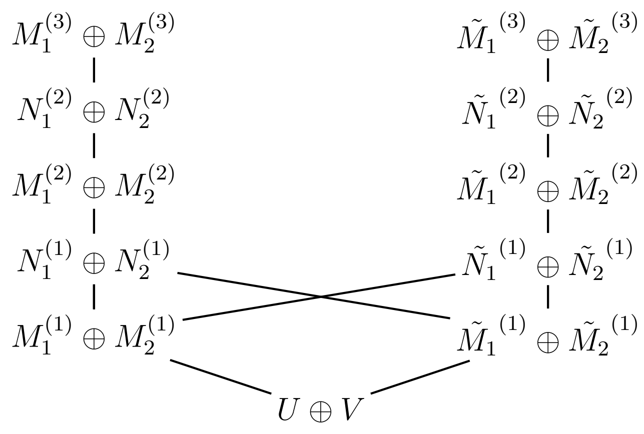


fig. 3.6: Degeneration-Diagram of $U \oplus V$ after 1st till 4th step

4 Deformations of $U \oplus V$ for preprojective U and regular V

4.1 First observations and notations

In this chapter we want to study deformations of $U \oplus V$, where U is an indecomposable preprojective A-module and V an indecomposable regular A-module. Due to the tilting-theory we can always assume that U is the only simple projective.

So from now on let U be simple projective and $V \in \mathcal{T}_\mu$, where $\mathcal{T}_\mu$ is a tube of period p.

Consider an A-module $M = \bigoplus_{i=1}^{n} M_i$, where all M_i are indecomposable, and assume $M < U \oplus V$. This implies for all $Q \in \mathcal{I}$:

$$[Q, M] \leq [Q, U] + [Q, V] = 0.$$

Hence no M_i can be preinjective and therefore M decomposes into its preprojective and regular parts

$$M = M_{\mathcal{P}} \oplus M_{\mathcal{R}}.$$

Because of $\partial(M_{\mathcal{P}}) = [X, M]^1 - [X, M] = [X, U]^1 - [X, U] = \partial(U)$ for an indecomposable regular module X with $\underline{\dim}\, X = \underline{\delta}$, we have $M_{\mathcal{P}} \neq 0$. If we assume now that $M_{\mathcal{R}} \neq 0$ and take S to be the regular socle of a summand of $M_{\mathcal{R}}$, then

$$1 \leq [S, M] = [S, M_{\mathcal{R}}] \leq [S, U \oplus V] = [S, V] \leq 1$$

yields that $M_{\mathcal{R}}$ is indecomposable and that the regular socles of $M_{\mathcal{R}}$ and V coincide. We write from now on M_R in preference to $M_{\mathcal{R}}$. Thus M_R is a regular submodule of V and consequently there exists a short exact sequence

$$\Sigma : 0 \to M_R \to V \xrightarrow{\pi} R \to 0 \qquad \text{and } R \text{ is again indecomposable regular.} \qquad (3)$$

Using the above notation the following reduction result is crucial:

Theorem 4.1:

$M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ is a minimal degeneration if and only if $M_{\mathcal{P}} \leq U \oplus R$ is a minimal degeneration.

This result is extremely useful, because combined with the following lemma it reduces the classification of the minimal deformations to a finite problem that can be attacked by a computer. The lemma in turn is an adaption of lemma 3 in [5]. With the previous notations we get:

Lemma 4.2:

Assume that $M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ is a minimal degeneration, then $\underline{\dim}\, R \leq \underline{\delta}$ holds.

Proof. Because $M = M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ is a minimal degeneration, we get a non-split short exact sequence $0 \to U \xrightarrow{\varepsilon} M \to V \to 0$. Decompose now ε into $\binom{\varepsilon_1}{\varepsilon_2}$. If $\varepsilon_1 = 0$, then we get $V = M_{\mathcal{P}} \oplus \mathrm{cokern}(\varepsilon_2)$, which implies $V = M_{\mathcal{P}}$, a contradiction. So we have $\varepsilon_1 \neq 0$, whence ε_1 injective, because U is simple. According to that we get the following exact commutative diagram

$$\begin{tikzcd}
 & & & & \Sigma & \\
 & & & 0 \arrow[d] & 0 \arrow[d] & \\
 & & & M_R \arrow[r, "\simeq"] \arrow[d] & M_R \arrow[d] & \\
 & 0 \arrow[r] & U \arrow[r, "\varepsilon"] \arrow[d, "id"'] & M \arrow[r] \arrow[d] & V \arrow[r] \arrow[d] & 0 \\
\sigma_1 : & 0 \arrow[r] & U \arrow[r, "\varepsilon_1"'] & M_{\mathcal{P}} \arrow[r] \arrow[d] & R \arrow[r] \arrow[d] & 0 \\
 & & & 0 & 0 &
\end{tikzcd}$$

Suppose now $\underline{\dim}\, R > \underline{\delta}$, then there exist two short exact sequences

$$0 \to R_1 \xrightarrow{\epsilon} R \to R_2 \to 0 \qquad \text{and} \qquad 0 \to \tilde{R}_1 \to R \xrightarrow{pr} \tilde{R}_2 \to 0$$

with $\underline{\dim}\, R_1 = \underline{\delta} = \underline{\dim}\, \tilde{R}_2$. This means that R_1, respectively $\tilde{R}_2$ is the regular uniserial submodule, respectively factor module of R that contains exactly once each simple belonging to $\mathcal{T}_\mu$. Now from σ_1 and Σ we get by obvious manipulations the following two diagrams:

$$\begin{tikzcd}
 & & & \sigma_3 & & \\
 & & & 0 \arrow[d] & 0 \arrow[d] & \\
\sigma_2 : & 0 \arrow[r] & U \arrow[r] \arrow[d, "id"'] & \tilde{M}_{\mathcal{P}} \arrow[r] \arrow[d] & R_1 \arrow[r] \arrow[d, "\epsilon"] & 0 \\
\sigma_1 : & 0 \arrow[r] & U \arrow[r] & M_{\mathcal{P}} \arrow[r] \arrow[d] & R \arrow[r] \arrow[d] & 0 \\
 & & & R_2 \arrow[r, "\simeq"'] \arrow[d] & R_2 \arrow[d] & \\
 & & & 0 & 0 &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & & \sigma_4 & & & \\
 & & 0 \arrow[d] & & \tilde{R}_1 \arrow[d] & \\
\Sigma : & 0 \arrow[r] & M_R \arrow[r] \arrow[d] & V \arrow[r] \arrow[d, "id"'] & R \arrow[r] \arrow[d, "pr"] & 0 \\
\sigma_5 : & 0 \arrow[r] & \tilde{M}_R \arrow[r] \arrow[d] & V \arrow[r] & \tilde{R}_2 \arrow[r] & 0 \\
 & & X \arrow[d] & & & \\
 & & 0 & & &
\end{tikzcd}$$

and

So we obtain that $\tilde{M}_{\mathcal{P}}$ is preprojective, because $M_{\mathcal{P}}$ is preprojective, $X \simeq \tilde{R}_1$ is regular and therefore $0 \neq \tilde{M}_R$ is regular, too. Our aim is now to conclude

$$M \leq \tilde{M} = \tilde{M}_{\mathcal{P}} \oplus \tilde{M}_R \leq U \oplus V \ .$$

First by equality of dimensionvectors for T injective we have

$$[U \oplus V, T] - [\tilde{M}, T] = 0 = [\tilde{M}, T] - [M, T]$$

and for T not injective we get

$$[U \oplus V, T] - [\tilde{M}, T] = \left[\tau^- T, U \oplus V\right] - [\tau^- T, \tilde{M}] \text{and} \tag{4}$$
$$[\tilde{M}, T] - [M, T] = [\tau^- T, \tilde{M}] - \left[\tau^- T, M\right] \ . \tag{5}$$

Both equations (4) and (5) are identical to 0 for T preinjective and ≥ 0 for T regular, since M_R embeds in $\tilde{M}_R$ and $\tilde{M}_R$ embeds in V (see σ_4 and σ_5). Lastly we look at the case where T is preprojective. Now $\mathrm{Hom}_{\mathrm{A}}(-, T)$ applies to σ_2 and σ_3

$$\mathrm{Hom}_{\mathrm{A}}(R_1, T) = 0 \to \mathrm{Hom}_{\mathrm{A}}(\tilde{M}_{\mathcal{P}}, T) \to \mathrm{Hom}_{\mathrm{A}}(U, T) \text{ and}$$
$$\mathrm{Hom}_{\mathrm{A}}(R_2, T) = 0 \to \mathrm{Hom}_{\mathrm{A}}(M_{\mathcal{P}}, T) \to \mathrm{Hom}_{\mathrm{A}}(\tilde{M}_{\mathcal{P}}, T)$$

and hence we conclude $M \leq \tilde{M} \leq U \oplus V$. By the minimality condition of $M \leq U \oplus V$ we can deduce, that either $M \simeq \tilde{M}$ or $\tilde{M} \simeq U \oplus V$ must hold. But the first case is a contradiction to σ_4 and $\tilde{R}_1 \neq 0$ and similarly in the second case we get a contradiction because of σ_5 and $\tilde{R}_2 \neq 0$. $\square$

4.2 Proof of Theorem 4.1

We start with the assumption, that $M_{\mathcal{P}} \leq U \oplus R$ is a minimal degeneration. Out of the minimality results the existence of a short exact sequence σ and together with Σ, in particular the pullback along π, we get the following commutative and exact diagram

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
 & & M_R \arrow[r, "\simeq"] \arrow[d] & M_R \arrow[d] & \\
\sigma' : 0 \arrow[r] & U \arrow[r] \arrow[d, "id"'] & E \arrow[r] \arrow[d] & V \arrow[r] \arrow[d, "\pi"] & 0 \\
\sigma : 0 \arrow[r] & U \arrow[r] & M_{\mathcal{P}} \arrow[r] \arrow[d] & R \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

.

As now $\mathrm{Ext}^1_{\mathrm{A}}(M_{\mathcal{P}}, M_R) = 0$ it must hold that $E \simeq M_{\mathcal{P}} \oplus M_R$. Hence as a result from σ' we get $M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ and we want to show that this is minimal. Without loss of generality let $M_R \neq 0$ and assume that there exists a module $M' = M'_{\mathcal{P}} \oplus M'_R \leq U \oplus V$, which is a minimal deformation, with $M \leq M'$ in addition. From the additivity of the defect it follows directly that $\partial(M_{\mathcal{P}}) = \partial(M'_{\mathcal{P}})$. Hence with Lemma 3.8(ii) in [12] it results, that M_R is a submodule of M'_R (so especially $M'_R \neq 0$).
To infer a contradiction we first look at the case where M_R is a proper submodule of M'_R and M'_R is a proper submodule of V. This assertion will lead to a regular module R'' with $0 \neq R'' \subset R$ and $M_{\mathcal{P}} \leq M'_{\mathcal{P}} \oplus R'' \leq U \oplus R$ which is a contradiction to the minimality condition of $M_{\mathcal{P}}$.
Consider the short exact sequence $\Sigma' : 0 \to M'_R \to V \to R' \to 0$, which follows from $M'_R \subset V$. As in the proof of Lemma 4.2 we have an exact sequence $0 \to U \to M'_{\mathcal{P}} \oplus M'_{\mathcal{R}} \to V \to 0$. Again we get a short exact sequence $\sigma' : 0 \to U \xrightarrow{\epsilon'} M'_{\mathcal{P}} \to R' \to 0$ and the embedding ϵ' of M_R in M'_R yields a short exact sequence σ'_4 and an epimorphism $R \xrightarrow{\pi'} R'$

$$
\begin{tikzcd}
 & \sigma_4' & & 0 \arrow[d] & \\
 & 0 \arrow[d] & & R'' \arrow[d] & \\
\Sigma : 0 \arrow[r] & M_R \arrow[r] \arrow[d, "\epsilon'"'] & V \arrow[r] \arrow[d, "id"'] & R \arrow[r] \arrow[d, "\pi'"] & 0 \\
\sigma_5' : 0 \arrow[r] & M_R' \arrow[r] \arrow[d] & V \arrow[r] & R' \arrow[r] \arrow[d] & 0 \\
 & X \arrow[d] & & 0 & \\
 & 0 & & &
\end{tikzcd}
$$

.

Especially $0 \neq X \simeq R''$ is indecomposable regular. Again the pullback from σ' along π' implies $M_{\mathcal{P}}' \oplus R'' \leq U \oplus R$. What remains to be shown is that $M_{\mathcal{P}} \leq M_{\mathcal{P}}' \oplus R''$ holds too. So let T be an indecomposable module. Then $\mathrm{Hom}_{\mathrm{A}}(T, -)$ applied to σ_4' yields an exact sequence $0 \to \mathrm{Hom}_{\mathrm{A}}(T, M_R) \to \mathrm{Hom}_{\mathrm{A}}(T, M_R') \to \mathrm{Hom}_{\mathrm{A}}(T, R'')$ and therewith

$$[T, M_R] \geq [T, M_R'] - [T, R''] \ .$$

Due to $M \leq M'$ it now follows for all T indecomposable:

$$
\begin{aligned}
[T, M_{\mathcal{P}}] + [T, M_R] &\leq [T, M_{\mathcal{P}}'] + [T, M_R'] - [T, R''] + [T, R''] \\
&\leq [T, M_{\mathcal{P}}'] + [T, M_R] + [T, R'']
\end{aligned}
$$

and therefore $[T, M_{\mathcal{P}}] \leq [T, M_{\mathcal{P}}'] + [T, R'']$, thus

$$M_{\mathcal{P}} \leq M_{\mathcal{P}}' \oplus R'' \ .$$

As $M_{\mathcal{P}} \leq U \oplus R$ is minimal we get from $M_{\mathcal{P}} \leq M_{\mathcal{P}}' \oplus R'' \leq U \oplus R$ either $M_{\mathcal{P}} \simeq M_{\mathcal{P}}' \oplus R''$ or $M_{\mathcal{P}}' \oplus R'' \simeq U \oplus R$. The first case is impossible because of $R'' \neq 0$ and the second because of $R'' \not\simeq R$.
Consequently it must either be that $M_R = M_R'$ or $M_R' = V$. In the case of $M_R = M_R'$ the split of M_R' yields a short exact sequence $0 \to U \to M_{\mathcal{P}}' \to R \to 0$ and therewith $M_{\mathcal{P}}' \leq U \oplus R$ on the one side. On the other side it results from $M_{\mathcal{P}} \oplus M_R = M \leq M' = M_{\mathcal{P}}' \oplus M_R'$ that $M_{\mathcal{P}} \leq M_{\mathcal{P}}'$. Due to the minimality of $M_{\mathcal{P}}$ it follows that $M_{\mathcal{P}} \simeq M_{\mathcal{P}}'$, thus $M \simeq M'$ too. The case of $M_R' = V$ leads immediately to $M' \simeq U \oplus V$.

Now we consider the reverse direction, so let $M = M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ be a minimal degeneration and look at the exact sequences $0 \to M_R \to V \to R \to 0$ and $0 \to U \to M_{\mathcal{P}} \to R \to 0$. Suppose $M_{\mathcal{P}} \leq U \oplus R$ is not minimal. Then there are proper degenerations $M_{\mathcal{P}} < M'_{\mathcal{P}} \oplus R' < U \oplus R$ where the last degeneration is minimal. So there are exact sequences $0 \to U \to M'_{\mathcal{P}} \oplus R' \to R \to 0$ and $0 \to R' \to R \to R'' \to 0$ where the case $R' = 0$ is possible. Let M'_R be the kernel of the composition of epimorphisms $V \to R \to R''$. We claim that $M_{\mathcal{P}} \oplus M_R \leq M'_{\mathcal{P}} \oplus M'_R \leq U \oplus V$ holds. Indeed, all three modules have the same dimension vector. For a preinjectice indecomposable module T we have

$$[T, M_{\mathcal{P}} \oplus M_R] = [T, M'_{\mathcal{P}} \oplus M'_R] = [T, U \oplus V] = 0 \ .$$

For regular T we have

$$[T, M_{\mathcal{P}} \oplus M_R] \leq [T, M'_{\mathcal{P}} \oplus M'_R] \leq [T, U \oplus V] = 0 \ ,$$

because of $M_R \subseteq M'_R \subseteq V$.
For preprojective T we have $[M_{\mathcal{P}} \oplus M_R, T] = [M_{\mathcal{P}}, T] \leq [M'_{\mathcal{P}} \oplus M'_R, T] \leq [U \oplus V, T]$ because of $M_{\mathcal{P}} \leq M'_{\mathcal{P}} \oplus R' \leq U \oplus R$. Using the nice formula of Auslander-Reiten from 2.2 we conclude $[T, M_{\mathcal{P}} \oplus M_R] \leq [T, M'_{\mathcal{P}} \oplus M'_R] \leq [T, U \oplus V]$.
By minimality of $M_{\mathcal{P}} \oplus M_R \leq U \oplus V$ we obtain $M_{\mathcal{P}} \oplus M_R \simeq M'_{\mathcal{P}} \oplus M'_R$ or $M'_{\mathcal{P}} \oplus M'_R \simeq U \oplus V$. The first case gives $M_{\mathcal{P}} \simeq M'_{\mathcal{P}}$ contradicty $M_{\mathcal{P}} < M'_{\mathcal{P}} \oplus R'$. The second case implies $M'_{\mathcal{P}} \simeq U$ and $M'_R \simeq V$, whence $R' \simeq R$ and $M'_{\mathcal{P}} \oplus R' \simeq U \oplus R$. Both contradictions shows that $M_{\mathcal{P}} \leq U \oplus R$ is minimal.

4.3 The Codimension

Following Theorem 4.1 the question comes up, what happens with the codimensions for this reduction. The answer is given in Lemma 4.3: They remain the same!

Lemma 4.3:

Under the assumptions of Theorem 4.1 the codimensions on both sides coincide.

Proof. We have to check the following equation

$$\begin{aligned}
& [U \oplus V, U \oplus V] - [M_{\mathcal{P}} \oplus M_r, M_{\mathcal{P}} \oplus M_r] = [U \oplus R, U \oplus R] - [M_{\mathcal{P}}, M_{\mathcal{P}}] \\
\Leftrightarrow \quad & [U, V] + [V, V] - [M_{\mathcal{P}}, M_r] - [M_R, M_R] = [U, R] + [R, R] \\
\Leftrightarrow \quad & [U, M_R] - [M_{\mathcal{P}}, M_R] = [R, R] + [M_R, M_R] - [V, V]
\end{aligned}$$

Because there exists a short exact sequence $0 \to U \to M_{\mathcal{P}} \to R \to 0$, we get via the Euler form the equation we want to verify

$$-\langle \underline{\dim}\, R, \underline{\dim}\, M_R \rangle = [R, R] + [M_R, M_R] - [V, V] \tag{6}$$

On the right side of (6) we get for $l(V) = k_V p + r_V$ and $l(M_R) = k_{M_R} p + r_{M_R}$, where $0 \leq r_{M_R}, r_V < p$, that

$$[M_R, M_R] = \begin{cases} k_{M_R} + 1 & , \text{for } r_{M_R} > 0 \\ k_{M_R} & , \text{for } r_{M_R} = 0 \end{cases} \quad \text{and} \quad [V, V] = \begin{cases} k_V + 1 & , \text{for } r_V > 0 \\ k_V & , \text{for } r_V = 0 \end{cases} \tag{7}$$

Furthermore lemma 4.2 tells us, that

$$0 < l(R) = l(V) - l(M_R) = (k_V - k_{M_R})p + r_V - r_{M_R} \leq p$$

Now we consider two cases, namely $\underline{\dim}(R) = \underline{\delta}$ and $\underline{\dim}(R) < \underline{\delta}$. In the first case we have $k_V - k_{M_R} = 1$ and $r_V = r_{M_R}$, thus the left side of (6) is nothing more than $\partial(M_R) = 0$ and on the right side we get with(7) and $[R, R] = 1$ ($\underline{\dim}\, R \leq \underline{\delta}$) the same. Now have a look at the second case. From $\underline{\dim}\, R < \underline{\delta}$ we infer $[R, R]^1 = 0$ and $[R, V]^1 = [\tau^- V, R] = 0$ (because $\mathrm{Top}(V) = \mathrm{Top}(R)$ and $l(R) < p$, so $\mathrm{Top}(\tau^- V)$ is not a regular composition factor of R). Together with the short exact sequence $\Sigma : 0 \to M_R \to V \to R \to 0$ we obtain $-\langle \underline{\dim}\, R, \underline{\dim}\, M_R \rangle = [R, R] - [R, V]$, thus (6) reduces to

$$[R, V] = [V, V] - [M_R, M_R] \; .$$

But this is obviously true, as Lemma 3.1 ($l(R) < l(V)$) and Σ implies $[R, V] = l_S(R) = l_S(V) - l_S(M_R) = [V, V] - [M_R, M_R]$, where $S = \mathrm{Soc}(V) = \mathrm{Soc}(M_R)$. □

Since we only have to look at the finitely many modules V with $\underline{\dim}\, V \leq \underline{\delta}$ and finitely many U, such that U is the only simple projective, we get with the help of a computer the following statement for minimal degenerations $M < U \oplus V$.

Theorem 4.4:
If $M < U \oplus V$ is a minimal degeneration, then the codimension is at most 2.

In a forthcoming paper we give a theoretical argument for this theorem!

4.4 Deformations and extensions

The next aim is to describe for V with $\underline{\dim}\, V \leq \underline{\delta}$ the set

$$\mathcal{E} = \{M \mid \exists \text{ exact sequence } 0 \to U \to M \to V \to 0\}$$

of all extensions inside the set of all deformations of $U \oplus V$. First we give some sufficient constraints for the existence of an exact sequences $0 \to U \to M \to V \to 0$.
For the case of $M = M_{\mathcal{P}} \oplus M_R \leq U \oplus V$, where $\underline{\dim}\, V < \underline{\delta}$ we refer to Lemma 4.4 in [3], which is the following result.

Lemma 4.5:
Let a module M and an exact sequence $0 \to U \to N \to V \to 0$ be given such that the following three conditions are satisfied:

1. *The orbit of V is open,*
2. $[U, N] = [U, M]$,
3. *M degenerates to N .*

Then there is an exact sequence $0 \to U \to M \to V \to 0$

So we get for this case that the condition M degenerates to $U \oplus V$ is equivalent to the existence of an sequence $0 \to U \to M \to V \to 0$. Particularly we get from $\underline{\dim}\, V < \underline{\delta}$ that $0 = \mathrm{Ext}^1_{\mathrm{A}}(V, V) = T_v Mod^d_A / T_v \overline{\mathcal{O}(v)}$, so the orbit of V is open. Thus $N = U \oplus V$ in Lemma 4.5 yields the equivalence.
For $\underline{\dim}\, V = \underline{\delta}$ we get a further sufficient constraint for the existence of an exact sequence by using the dual Version of Theorem 2.4(a) in [3], which is the following statement.

Lemma 4.6:
Let M, N and V be A-modules such that M degenerates to N and $[M,V] = [N,V]$. Then we have: If there exists an epimorphism $N \xrightarrow{\pi} V \to 0$, then there exists an epimorphism $M \xrightarrow{\tilde{\pi}} V \to 0$ too.

With this we will see now, that every degeneration $M \leq U \oplus V$, which has a path in the degeneration diagram of $U \oplus V$ to a minimal deformation $N \leq U \oplus V$, where N is preprojective, is a middle term of a short exact sequence between U and V. So we define the set $\mathcal{K}$ to be

$$\{M \leq U \oplus V \mid M \text{ has a path to a minimal preprojective deformation or } M_{\mathcal{R}} \neq 0\}$$

and see with the next statement that $\mathcal{K} \subseteq \mathcal{E}$.

Lemma 4.7:

1. *If $M \leq N \leq U \oplus V$ with $N < U \oplus V$ minimal and N is preprojective, then there exists a short exact sequence $0 \to U \to M \to V \to 0$.*

2. *If $M \leq U \oplus V$ with $M_R \neq 0$ and $\underline{\dim}\, V = \underline{\delta}$, then there exists an exact sequence $0 \to U \to M \to V \to 0$.*

Proof. To (1): As $N \leq U \oplus V$ is minimal, we get a short exact sequence $0 \to U \to N \to V \to 0$. Hence we only have to prove that $[M,V] = [N,V]$ holds, then Lemma 4.6 gives the assertion. Because N is preprojective and $M \leq N$ holds, M is preprojective too. Thus we get

$$[M,V] - [N,V] = [\tau^- V, M] - [\tau^- V, N] = 0\ ,$$

as $\tau^- V$ is regular.
To (2): Pick a minimal degeneration $N \leq U \oplus V$ such that $M \leq N$ holds. Then M_R is a submodule of N_R and N_R is a proper submodule of V, so $l(M_R) \leq l(N_R) < p = l(V)$ holds. Again we deduce from $[M,V] = [N,V]$ the assertion by using Lemma 4.6. Since $\underline{\dim}\, M = \underline{\dim}\, N$ we get with the Euler form

$$\begin{aligned} & \langle \underline{\dim}\, N, \underline{\dim}\, V \rangle = \langle \underline{\dim}\, M, \underline{\dim}\, V \rangle \\ \Leftrightarrow \quad & [N,V] - [M,V] = [N,V]^1 - [M,V]^1 \end{aligned}$$

and $[M,V]^1 = [\tau^- V, M_R] = 0 = [\tau^- V, N_R] = [N,V]^1$. □

So if we want to answer the question, which deformations M of $U \oplus V$ are not extensions of U and V in the case of $\underline{\dim}\, V = \underline{\delta}$, then, following Lemma 4.7, we have to examine those preprojective M, that have no path in the degeneration diagram of $U \oplus V$ to a minimal $N \leq U \oplus V$ with N preprojective. Consider now two deformations M and M' of $U \oplus V$ with the just mentioned properties. Then it results from $M \leq M'$ and M being no extension of U and V, that M' is no extension of U and V either, because $[M, V] = [M', V]$ holds. Thus in the first instance we consider the "highest" deformations M in the degeneration diagram. Later on we discuss an example for the quiver $\tilde{E}_7$ on the next paragraph. Sometimes there is the case, that only one deformation M is to verify and that it is indecomposable. Thanks to Lemma 6.6 in [3] we have a nice condition to handle this.

Lemma 4.8:

Let M be an indecomposable preprojective module such that M degenerates to $U \oplus V$ and denote by S the regular socle of V. Then the following two statements are equivalent:

1. $[S, U]^1 \neq 0$
2. *There exists an exact sequence* $0 \to U \to M \to V \to 0$.

The examination of the deformations $M \notin \mathcal{K}$ shows that the following holds

Theorem 4.9:

Using the above definitions we have:

$$\mathcal{K} = \mathcal{E}$$

in all cases.

Unfortunately, we have no theoretical argument for this. Observe however that for homogeneous V the set $\mathcal{K} = \mathcal{E}$ consists already $\mathcal{D}$, the set of all deformations of $U \oplus V$. But up to the case, where $\mathcal{E}$ is only the orbit of an indecomposable, we have always $\mathcal{E} \subsetneq \mathcal{D}$ for V is not homogeneous. Furthermore there are cases, where no minimal deformation M of $U \oplus V$ with M is preprojective exists, because for all $M \in \mathcal{E}$ it holds that $M_R \neq 0$. See for this for example the cases of $\tilde{D}_n (n \geq 4)$ or $\tilde{E}_i (i = 6, 7, 8)$ in the appendix. Moreover the "highest" deformations, which have to examine, are not always on the same level in the degeneration diagram. For example the case of

$V = S_2(4)$ (see figure 4.2) in the next subsection 4.5 shows this.
As the following example shows, the verification of this theorem requires a lot of non-routine calculations. This is due to the fact that we do not know any efficient algorithms to decide whether a given module V is a quotient of another module M or whether M is an extension of V by U.
Note that the following example shows that the set of extensions is not a subvariety of the set of all deformations but only a constructible subset.

4.5 Example: $\tilde{E}_7$

Now we want to consider the quiver $\tilde{E}_7$ with the following orientation

$$\begin{tikzcd}[column sep=small] & & & c \arrow[d] & & & \\ a_1 \arrow[r] & a_2 \arrow[r] & a_3 \arrow[r] & d & b_3 \arrow[l] & b_2 \arrow[l] & b_1 \arrow[l] \end{tikzcd}$$

Thus we have three regular tubes with period $p > 1$, in particular $p \in \{2, 3, 4\}$, and we denote by $S_1, \ldots, S_p$ the regular simples of the respective tube and a module $\tau^n P(q)$ shorter by $\tau^n q$.

4.5.1 Period 4

First we study the case of period $p = 4$. For the dimension vectors of the regular simples we have:

$$\underline{\dim}\, S_1 = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{smallmatrix}\right] \quad \underline{\dim}\, S_2 = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$$

$$\underline{\dim}\, S_3 = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right] \quad \underline{\dim}\, S_4 = \left[\begin{smallmatrix} & & & 0 & & & \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$$

In figure 4.1 we see the degeneration diagram of $U \oplus V$, where $U = d$ and $V = S_1(4)$. We mark with $\square$ the deformations of $U \oplus V$ with a nonzero regular part and with $\blacksquare$ the preprojective deformations, which have no path in the degeneration diagram of $U \oplus V$ in a minimal $N \leq U \oplus V$ with N preprojective. The "highest" deformations M_i have the number i inside of $\blacksquare$. So we want to verify that

$$M_1 = \tau^{-3} a_3 \oplus a_1 \text{ and}$$
$$M_2 = \tau^{-3} b_1 \oplus \tau^{-2} b_3$$

are no extensions of U and V. To get this statement we use that every epi from M_i to V iduces an epi from M_i to every quotient of V, especially for the regular top S_4 of V.
For M_1 we get $\mathrm{Hom}_{\mathrm{A}}(M_1, S_4) \simeq \mathrm{Hom}_{\mathrm{A}}(a_1, S_4)$, because $[\tau^{-3}a_3, S_4] = [a_3, \tau^3 S_4] =$

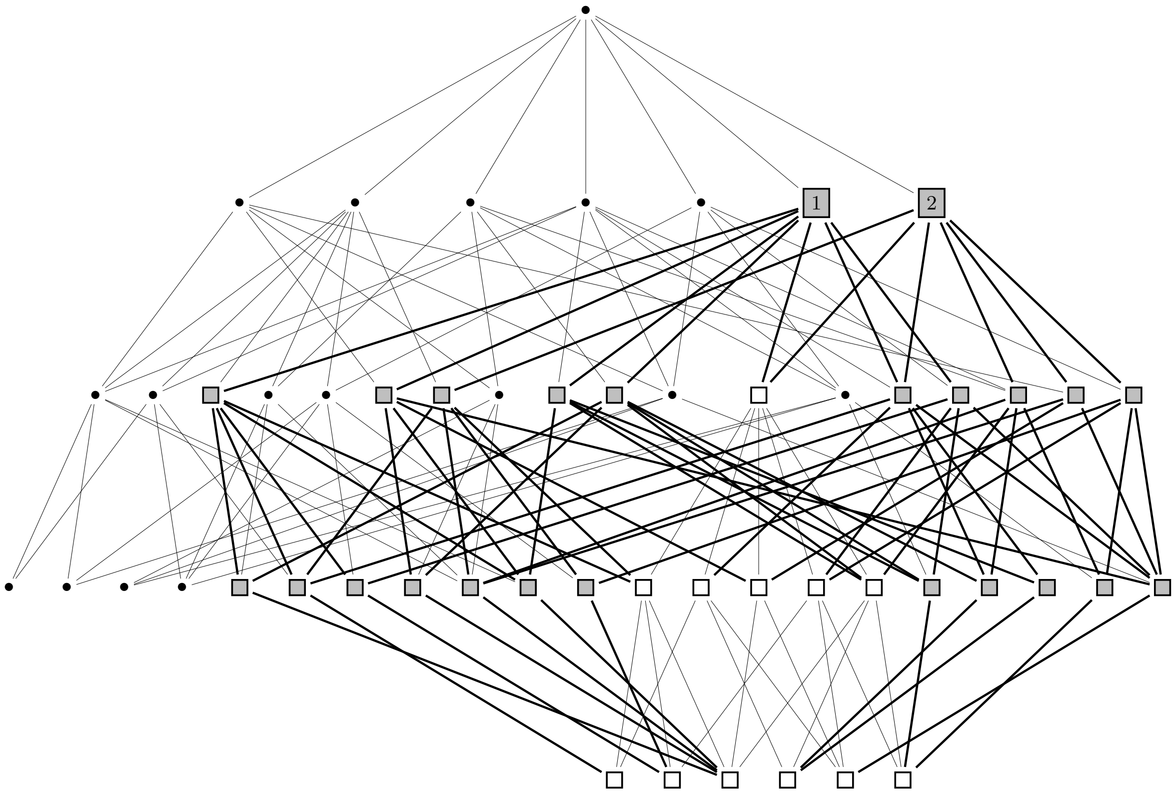


fig. 4.1: Degeneration diagram of $d \oplus S_1(4)$

$[a_3, S_1] = 0$. But $\underline{\dim}\, a_1 = \left[\begin{smallmatrix} & & & 0 & & & \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right] < \underline{\dim}\, S_4$, so there is no epi $a_1 \to S_4$.
For M_2 we get $\mathrm{Hom}_{\mathrm{A}}(M_2, S_4) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}b_3, S_4)$, because $[\tau^{-3}b_1, S_4] = [b_1, \tau^3 S_4] = [b_1, S_1] = 0$. But $\underline{\dim}\, \tau^{-2}b_3 = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 2 & 3 & 2 & 1 & 1 \end{smallmatrix}\right]$, thus $(\underline{\dim}\, \tau^{-2}b_3)_{a_1} = 0 < 1 = (\underline{\dim}\, S_4)_{a_1}$. So we conclude that every module signed with $\blacksquare$ in figure 4.1 is not an extension of $d \oplus S_1(4)$.

Next we take a look at the case where $V = S_2(4)$. There we see, that it is not true, that the "highest" modules, which have at least to be examine, are all on the same level at the degeneration diagram. Figure 4.2 shows the corresponding degeneration

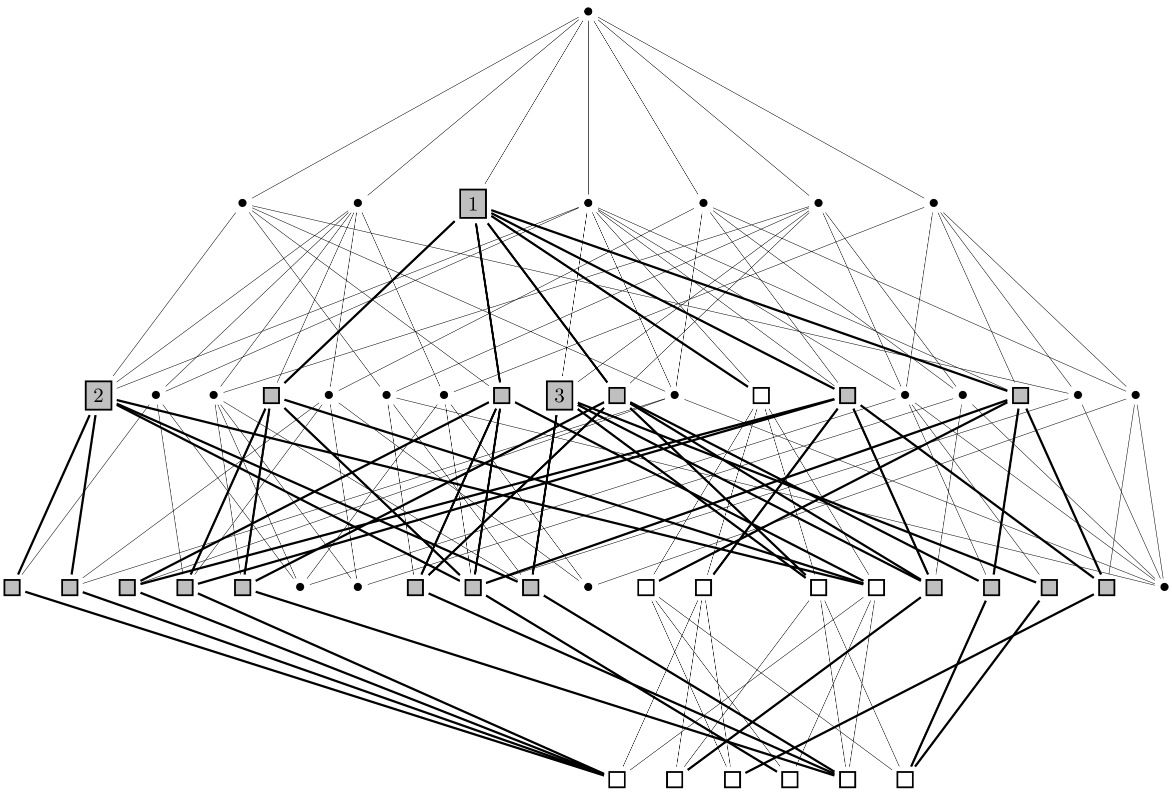


fig. 4.2: Degeneration diagram of $d \oplus S_2(4)$

diagram. There we have to consider three modules M_i, namely

$$\begin{aligned} M_1 &= \tau^{-3}a_2 \oplus \tau^{-}a_2 \ , \\ M_2 &= \tau^{-4}a_2 \oplus b_2 \text{ and} \\ M_3 &= \tau^{-4}b_1 \oplus \tau^{-2}c \oplus a_1 \ . \end{aligned}$$

For M_1 we again use the regular top S_1 of $S_2(4)$ and obtain that $\mathrm{Hom}_{\mathrm{A}}(M_1, S_1) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-}a_2, S_1)$, because $[\tau^{-3}a_2, S_1] = [a_2, \tau^3 S_1] = [a_3, S_2] = 0$. But $\underline{\dim}\,\tau^{-}a_2 = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&1&1&2&1&0&0\end{smallmatrix}\right]$, thus $(\underline{\dim}\,\tau^{-}a_2)_{b_2} < (\underline{\dim}\,S_1)_{b_2}$. Similarly we proceed with M_2 and again it is no extension. For M_3 this argument does not work, because there exists an epimorphism from M_3 to S_1. So we regard the quotient $S_4(2)$ of $S_2(4)$. Then $\underline{\dim}\,S_4(2) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&1&1&2&2&1&0\end{smallmatrix}\right]$ and $\mathrm{Hom}_{\mathrm{A}}(M_3, S_4(2)) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}c \oplus a_1, S_4(2))$, because $[\tau^{-4}b_1, S_4(2)] = [b_1, S_4(2)] = 0$. But $\underline{\dim}\,\tau^{-2}c \oplus a_1 = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&2&2&3&1&1&0\end{smallmatrix}\right]$, thus $(\underline{\dim}\,\tau^{-2}c \oplus a_1)_{b_3} < (\underline{\dim}\,S_4(2))_{b_3}$. The case of $V = S_3(4)$, respectively $V = S_4(4)$, is symmetric to $S_1(4)$, respectively

$V = S_2(4)$.

4.5.2 Period 3

For the tube of period 3 we have the following dimension vectors for the regular simples

$$\underline{\dim}\, S_1 = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right] \quad \underline{\dim}\, S_2 = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right] \quad \underline{\dim}\, S_3 = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 \end{smallmatrix}\right].$$

The case of $V = S_1(3)$ can be handled only with the observation of the dimension vector of the regular top S_3 of $S_1(3)$, so we do not discuss this and go on with $V = S_2(3)$. There we have again three modules to verify, namely

$$\begin{aligned} M_1 &= \tau^{-3} a_2 \oplus \tau^{-} a_2 \ , \\ M_2 &= \tau^{-3} b_2 \oplus \tau^{-} b_2 \text{ and} \\ M_3 &= \tau^{-3} a_1 \oplus \tau^{-} c \oplus \tau^{-3} b_1 \ . \end{aligned}$$

The regular top of $S_2(3)$ is S_1 and we get $\mathrm{Hom}_{\mathrm{A}}(M_1, S_1) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-} a_2, S_1)$, where $\underline{\dim}\, \tau^{-} a_2 = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 2 & 1 & 0 & 0 \end{smallmatrix}\right]$. Furthermore we take a quick glance at the orientation of our quiver and conclude that every $f = (f_{a_1}, f_{a_2}, f_{a_3}, f_d, f_{b_3}, f_{b_2}, f_{b_1}, f_c) \in \mathrm{Hom}_{\mathrm{A}}(\tau^{-} a_2, S_1)$ fulfills $f_{a_3} = 0$. But $(\underline{\dim}\, S_1)_{a_3} = 1$, thus f is never a epimorphism. For M_2 we get the same (it is symmetrical to M_1) and for M_3 we check the dimension vectors of $\tau^{-} c$ and S_1 like before.
Finally the case of $V = S_3(3)$ affords two modules

$$\begin{aligned} M_1 &= \tau^{-3} a_1 \oplus \tau^{-2} a_2 \text{ and} \\ M_2 &= \tau^{-3} b_1 \oplus \tau^{-2} b_2 \ , \end{aligned}$$

which are symmetrical to one other. Again we take the regular top S_2 of $S_3(3)$ and find for every $f \in \mathrm{Hom}_{\mathrm{A}}(M_1, S_2)$ that $f_{a_2} = 0$, but $(\underline{\dim}\, S_2)_{a_2} = 1$.

4.5.3 Period 2

The dimension vectors for the regular simples of the tube of period 2 are given by

$$\underline{\dim}\, S_1 = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 1 & 2 & 2 & 1 & 1 \end{smallmatrix}\right] \quad \underline{\dim}\, S_2 = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right].$$

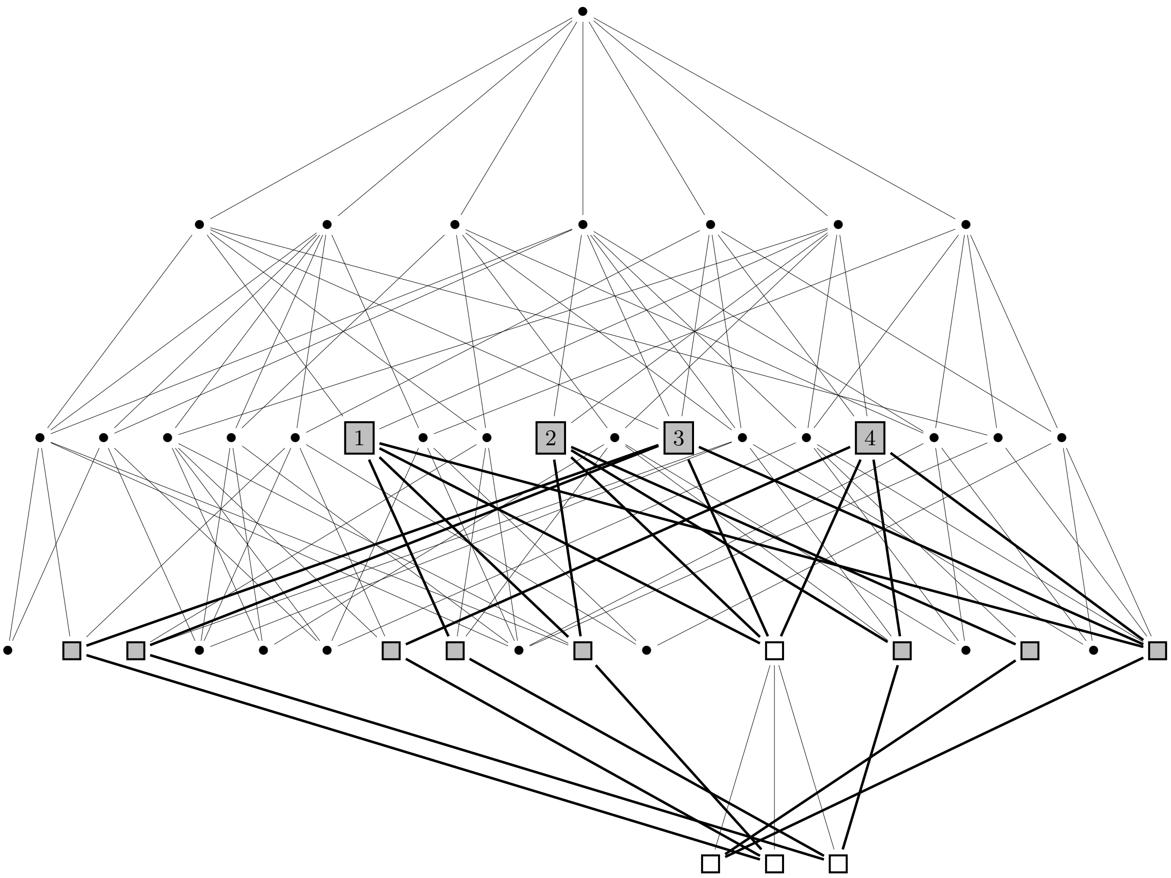


fig. 4.3: Degeneration diagram of $d \oplus S_1(2)$

We only discuss $V = S_1(2)$. The corresponding degeneration diagram is figure 4.3. For this case we have to check four modules

$$\begin{aligned} M_1 &= \tau^{-3}a_1 \oplus \tau^{-2}a_2 \oplus a_1 \ , \\ M_2 &= \tau^{-4}b_1 \oplus \tau^{-2}c \oplus a_1 \ \ , \\ M_3 &= \tau^{-6}b_1 \oplus \tau^{-}b_3 \qquad \text{and} \\ M_4 &= \tau^{-5}a_1 \oplus \tau^{-}b_2 \oplus a_1 \ . \end{aligned}$$

We have $\mathrm{Hom}_{\mathrm{A}}(M_1, S_2) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}a_2 \oplus a_1, S_2)$, but if we regard intently the representations of $\tau^{-2}a_2 \oplus a_1$ and S_2, we see that every $f \in \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}a_2 \oplus a_1, S_2)$ complies $f_c = 0$. So f is never an epimorphism, because $(\underline{\dim}\, S_2)_c = 1$. Particularly we get a representation of $\tau^{-2}a_2 \oplus a_1$ by the following

$$\begin{tikzcd}
 & & & k \arrow[d, "\left[\begin{smallmatrix}1\\1\\0\end{smallmatrix}\right]"] & & & \\
k \arrow[r, "1"] & k \arrow[r, "\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]"] & k^2 \arrow[r, "\left[\begin{smallmatrix}1&0\\0&0\\0&1\end{smallmatrix}\right]"] & k^3 & k^2 \arrow[l, "\left[\begin{smallmatrix}1&0\\0&1\\0&0\end{smallmatrix}\right]"'] & k \arrow[l, "\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]"'] & 0 \arrow[l, "0"']
\end{tikzcd}$$

and for S_2 we have

$$\begin{tikzcd}
 & & & k \arrow[d, "\left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]"] & & & \\
k \arrow[r, "1"] & k \arrow[r, "\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]"] & k^2 \arrow[r, "\left[\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right]"] & k^2 & k \arrow[l, "\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]"'] & k \arrow[l, "1"'] & 0 \arrow[l, "0"']
\end{tikzcd} .$$

Using the commutativity conditions we calculate for $f \in \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}a_2 \oplus a_1, S_2)$:

$$f_{a_1} = f_{a_2} = x \ , \ f_{a_3} = \left[\begin{smallmatrix}0&x\\y&0\end{smallmatrix}\right] \ , \ f_d = \left[\begin{smallmatrix}0&0&x\\y&z&0\end{smallmatrix}\right] \ , \ f_{b_3} = \left[\begin{smallmatrix}y&z\end{smallmatrix}\right] \text{ and } f_{b_2} = z \ ,$$

where $x, y, z \in k$. But now we get $f_d \left[\begin{smallmatrix}1\\1\\0\end{smallmatrix}\right] = \left[\begin{smallmatrix}0\\y+z\end{smallmatrix}\right] = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right] f_c$. This implies $f_c = 0$.

Next we want to study M_2. Now we have $\mathrm{Hom}_{\mathrm{A}}(M_2, S_2) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}c \oplus a_1, S_2)$ and we choose the following representation of $\tau^{-2}c \oplus a_1$:

$$\begin{tikzcd}
 & & & k \arrow[d, "\left[\begin{smallmatrix}1\\1\\0\end{smallmatrix}\right]"] & & & \\
k \arrow[r, "\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]"] & k^2 \arrow[r, "\left[\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right]"] & k^2 \arrow[r, "\left[\begin{smallmatrix}1&0\\0&0\\0&1\end{smallmatrix}\right]"] & k^3 & k \arrow[l, "\left[\begin{smallmatrix}0\\1\\0\end{smallmatrix}\right]"'] & k \arrow[l, "1"'] & 0 \arrow[l, "0"']
\end{tikzcd} .$$

Set for $f \in \mathrm{Hom}_{\mathrm{A}}(\tau^{-2}c \oplus a_1, S_2)$:

$$f_{a_1} = x \ , \ f_{a_2} = \left[\begin{smallmatrix}y&z\end{smallmatrix}\right] \ ,$$

where $x, y, z \in k$. Then we get $f_{a_3} = \left[\begin{smallmatrix}y&z\\0&0\end{smallmatrix}\right]$, but if f is an epimorphism, then f_{a_3} must be an isomorphism, which is never satisfied.
Finally we see that $\mathrm{Hom}_{\mathrm{A}}(M_3, S_2) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-}b_3, S_2)$ and $\mathrm{Hom}_{\mathrm{A}}(M_4, S_2) \simeq \mathrm{Hom}_{\mathrm{A}}(\tau^{-}b_2 \oplus a_1, S_2)$, thus by compare again the dimension vectors with $\underline{\dim}\, S_2$ we conclude, that there is no epimorphism too.

5 Some geometric aspects

The object of this section is to elucidate briefly how to analyse singularities in the case of A being a tame path algebra. We therefore recall the definition of smooth equivalence occurring in orbit closures of modules .

5.1 Types of singularities

In the first instance let A be a finitely generated associative k-algebra with unit.

Definition 5.1:
Two pointed varieties (X, x) and (Y, y) are called SMOOTHLY EQUIVALENT if there are smooth morphisms of pointed varieties $\lambda : (Z, z) \to (X, x)$ and $\mu : (Z, z) \to (Y, y)$. This yields an equivalence relation $\sim$ and the equivalence classes are called TYPES OF SINGULARITIES.

Furthermore we denote by Sing(M, N) for A-modules M and N with $M \leq_{deg} N$ the type of ($\overline{\mathcal{O}(m)}$,n). The next theorem gives a first reduction for the examination of Sing(M,N)(see [2]).

Theorem 5.2:
Let S, M, M', Q and Q' be finite dimensional modules such that the following conditions are fulfilled:

1. $[S, M] = [S, M']$.
2. $[Q, S]^1 - [Q, S] = [Q', S]^1 - [Q', S]$.
3. *Q is the generic quotient of M by S, and M is the generic extension of Q by S.*
4. *Q' is a quotient of M' by S*

Then M degenerates to M' if and only if Q degenerates to Q' and in that case Sing(M, M') $\sim$ Sing(Q, Q') holds.

In the case of finite dimensional modules over a finite dimensional path algebra we therewith get the following statement.

Lemma 5.3:

Let U, V and M be modules over a finite dimensional path algebra A such that U is indecomposable preprojective, V is indecomposable regular and $M = M_{\mathcal{P}} \oplus M_{\mathcal{R}}$, whereas $M_{\mathcal{R}}$ is a regular submodule of V and there exists a exact sequence $0 \to U \to M \to V \to 0$. Then

$$\mathit{Sing}(M, U \oplus V) \sim \mathit{Sing}(M_{\mathcal{P}}, U \oplus R)$$

holds, whereas R is the quotient of V by $M_{\mathcal{R}}$.

In paragraph 4.4 we have shown, that in the cases of $M \leq U \oplus V$ with $M_{\mathcal{R}} \neq 0$, which are to be examined, the conditions of Lemma 5.3 are always fulfilled.

Proof. Let S be the regular socle of V. In the notation of theorem 5.2 we now set $M' = U \oplus V$, $Q = (M_{\mathcal{P}} \oplus M_{\mathcal{R}})/S = M_{\mathcal{P}} \oplus (M_{\mathcal{R}}/S)$ and $Q' = U \oplus (V/S)$. The conditions of theorem 5.2 are then fulfilled, as (1.) $[S, M] = [S, M'] = 1$, (2.) holds always for path algebras by using the Euler form, as $\underline{\dim}\, Q = \underline{\dim}\, Q'$, (3.) Q is generic quotient, since it is the only quotient of M by S and M is the generic extension of Q by S, as $\mathrm{Hom}_A(S, M_{\mathcal{P}}) = 0$ and (4.) by definition of Q'. Thus $\mathrm{Sing}(M, U \oplus V) \sim \mathrm{Sing}(M_{\mathcal{P}} \oplus (M_{\mathcal{R}}/S), U \oplus (V/S))$ holds. Furthermore the embeddings of S in $M_{\mathcal{R}}$ and $M_{\mathcal{R}}$ in V lead to the following diagram with exact rows and columns:

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
 & & S \arrow[r, "\mathrm{id}"] \arrow[d] & S \arrow[d] & \\
0 \arrow[r] & U \arrow[r] \arrow[d, "\mathrm{id}"'] & M_{\mathcal{P}} \oplus M_{\mathcal{R}} \arrow[r] \arrow[d] & V \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & U \arrow[r] & M_{\mathcal{P}} \oplus (M_{\mathcal{R}}/S) \arrow[r] \arrow[d] & V/S \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

So we come upon the initial situation again and can go on with the same argumentation till $M_{\mathcal{R}} = 0$. □

Next we want to show that in the case of path algebras kQ we can reduce our examination to representations of Q. We therefore use a general reduction, that results from the following consideration (see for example [4] example 5.18).

First of all let B, generated by $b_1, \ldots, b_r$, be a subalgebra of A, which is generated by

$a_1, \dots, a_n$. Then there exist for any b_i a polynomial P_i in not-commuting variables, such that $b_i = P_i(a_1, \dots, a_n)$ holds. Now regard $\pi : Mod_A^d \to Mod_B^d$, given by evaluating the P_i's, then π is a Gl_d-equivariant morphism. Thus we get as a consequence of 7.8 in [6]

Lemma 5.4:

Use the above notations and take $n \in Mod_B^d$ with stabilizer $H = Stab_G(n)$. If we denote by F the preimage of n under π, then π induces a bijection between the H-invariant subsets of F and the G-invariant subsets of the preimage of $\mathcal{O}(n)$ under π. This bijection respects inclusions, closures and codimensions, we especially get for $m, m' \in F$:

$$m' \in \overline{Gm} \qquad \Leftrightarrow \qquad m' \in \overline{Hm}$$

and in this case Sing($\overline{Gm}$, m') $\sim$ Sing($\overline{Hm}$, m') holds.

We want to adapt this to Mod_A^d and $Rep_Q^{\underline{d}}$, where $A = kQ$, $\underline{d} = (d_1, \dots, d_n)$ with $\sum_{i=1}^n d_i = d$ ($Q_0 = (1, \dots, n)$) and $Rep_Q^{\underline{d}}$ denotes the representations to a fixed dimension vector $\underline{d}$. Therefore consider the orthogonal primitive idempotents e_i and the subalgebra $B = \bigoplus_{i=1}^{n} ke_i$ of A. Then every G-orbit in Mod_B^d is closed, as all B-modules are closed, because they are semi-simple. Hence the G-orbits are the connected components of Mod_B^d. Now we look at the surjective π from Lemma 5.4, then the preimages of the G-orbits in Mod_B^d are the connected components of Mod_A^d (on the one hand the preimages of the G-orbits are open and closed, on the other hand it holds for every M in the preimage that $M \leq_{deg} \bigoplus_{i=1}^{n} S_i^{d_1}$, where S_i denotes the simple to the point $i \in Q_0$).
In particular we have:
$Mod_B^d = \left\{ \tilde{E}_{\underline{d}} = \left(\tilde{E}_1, \dots, \tilde{E}_n \right) \mid \tilde{E}_i \in k^{d \times d}, \sum_{i=1}^n \tilde{E}_i = E_d, \tilde{E}_i \tilde{E}_j = \delta_{ij} \tilde{E}_i \right\}$.
Now choose the obvious normal form for $\tilde{E}_{\underline{d}}$ and plug it for n in Lemma 5.4. Then H is the product $\prod_{i=1}^n Gl_{d_i}$. Thus the examination of ($\overline{Hm}$, m') corresponds to the examination of representations of Q.
As the last reduction of the problem we now introduce a definition and give a description of a transversal slice in the special case of quiver representations.

5.2 Transversal slices

Let $A = kQ$ be a finite dimensional path algebra over a quiver Q.

<u>Definition</u> 5.5:
Let $\underline{d}$ be an element of $\mathbb{N}^{|Q_0|}$, $D \in Rep_Q(\underline{d})$ and $G(\underline{d}) = \prod_{i \in Q_0} Gl_{d_i}$. Then a TRANSVERSAL SLICE ON D in $Rep_Q(\underline{d})$ is a subspace S of $Rep_Q(\underline{d})$ such that the map $\mu : G(\underline{d}) \times (D + S) \to Rep_Q(\underline{d})$ with $((g_i)_{i \in Q_0}, (M_\alpha)_{\alpha \in Q_1}) \mapsto (\ldots, g_{s(\alpha)} M(\alpha) g^{-1}_{n(\alpha)}, \ldots)$ induces a bijective map on the tangent spaces at the point $(1, D)$, respectively D.

For the motivation of this definition note that $G(\underline{d}) \times (D + S)$, as well as $Rep_Q(\underline{d})$ are smooth. Hence μ is smooth in an open neighberhood of $(1, D)$ and D, if the induced map on the tangent spaces is bijective. Assuming this we get for a degeneration $D' \leq_{deg} D$ in $Rep_Q(\underline{d})$ the following diagram:

$$\begin{tikzcd}
G(\underline{d}) \times (D+S) \arrow[r, "\mu"] \arrow[d, phantom, "\cup"] & Rep_Q(\underline{d}) \arrow[d, phantom, "\cup"] \\
G(\underline{d}) \times (\overline{\mathcal{O}(D')} \cap (D+S)) = \mu^{-1}(\overline{\mathcal{O}(D')}) \arrow[r, "smooth"] \arrow[d, "smooth"] & \overline{\mathcal{O}(D')} \\
\overline{\mathcal{O}(D')} \cap (D+S) &
\end{tikzcd}$$

This shows $\mathrm{Sing}(\overline{D'},\, D) \sim \mathrm{Sing}(\overline{\mathcal{O}(D')} \cap (D+S),\, D)$. We are left with the question, how to calculate such a subspace S. For that purpose we consider the tangent space $T_D Rep_Q(\underline{d})$ at the point D and denote by T_μ the induced tangent map at the point $(1, 0)$. Now it holds that

$$T_\mu(T_1 G(\underline{d}) \times 0) = \mathcal{B}(D, D) \subseteq \mathcal{Z}(D, D) = T_D Rep_Q(\underline{d}) = Rep_Q(\underline{d})$$

where $\mathcal{B}(D, D)$, respectively $\mathcal{Z}(D, D)$, denotes the coboundaries of D, respectively cocycles of D. Thus we get a subspace S having the required characteristics, by calculating a complement for $\mathcal{B}(D, D)$ in $\mathcal{Z}(D, D)$.

In those cases in which we want to use this technique, D will be a representation of a tame quiver Q, that corresponds to a kQ-module $U \oplus V$, whereat U is indecomposable preprojective and V is indecomposable regular with $\underline{\dim}\, V = \underline{\delta}$. Thus D decomposes into $D_U \oplus D_V$ and with this we infer from $\mathrm{Ext}_Q(D_U, D_U) = \mathrm{Ext}_Q(D_U, D_V) = 0$ that:

$$\begin{aligned}\mathcal{Z}(D,D)/\mathcal{B}(D,D) &= \mathrm{Ext}_Q(D,D) = \mathrm{Ext}_Q(D_V, D_U) \oplus \mathrm{Ext}_Q(D_V, D_V) \\ &= (\mathcal{Z}(D_V, D_U)/\mathcal{B}(D_V, D_U)) \oplus (\mathcal{Z}(D_V, D_V)/\mathcal{B}(D_V, D_V))\end{aligned}$$

So it especially suffices to find complements for these direct summands of $\mathcal{B}(D,D)$.

We now denote by e an extension vertex of Q, i.e. $\underline{\delta}_e = 1$. Therefore the quiver Q' obtained by deleting e is a corresponding Dynkin quiver to Q and there is exactly one arrow $\alpha \in Q_1$ with either $s(\alpha) = e$ or $n(\alpha) = e$. Consider the embedding $\nu : Q' \to Q$. Because V is indecomposable, we have $0 \neq V(\alpha) \in k^2$. Now ν and D_V imply a representation D'_V of Q'. Assume that D'_V is again indecomposable. Then $\mathrm{Ext}'_Q(D'_V, D'_V) = 0$, since Q' is Dynkin, and $\underline{\dim}\, V = \underline{\delta}$ yields $\mathrm{Ext}_Q(D_V, D_V) = k$. Hence we get a commutative and exact diagram:

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & 0 \arrow[d] & \\
0 \arrow[r] & kV(\alpha) \arrow[r] \arrow[d] & k^2 \arrow[r] \arrow[d] & k \arrow[d, "\simeq"] & \\
0 \arrow[r] & \mathcal{B}(D_V, D_V) \arrow[r] \arrow[d] & \mathcal{Z}(D_V, D_V) \arrow[r] \arrow[d] & \mathrm{Ext}_Q(D_V, D_V) \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & \mathcal{B}(D'_V, D'_V) \arrow[r, "\simeq"] \arrow[d] & \mathcal{Z}(D'_V, D'_V) \arrow[r] \arrow[d] & 0 = \mathrm{Ext}_Q(D'_V, D'_V) \arrow[r] & 0 \\
 & 0 & 0 & &
\end{tikzcd}$$

Thus for every $b \notin kV(\alpha)$ it holds, that kb is a complement of $\mathcal{B}(D_V, D_V)$.

5.3 Example

Again we consider the quiver $\tilde{E}_7$ with the following orientation

$$\begin{tikzcd}
 & & & c \arrow[d, "\gamma"] & & & \\
a_1 \arrow[r, "\alpha_{a_1}"] & a_2 \arrow[r, "\alpha_{a_2}"] & a_3 \arrow[r, "\alpha_{a_3}"] & d & b_3 \arrow[l, "\beta_{b_3}"'] & b_2 \arrow[l, "\beta_{b_2}"'] & b_1 \arrow[l, "\beta_{b_1}"']
\end{tikzcd}$$

First we calculate a tranversal slice. We discus explicitly the case for $V = V_{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]}$, $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in \mathbb{P}^1$. Therefore consider the representation

$$\begin{tikzcd}
 & & & k^2 \arrow[d, "\left[\begin{smallmatrix} 1&0\\0&1\\0&1\\1&1 \end{smallmatrix}\right]"] & & & \\
k \arrow[r, "\left[\begin{smallmatrix} 1\\0 \end{smallmatrix}\right]"] & k^2 \arrow[r, "\left[\begin{smallmatrix} 1&0\\0&1\\0&0 \end{smallmatrix}\right]"] & k^3 \arrow[r, "\left[\begin{smallmatrix} 1&0&0\\0&1&0\\0&0&1\\0&0&0 \end{smallmatrix}\right]"] & k^4 & k^3 \arrow[l, "\left[\begin{smallmatrix} 0&0&0\\1&0&0\\0&1&0\\0&0&1 \end{smallmatrix}\right]"'] & k^2 \arrow[l, "\left[\begin{smallmatrix} 0&0\\1&0\\0&1 \end{smallmatrix}\right]"'] & k \arrow[l, "\left[\begin{smallmatrix} x\\y \end{smallmatrix}\right]"'] .
\end{tikzcd}$$

Then we get for $\mathcal{B}(V,U)$:

$$\mathcal{B}(V,U) = \left\{ [g_1,g_2,g_3,g_4]\begin{bmatrix} 1&0&0\\0&1&0\\0&0&1\\0&0&0 \end{bmatrix}, [g_1,g_2,g_3,g_4]\begin{bmatrix} 1&0\\0&1\\0&1\\1&1 \end{bmatrix}, [g_1,g_2,g_3,g_4]\begin{bmatrix} 0&0&0\\1&0&0\\0&1&0\\0&0&1 \end{bmatrix} \mid g_i \in k \right\}$$

$$\begin{aligned}
&= \{[g_1,g_2,g_3],[g_1+g_4,g_2+g_3+g_4],[g_2,g_3,g_4] \mid g_i \in k\} \quad \subset k^8 \\
&= \{g_1([1,0,0],[1,0],[0,0,0]) + g_2([0,1,0],[0,1],[1,0,0]) \\
&+ g_3([0,0,1],[0,1],[0,1,0]) + g_4([0,0,0],[0,1],[0,0,0]) \mid g_i \in k\}
\end{aligned}$$

and see that

$$\{[0,a,0],[b,c],[0,d,0] | a,b,c,d \in k\}$$

is a complement of $\mathcal{B}(V,U)$. For a complement of $\mathcal{B}(V,V)$ we have to choose a vector $w \notin k\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$. Thus we take $w = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$ for $y \neq 0$ and $w = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$ for $x \neq 0$. So finally we define the two tranversal slices

$$S_{y\neq 0} := \left\{ \underset{\alpha_1}{\left[\begin{smallmatrix} 1\\0 \end{smallmatrix}\right]}, \underset{\alpha_2}{\left[\begin{smallmatrix} 1&0\\0&1\\0&0 \end{smallmatrix}\right]}, \underset{\alpha_3}{\left[\begin{smallmatrix} 0&a&0\\1&0&0\\0&1&0\\0&0&1\\0&0&0 \end{smallmatrix}\right]}, \underset{\gamma}{\left[\begin{smallmatrix} b&c\\1&0\\0&1\\0&1\\1&1 \end{smallmatrix}\right]}, \underset{\beta_3}{\left[\begin{smallmatrix} 0&d&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1 \end{smallmatrix}\right]}, \underset{\beta_2}{\left[\begin{smallmatrix} 0&0\\1&0\\0&1 \end{smallmatrix}\right]}, \underset{\beta_1}{\left[\begin{smallmatrix} x+e\\y \end{smallmatrix}\right]} \;\middle|\; \begin{matrix} a,b,c, \\ d,e \in k \\ y \neq 0 \end{matrix} \right\}$$

and

$$S_{x\neq 0} := \left\{ \left[\begin{smallmatrix} 1\\0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 1&0\\0&1\\0&0 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0&a&0\\1&0&0\\0&1&0\\0&0&1\\0&0&0 \end{smallmatrix}\right], \left[\begin{smallmatrix} b&c\\1&0\\0&1\\0&1\\1&1 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0&d&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1 \end{smallmatrix}\right], \left[\begin{smallmatrix} 0&0\\1&0\\0&1 \end{smallmatrix}\right], \left[\begin{smallmatrix} x\\y+e \end{smallmatrix}\right] \;\middle|\; \begin{matrix} a,b,c, \\ d,e \in k \\ x \neq 0 \end{matrix} \right\}$$

Now we want to explain, how we find equations, which describe $\overline{\mathcal{O}(M)} \cap ((U \oplus V) + S)$ for a deformation M of $U \oplus V$.

For every indecomposable regular V with $\underline{\dim}\, V = \underline{\delta}$ and $U = Ae_d$ there exist seven deformations M_i of $U \oplus V$ of codimension 4, namely we have:

$$\begin{array}{llll}
M_1 = \tau^{-3}c \oplus \tau^{-2}c & & & , \\
M_2 = \tau^{-3}a_1 \oplus \tau^{-2}a_3 & \text{and} & M_5 = \tau^{-3}b_1 \oplus \tau^{-2}b_3 & , \\
M_3 = \tau^{-3}a_3 \oplus a_1 & \text{and} & M_6 = \tau^{-3}b_3 \oplus b_1 & , \\
M_4 = \tau^{-3}a_2 \oplus \tau^{-}a_2 & \text{and} & M_7 = \tau^{-3}b_2 \oplus \tau^{-}b_2 & .
\end{array}$$

As an example we discuss the case of $M_1 = X \oplus Y$, with $X = \tau^{-3}c$. Obviously, we have $[X, M_1] \neq 0$ and this holds by semi-continuity for all of $\overline{\mathcal{O}(M_1)}$. To find equations expressing $[X, M_1] \neq 0$ we take a projective resolution $\Sigma :\; 0 \to P_1 \xrightarrow{\varepsilon} P_0 \xrightarrow{\pi} X \to 0$, particularly

$$P_1 = Ae_d \oplus Ae_d \text{ and } P_0 = Ae_{a_1} \oplus Ae_{a_3} \oplus Ae_{b_3} \oplus Ae_{b_1} \oplus Ae_c.$$

Then we obtain $\ker\pi_d \simeq k\,\langle \alpha_3 - \beta_3, \alpha_3\alpha_2\alpha_1 + \alpha_3 + \beta_3\beta_2\beta_1 - \gamma \rangle$. If we now apply $\mathrm{Hom}_{\mathrm{A}}(-, M_1)$ to Σ, we get an exact sequence

$$0 \to \mathrm{Hom}_{\mathrm{A}}(X, M_1) \to \mathrm{Hom}_{\mathrm{A}}(P_0, M_1) \xrightarrow{\mathrm{Hom}_{\mathrm{A}}(\varepsilon, M_1)} \mathrm{Hom}_{\mathrm{A}}(P_1, M_1) \to \mathrm{Ext}^1_{\mathrm{A}}(X, M_1) \to 0$$

.

Furthermore $[X, M_1] = [X, X] = 1$ and $[X, M_1]^1 = [X, Y]^1 = 1$, thus $[P_0, M_1] = [P_1, M_1]$, but $\mathrm{Hom}_{\mathrm{A}}(\varepsilon, M_1)$ is not an isomorphism and we get that $\det\mathrm{Hom}_{\mathrm{A}}(\varepsilon, M_1) = 0$. More precisely we have $\mathrm{Hom}_{\mathrm{A}}(P_0, M_1) \simeq e_{a_1}M_1 \oplus e_{a_3}M_1 \oplus e_{b_3}M_1 \oplus e_{b_1}M_1 \oplus e_cM_1$ and $\mathrm{Hom}_{\mathrm{A}}(P_1, M_1) \simeq e_dM_1 \oplus e_dM_1$, so we get

$$\det \mathrm{Hom}_{\mathrm{A}}(\varepsilon, M_1) = \det \begin{bmatrix} \alpha_3\alpha_2\alpha_1 & \alpha_3 & 0 & \beta_3\beta_2\beta_1 & -\gamma \\ 0 & \alpha_3 & -\beta_3 & 0 & 0 \end{bmatrix} = 0 \;.$$

Thus we have

$$\overline{\mathcal{O}(M_1)} \subseteq \mathcal{N}\left(\det \begin{bmatrix} \alpha_3\alpha_2\alpha_1 & \alpha_3 & 0 & \beta_3\beta_2\beta_1 & -\gamma \\ 0 & \alpha_3 & -\beta_3 & 0 & 0 \end{bmatrix}\right) \;,$$

where $\mathcal{N}(P)$ for a polynomial P means the zero set of P. Similarly we calculate conditions for the other M_i's and get the following table 1. Now we want explicitly calculate equations, which describe $\overline{\mathcal{O}(M_i)} \cap ((U \oplus V) + S)$ and discuss the case of $\overline{\mathcal{O}(M_1)} \cap ((U \oplus V) + S_{y \neq 0})$. The condition leads to:

Deformation	Condition
$M_1 = \tau^{-3}c \oplus \tau^{-2}c$	$\det \begin{bmatrix} \alpha_3\alpha_2\alpha_1 & \alpha_3 & 0 & \beta_3\beta_2\beta_1 & -\gamma \\ 0 & \alpha_3 & -\beta_3 & 0 & 0 \end{bmatrix} = 0$
$M_2 = \tau^{-3}a_1 \oplus \tau^{-2}a_3$	$\det \begin{bmatrix} \gamma & \beta_3\beta_2\beta_1 & \alpha_3\alpha_2 \end{bmatrix} = 0$
$M_3 = \tau^{-3}a_3 \oplus a_1$	$\det \begin{bmatrix} \gamma & \beta_3 \end{bmatrix} = 0$
$M_4 = \tau^{-3}a_2 \oplus \tau^{-}a_2$	$\det \begin{bmatrix} \alpha_3 & \beta_3\beta_2 \end{bmatrix} = 0$

Table 1: Conditions

$$\det \begin{bmatrix} 0 & 0 & a & 0 & 0 & 0 & 0 & d(x+e) & -b & -c \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & x+e & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & y & -1 & -1 \\ 0 & 0 & a & 0 & 0 & -d & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} = 0,$$

and we get the following equation

$$\begin{aligned} 0 &= adx + ade + d^2x + d^2e + dby - dbe - cdx - cde + bay - bdy - adx - ade \\ &= (d^2 + db - cd)(x+e) + (ba - bd)y \\ &= d(d+b-c)(x+e) + b(a-d)y \ . \end{aligned}$$

Now we use the variable substitutions (note that $y \neq 0$)

$$\begin{aligned} d &\mapsto d' \\ d+b-c &\mapsto c' \\ x+e &\mapsto e' \\ b &\mapsto b' \\ (a-d)y &\mapsto a' \end{aligned}$$

and get

$$d'c'e' + b'a' = 0 \ .$$

Note that this polynomial is irreducible. So we see that $\overline{\mathcal{O}(M_1)} \cap ((U \oplus V) + S_{y\neq 0})$ is an irreducible subset of $\mathcal{N}(d'c'e' + b'a')$, whence we have found reduced equations for $\overline{\mathcal{O}(M_1)} \cap ((U \oplus V) + S_{y\neq 0})$.

In our case the occurring singularity coincides with the invariant ring of the three dimensional torus acting by conjugation on the set of 3×3- matrices with diagonal entries zero, as explained to us by M. Reineke. After a variable substitution we see that the cases of $S_{y\neq 0}$ and $S_{x\neq 0}$ are equivalent. The results of the computation (after variable substitutions) are given in tabel 2. As this simple example shows, the thorough

Deformation	Condition
$M_1 = \tau^{-3}c \oplus \tau^{-2}c$	$d'c'e' + b'a' = 0$
$M_2 = \tau^{-3}a_1 \oplus \tau^{-2}a_3$	$a'b' + c' = 0$
$M_3 = \tau^{-3}a_3 \oplus a_1$	$a' - b' = 0$
$M_4 = \tau^{-3}a_2 \oplus \tau^{-}a_2$	$a' = 0$

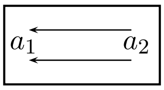
Table 2: Explicit equations

analysis of all the occurring singularities requires lots of partially delicate calculations with polynomials. For $\tilde{E}_6$ we did all necessary calculations but there were no new singularities showing up.

A Degeneration diagrams

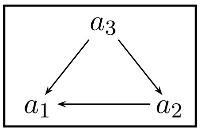
In the appendix we use the same notation as in the example (see section 4.5). The given degeneration diagrams are ordered from top to bottom, which means, that the deformations at the the top have higher codimension as at the bottom. The steps between the immediate neighbouring different levels are always codimension one steps. So it suffices to give the codimension of the root. To determine the inhomogeneous, irreducible modules $V = S_i(j)$, we give always at first the dimension vectors of the regular simples of the regarded tube, but shorter by only writing S_i instead of $\underline{\dim}\, S_i$. Note, that thanks to Lemma 4.2 and Theorem 4.1, it is enough to look at the degeneration diagrams of the $U \oplus V$, with $\underline{\dim}\, V = \underline{\delta}$.
Up to the case of $\tilde{A}_n$ the orientation of the underlying graph is uniquely determined by specification of U, because U is the only sink in the quiver. Apart from U with $\partial(U) = -5$ and $\partial(U) = -6$, we give all degeneration diagrams up to symmetries. The other two cases of U yields too large degeneration diagrams, thus we only specify the minimal deformations for these instances.

A.1 $\tilde{A}1$



In this case we have only tubes of period 1 and there is only one deformation of $U \oplus V$, where $\underline{\dim}\, V = \delta$, namely a_2. The codimension is 2.

A.2 $\tilde{A}2$



A.2.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, a_3) = 2$$

Period 2:	$S_1(1) = [{}_1{}^1{}_0]$ $S_2(1) = [{}_0{}^0{}_1]$

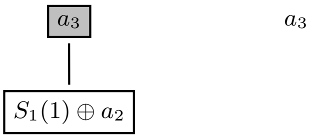
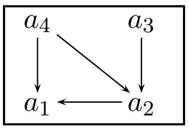


fig. A.1: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.3 $\tilde{A}3$



A.3.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, a_4) = 2$$

Period 3:	$S_1(1) = \left[\begin{smallmatrix}1&0\\1&0\end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix}0&0\\0&1\end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix}0&1\\0&0\end{smallmatrix}\right]$

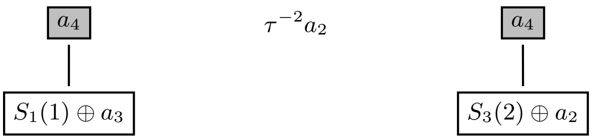
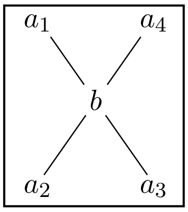


fig. A.2: Degeneration diagrams of $a_1 \oplus S_i(3)$, $i \in \{1, 2, 3\}$ in Period 3

A.4 $\tilde{D}4$



A.4.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-2}a_2) = 2$$

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 1 & & 1 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 1 \\ & 1 & \\ 0 & & 0 \end{bmatrix}$

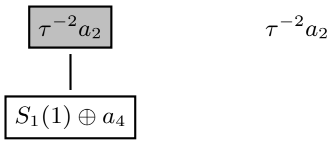


fig. A.3: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.4.2 $\mathbf{U = b}$

$$\mathbf{Codim}(U \oplus V, \tau^{-}b) = 3$$

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 1 & & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 1 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$

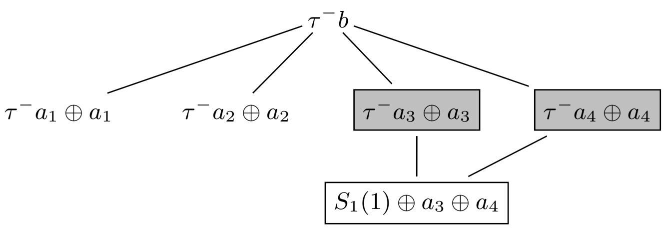
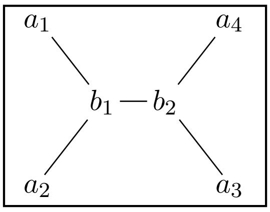


fig. A.4: Degeneration diagram of $b \oplus S_1(2)$ in Period 2

A.5 $\tilde{D}5$



A.5.1 $\mathbf{U = a_1}$

Codim$(U \oplus V, \tau^{-2}a_2) = 2$

Period 3: $S_1(1) = \begin{bmatrix} 1 & & 1 \\ & 1\ 1 & \\ 0 & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 1\ 0 & \\ 1 & & 0 \end{bmatrix}$ $S_3(1) = \begin{bmatrix} 0 & & 0 \\ & 0\ 1 & \\ 0 & & 0 \end{bmatrix}$

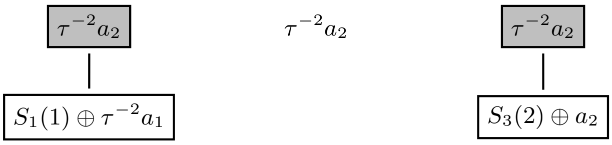


fig. A.5: Degeneration diagrams of $a_1 \oplus S_i(3)$, $i \in \{1, 2, 3\}$ in Period 3

Period 2: $S_1(1) = \begin{bmatrix} 1 & & 0 \\ & 1\ 1 & \\ 1 & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & 1 \\ & 1\ 1 & \\ 0 & & 0 \end{bmatrix}$

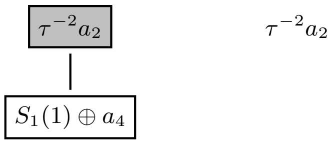


fig. A.6: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.5.2 $\mathbf{U = b_1}$

Codim$(U \oplus V, \tau^{-}b_2) = 3$

Period 3: $S_1(1) = \begin{bmatrix} 1 & & 0 \\ & 1\ 0 & \\ 1 & & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 0\ 1 & \\ 0 & & 0 \end{bmatrix}$ $S_3(1) = \begin{bmatrix} 0 & & 1 \\ & 1\ 1 & \\ 0 & & 1 \end{bmatrix}$

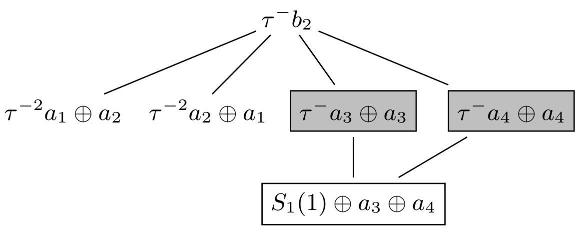


fig. A.7: Degeneration diagram of $b_1 \oplus S_1(3)$ in Period 3

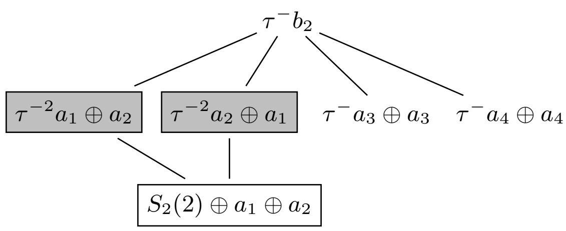


fig. A.8: Degeneration diagram of $b_1 \oplus S_2(3)$ in Period 3

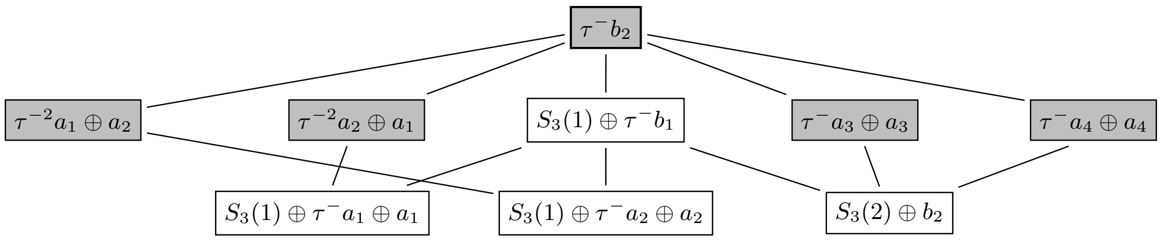


fig. A.9: Degeneration diagram of $b_1 \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = \begin{bmatrix} 1 & & & 0 \\ & 1 & 1 & \\ 0 & & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & 1 \\ & 1 & 1 & \\ 1 & & & 0 \end{bmatrix}$

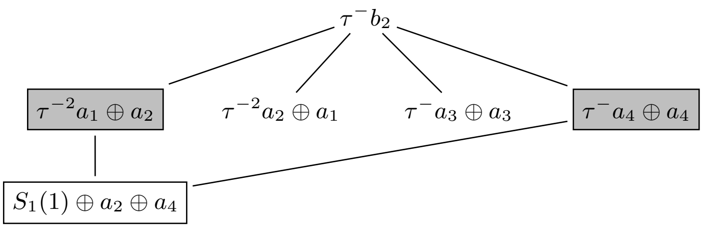
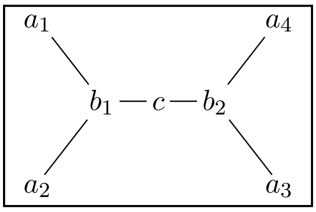


fig. A.10: Degeneration diagram of $b_1 \oplus S_1(2)$ in Period 2

A.6 $\tilde{D}6$



A.6.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-4}a_1) = 2$$

Period 4:	$S_1(1) = \begin{bmatrix} 1 & & & & 1 \\ & 1 & 1 & 1 & \\ 0 & & & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & 0 \\ & 1 & 0 & 0 & \\ 1 & & & & 0 \end{bmatrix}$
	$S_3(1) = \begin{bmatrix} 0 & & & & 0 \\ & 0 & 1 & 0 & \\ 0 & & & & 0 \end{bmatrix}$ $S_4(1) = \begin{bmatrix} 0 & & & & 0 \\ & 0 & 0 & 1 & \\ 0 & & & & 0 \end{bmatrix}$

fig. A.11: Degeneration diagrams of $a_1 \oplus S_i(4)$, $i \in \{1, 2, 3, 4\}$ in Period 4

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & & & 0 \\ & 1 & 1 & 1 & \\ 1 & & & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & 1 \\ & 1 & 1 & 1 & \\ 0 & & & & 0 \end{bmatrix}$

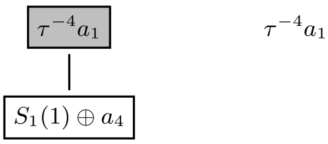


fig. A.12: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.6.2 $\mathbf{U = b_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-}b_2) = 3$$

Period 4:	$S_1(1) = \begin{bmatrix} 1 & & & & 0 \\ & 1 & 0 & 0 & \\ 1 & & & & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & 0 \\ & 0 & 1 & 0 & \\ 0 & & & & 0 \end{bmatrix}$
	$S_3(1) = \begin{bmatrix} 0 & & & & 0 \\ & 0 & 0 & 1 & \\ 0 & & & & 0 \end{bmatrix}$ $S_4(1) = \begin{bmatrix} 0 & & & & 1 \\ & 1 & 1 & 1 & \\ 0 & & & & 1 \end{bmatrix}$

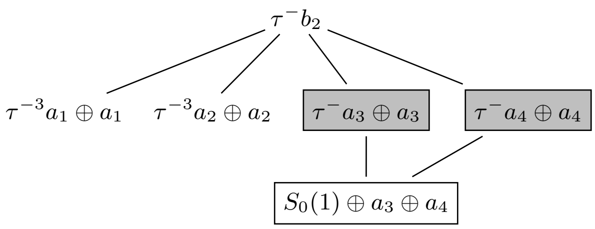


fig. A.13: Degeneration diagram of $b_1 \oplus S_1(4)$ in Period 4

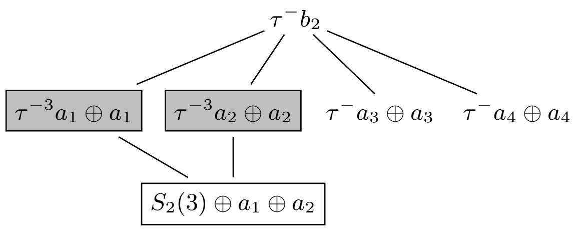


fig. A.14: Degeneration diagram of $b_1 \oplus S_2(4)$ in Period 4

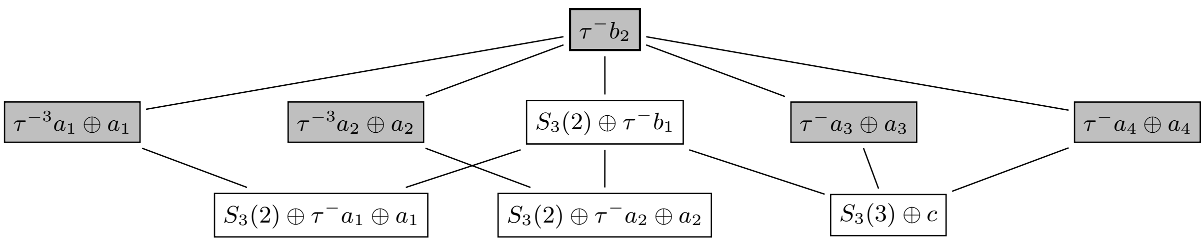


fig. A.15: Degeneration diagram of $b_1 \oplus S_3(4)$ in Period 4

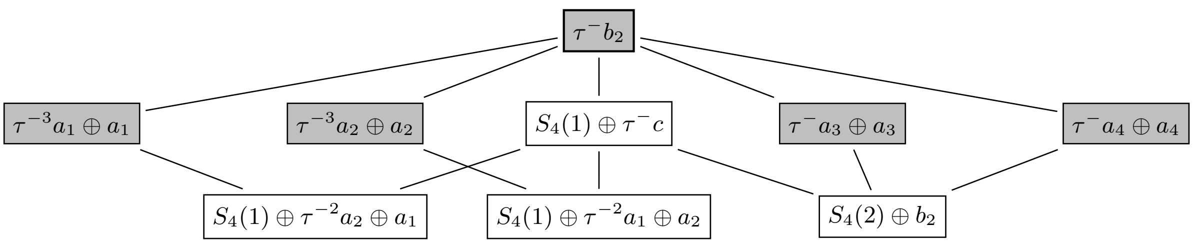


fig. A.16: Degeneration diagram of $b_1 \oplus S_4(4)$ in Period 4

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & & & 0 \\ & 1 & 1 & 1 & \\ 0 & & & & 1 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & & & 1 \\ & 1 & 1 & 1 & \\ 1 & & & & 0 \end{bmatrix}$

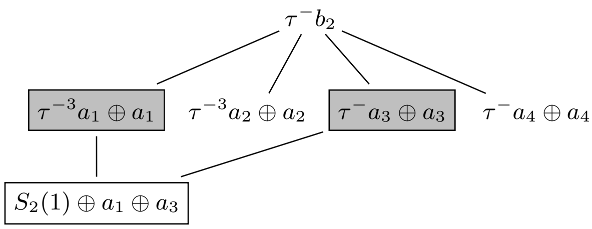


fig. A.17: Degeneration diagram of $b_1 \oplus S_2(2)$ in Period 2

A.6.3 $\mathbf{U = c}$

$$\mathbf{Codim}(U \oplus V, \tau^{-2}c) = 3$$

Period 4:	$S_1(1) = \begin{bmatrix} 1 & & & & 0 \\ & 1 & 1 & 0 & \\ 1 & & & & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & 0 \\ & 0 & 0 & 1 & \\ 0 & & & & 0 \end{bmatrix}$ $S_3(1) = \begin{bmatrix} 0 & & & & 1 \\ & 0 & 1 & 1 & \\ 0 & & & & 1 \end{bmatrix}$ $S_4(1) = \begin{bmatrix} 0 & & & & 0 \\ & 1 & 0 & 0 & \\ 0 & & & & 0 \end{bmatrix}$

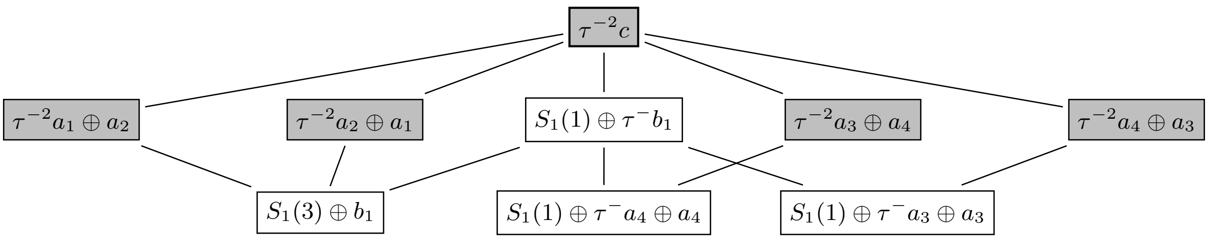


fig. A.18: Degeneration diagram of $c \oplus S_1(4)$ in Period 4

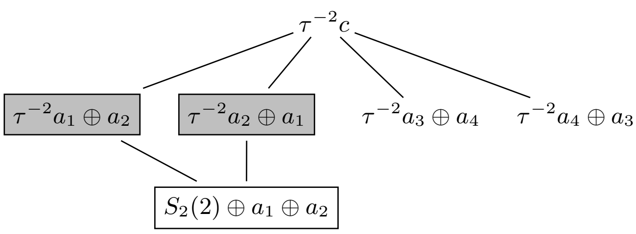


fig. A.19: Degeneration diagram of $c \oplus S_2(4)$ in Period 4

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & & & 1 \\ & 1 & 1 & 1 & \\ 0 & & & & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & 0 \\ & 1 & 1 & 1 & \\ 1 & & & & 1 \end{bmatrix}$

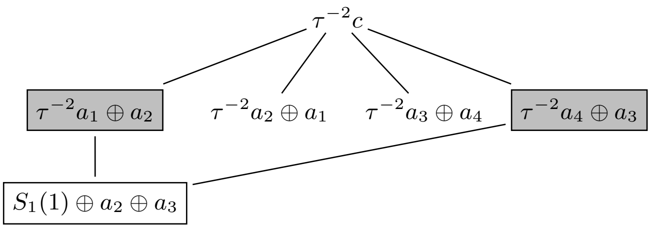
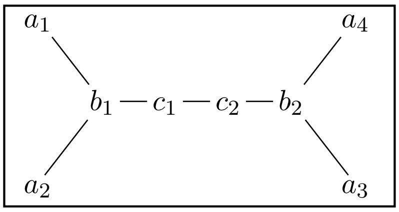


fig. A.20: Degeneration diagram of $c \oplus S_1(2)$ in Period 2

A.7 $\tilde{D}7$



A.7.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-4}a_2) = 2$$

Period 5:

$$S_1(1) = \begin{bmatrix} 1 & & & & & 1 \\ & 1 & 1 & 1 & 1 & \\ 0 & & & & & 1 \end{bmatrix} \quad S_2(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 1 & 0 & 0 & 0 & \\ 1 & & & & & 0 \end{bmatrix}$$

$$S_3(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 0 & 1 & 0 & 0 & \\ 0 & & & & & 0 \end{bmatrix} \quad S_4(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 0 & 0 & 1 & 0 & \\ 0 & & & & & 0 \end{bmatrix} \quad S_5(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 0 & 0 & 0 & 1 & \\ 0 & & & & & 0 \end{bmatrix}$$

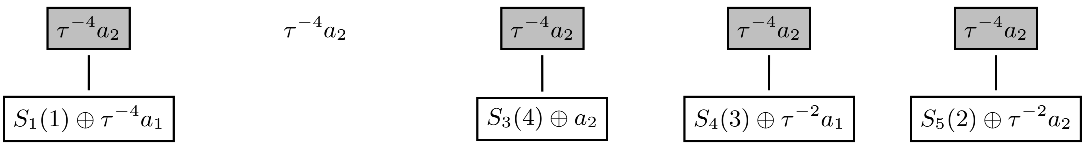


fig. A.21: Degeneration diagrams of $a_1 \oplus S_i(5)$, $i \in \{1, 2, 3, 4, 5\}$ in Period 5

Period 2:

$$S_1(1) = \begin{bmatrix} 1 & & & & & 0 \\ & 1 & 1 & 1 & 1 & \\ 1 & & & & & 1 \end{bmatrix} \quad S_2(1) = \begin{bmatrix} 0 & & & & & 1 \\ & 1 & 1 & 1 & 1 & \\ 0 & & & & & 0 \end{bmatrix}$$

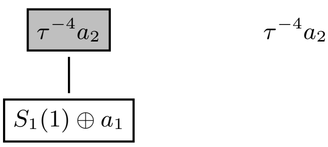


fig. A.22: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.7.2 $\mathbf{U = b_1}$

$$\mathbf{Codim}(U \oplus V, \tau^- b_2) = 3$$

Period 5:

$$S_1(1) = \begin{bmatrix} 0 & & 0 \\ & 0\ 1\ 0\ 0 & \\ 0 & & 0 \end{bmatrix} \quad S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 0\ 0\ 1\ 0 & \\ 0 & & 0 \end{bmatrix}$$

$$S_3(1) = \begin{bmatrix} 0 & & 0 \\ & 0\ 0\ 0\ 1 & \\ 0 & & 0 \end{bmatrix} \quad S_4(1) = \begin{bmatrix} 0 & & 1 \\ & 1\ 1\ 1\ 1 & \\ 0 & & 1 \end{bmatrix} \quad S_5(1) = \begin{bmatrix} 1 & & 0 \\ & 1\ 0\ 0\ 0 & \\ 1 & & 0 \end{bmatrix}$$

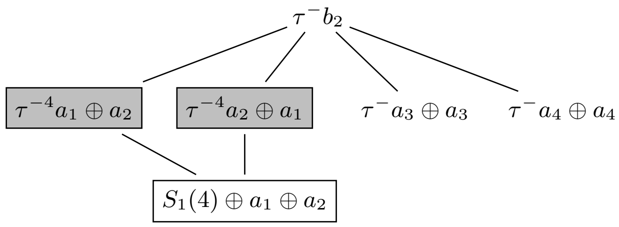


fig. A.23: Degeneration diagram of $b_1 \oplus S_1(5)$ in Period 5

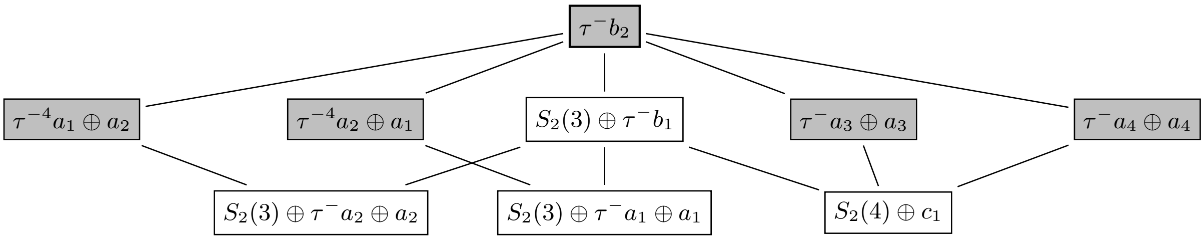


fig. A.24: Degeneration diagram of $b_1 \oplus S_2(5)$ in Period 5

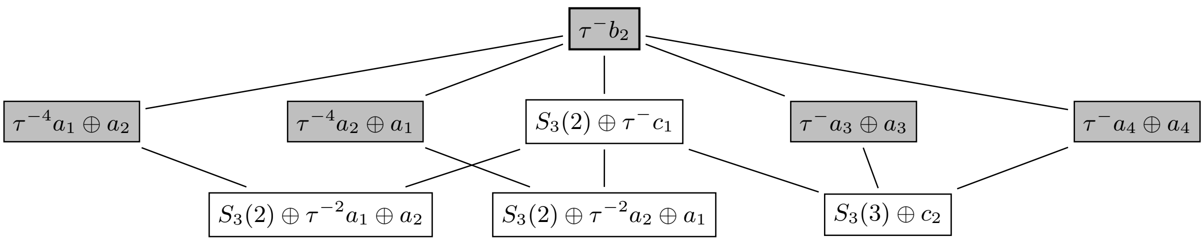


fig. A.25: Degeneration diagram of $b_1 \oplus S_3(5)$ in Period 5

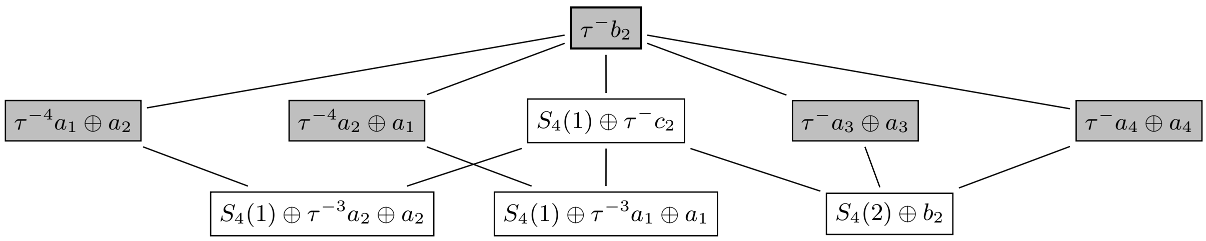


fig. A.26: Degeneration diagram of $b_1 \oplus S_4(5)$ in Period 5

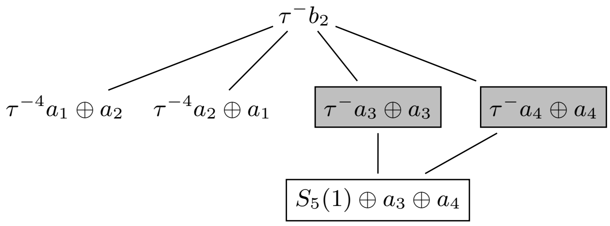


fig. A.27: Degeneration diagram of $b_1 \oplus S_5(5)$ in Period 5

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & & & & 0 \\ & 1 & 1 & 1 & 1 & \\ 0 & & & & & 1 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & & & & 1 \\ & 1 & 1 & 1 & 1 & \\ 1 & & & & & 0 \end{bmatrix}$

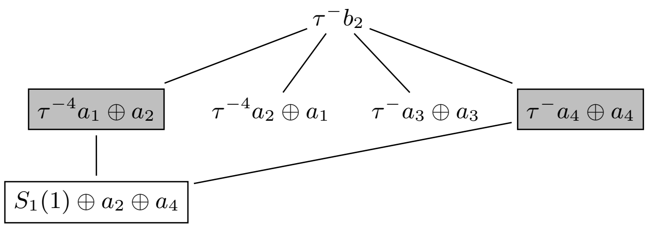


fig. A.28: Degeneration diagram of $b_1 \oplus S_1(2)$ in Period 2

A.7.3 $\mathbf{U} = \mathbf{c}_1$

$$\mathbf{Codim}(U \oplus V, \tau^{-2}c_2) = 3$$

Period 5:

$$S_1(1) = \begin{bmatrix} 1 & & & & & 0 \\ & 1 & 1 & 0 & 0 & \\ 1 & & & & & 0 \end{bmatrix} \quad S_2(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 0 & 0 & 1 & 0 & \\ 0 & & & & & 0 \end{bmatrix}$$

$$S_3(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 0 & 0 & 0 & 1 & \\ 0 & & & & & 0 \end{bmatrix} \quad S_4(1) = \begin{bmatrix} 0 & & & & & 1 \\ & 0 & 1 & 1 & 1 & \\ 0 & & & & & 1 \end{bmatrix} \quad S_5(1) = \begin{bmatrix} 0 & & & & & 0 \\ & 1 & 0 & 0 & 0 & \\ 0 & & & & & 0 \end{bmatrix}$$

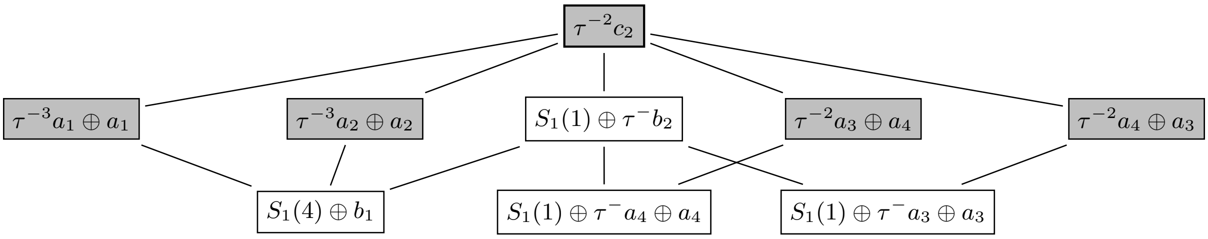


fig. A.29: Degeneration diagram of $c_1 \oplus S_1(5)$ in Period 5

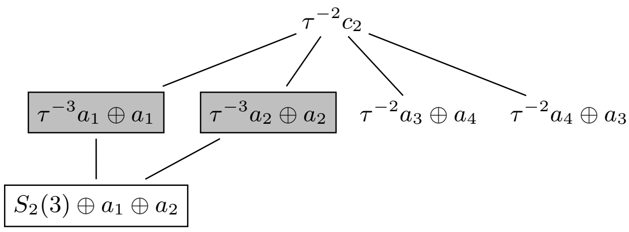


fig. A.30: Degeneration diagram of $c_1 \oplus S_2(5)$ in Period 5

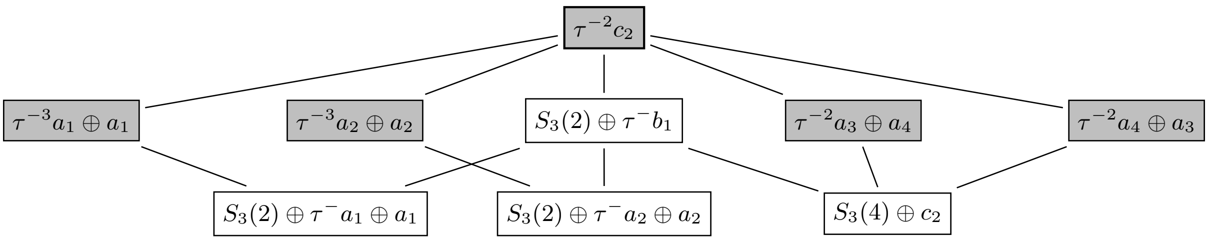


fig. A.31: Degeneration diagram of $c_1 \oplus S_3(5)$ in Period 5

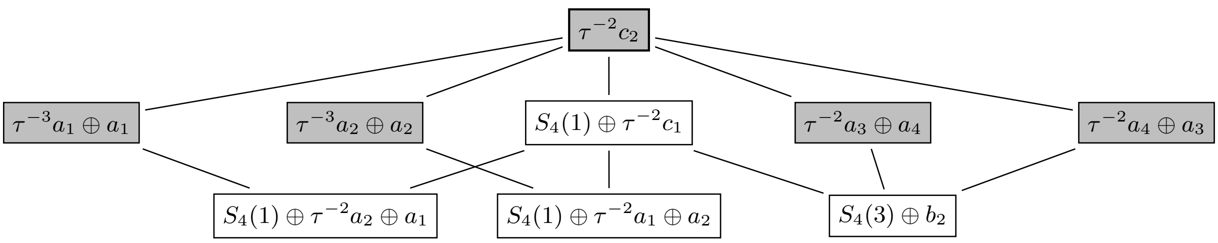


fig. A.32: Degeneration diagram of $c_1 \oplus S_4(5)$ in Period 5

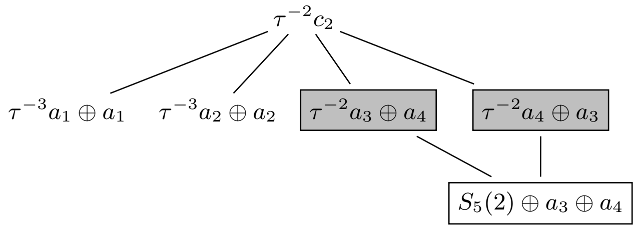


fig. A.33: Degeneration diagram of $c_1 \oplus S_5(5)$ in Period 5

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & 1 \\ & 1\;1\;1\;1 & \\ 0 & & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 1\;1\;1\;1 & \\ 1 & & 1 \end{bmatrix}$

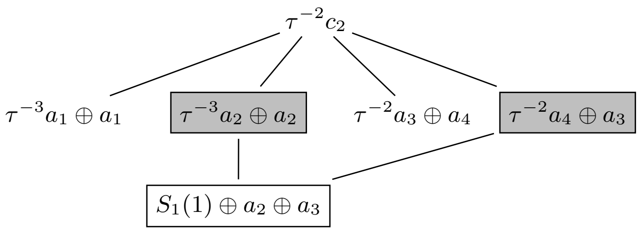
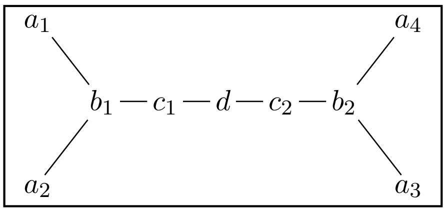


fig. A.34: Degeneration diagram of $c_1 \oplus S_1(2)$ in Period 2

A.8 $\tilde{D}8$



A.8.1 $\mathbf{U} = \mathbf{a}_1$

$$\mathbf{Codim}(U \oplus V, \tau^{-6}a_1) = 2$$

Period 6:	$S_1(1) = \begin{bmatrix} 1 & & 1 \\ & 1\;1\;1\;1\;1 & \\ 0 & & 1 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 1\;0\;0\;0\;0 & \\ 1 & & 0 \end{bmatrix}$	$S_3(1) = \begin{bmatrix} 0 & & 0 \\ & 0\;1\;0\;0\;0 & \\ 0 & & 0 \end{bmatrix}$
	$S_4(1) = \begin{bmatrix} 0 & & 0 \\ & 0\;0\;1\;0\;0 & \\ 0 & & 0 \end{bmatrix}$	$S_5(1) = \begin{bmatrix} 0 & & 0 \\ & 0\;0\;0\;1\;0 & \\ 0 & & 0 \end{bmatrix}$	$S_6(1) = \begin{bmatrix} 0 & & 0 \\ & 0\;0\;0\;0\;1 & \\ 0 & & 0 \end{bmatrix}$

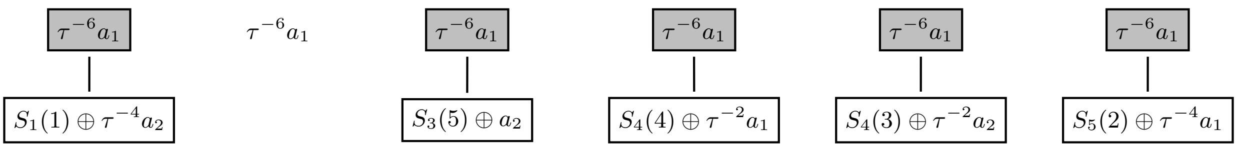


fig. A.35: Degeneration diagrams of $a_1 \oplus S_i(6)$, $i \in \{1, 2, 3, 4, 5, 6\}$ in Period 6

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & & & & & 0 \\ & 1 & 1 & 1 & 1 & 1 & \\ 1 & & & & & & 1 \end{bmatrix}$ $\quad S_2(1) = \begin{bmatrix} 0 & & & & & & 1 \\ & 1 & 1 & 1 & 1 & 1 & \\ 0 & & & & & & 0 \end{bmatrix}$

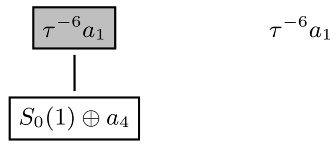


fig. A.36: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

A.8.2 $\mathbf{U = b_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-}b_2) = 3$$

Period 6:	$S_1(1) = \begin{bmatrix} 0 & & & & & & 0 \\ & 0 & 0 & 0 & 1 & 0 & \\ 0 & & & & & & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & & & & & 0 \\ & 0 & 0 & 0 & 0 & 1 & \\ 0 & & & & & & 0 \end{bmatrix}$	$S_3(1) = \begin{bmatrix} 0 & & & & & & 1 \\ & 1 & 1 & 1 & 1 & 1 & \\ 0 & & & & & & 1 \end{bmatrix}$
	$S_4(1) = \begin{bmatrix} 1 & & & & & & 0 \\ & 1 & 0 & 0 & 0 & 0 & \\ 1 & & & & & & 0 \end{bmatrix}$	$S_5(1) = \begin{bmatrix} 0 & & & & & & 0 \\ & 0 & 1 & 0 & 0 & 0 & \\ 0 & & & & & & 0 \end{bmatrix}$	$S_6(1) = \begin{bmatrix} 0 & & & & & & 0 \\ & 0 & 0 & 1 & 0 & 0 & \\ 0 & & & & & & 0 \end{bmatrix}$

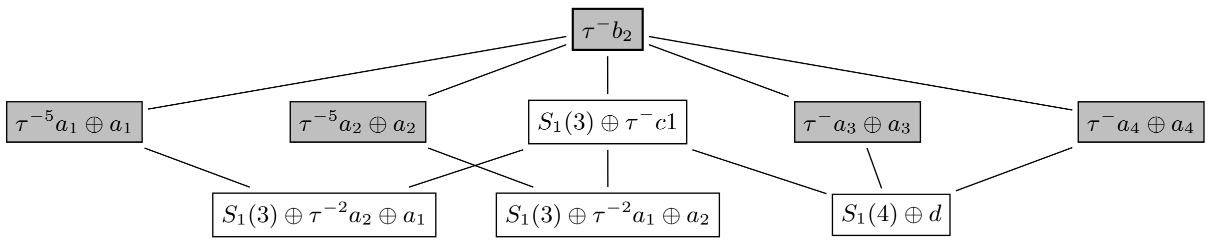


fig. A.37: Degeneration diagram of $b_1 \oplus S_1(6)$ in Period 6

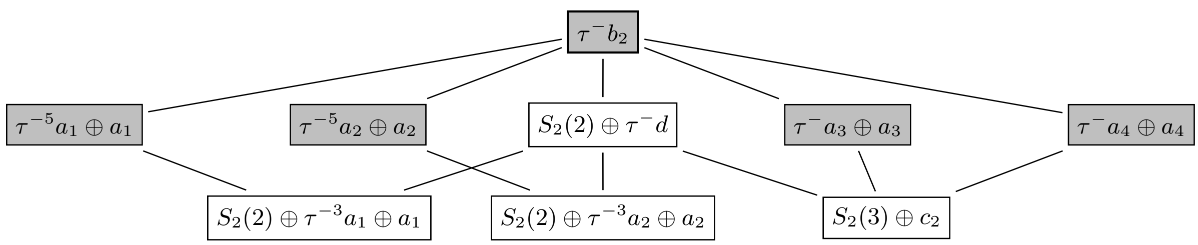


fig. A.38: Degeneration diagram of $b_1 \oplus S_2(6)$ in Period 6

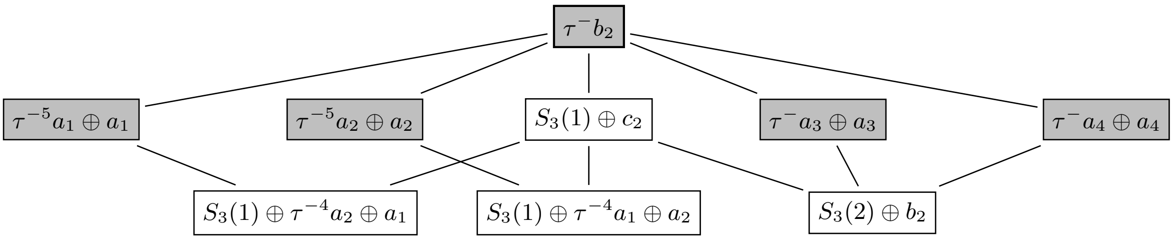


fig. A.39: Degeneration diagram of $b_1 \oplus S_3(6)$ in Period 6

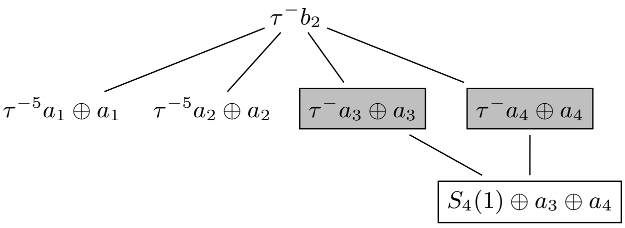


fig. A.40: Degeneration diagram of $b_1 \oplus S_4(6)$ in Period 6

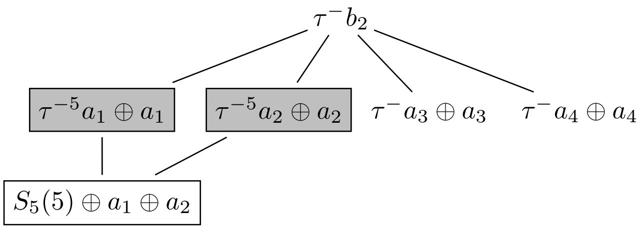


fig. A.41: Degeneration diagram of $b_1 \oplus S_5(6)$ in Period 6

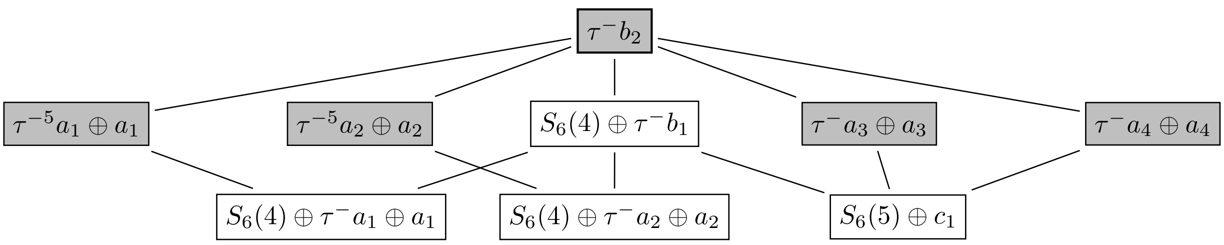


fig. A.42: Degeneration diagram of $b_1 \oplus S_6(6)$ in Period 6

Period 2: $S_1(1) = \begin{bmatrix} 1 & & & & & & 0 \\ & 1 & 1 & 1 & 1 & 1 & \\ 0 & & & & & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & & & 1 \\ & 1 & 1 & 1 & 1 & 1 & \\ 1 & & & & & & 0 \end{bmatrix}$

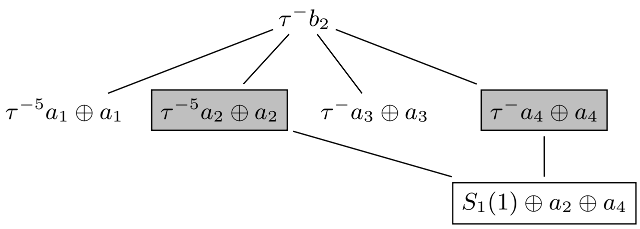


fig. A.43: Degeneration diagram of $b_1 \oplus S_1(2)$ in Period 2

A.8.3 $\mathbf{U = c_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-2}c_2) = 3$$

Period 6:	$S_1(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,0\,0\,1\,0 & \\ 0 & & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,0\,0\,0\,1 & \\ 0 & & 0 \end{bmatrix}$	$S_3(1) = \begin{bmatrix} 0 & & 1 \\ & 0\,1\,1\,1\,1 & \\ 0 & & 1 \end{bmatrix}$
	$S_4(1) = \begin{bmatrix} 0 & & 0 \\ & 1\,0\,0\,0\,0 & \\ 0 & & 0 \end{bmatrix}$	$S_5(1) = \begin{bmatrix} 1 & & 0 \\ & 1\,1\,0\,0\,0 & \\ 1 & & 0 \end{bmatrix}$	$S_6(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,0\,1\,0\,0 & \\ 0 & & 0 \end{bmatrix}$

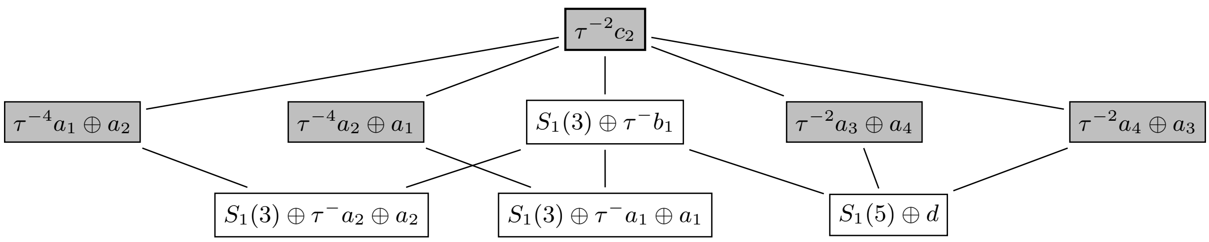


fig. A.44: Degeneration diagram of $c_1 \oplus S_1(6)$ in Period 6

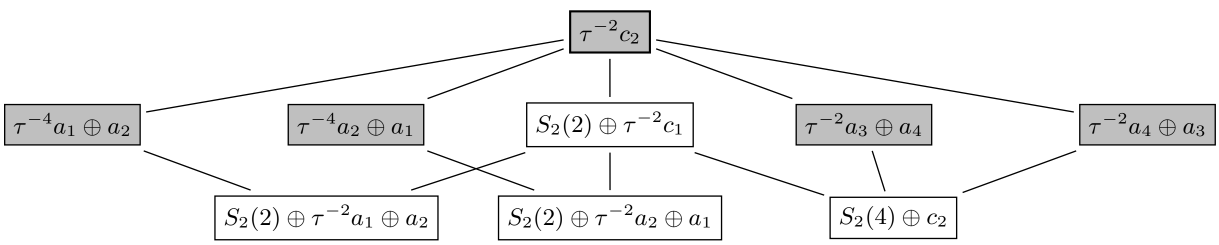


fig. A.45: Degeneration diagram of $c_1 \oplus S_2(6)$ in Period 6

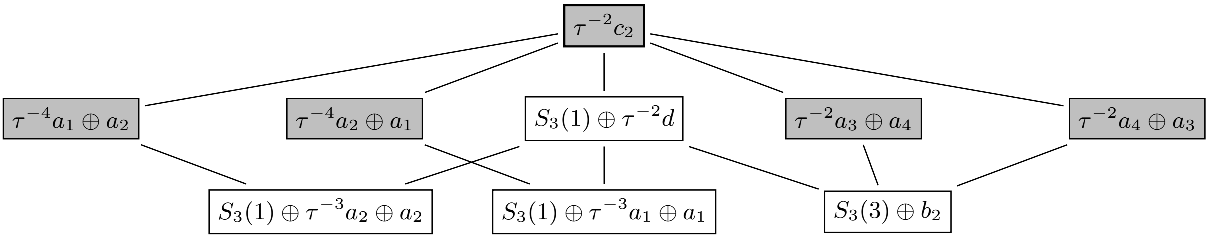


fig. A.46: Degeneration diagram of $c_1 \oplus S_3(6)$ in Period 6

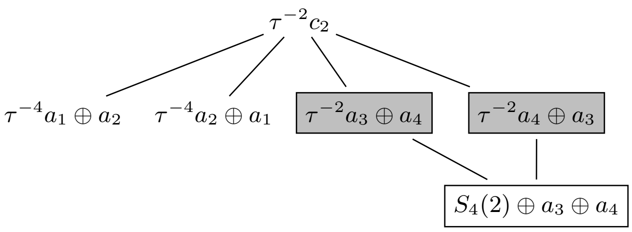


fig. A.47: Degeneration diagram of $c_1 \oplus S_4(6)$ in Period 6

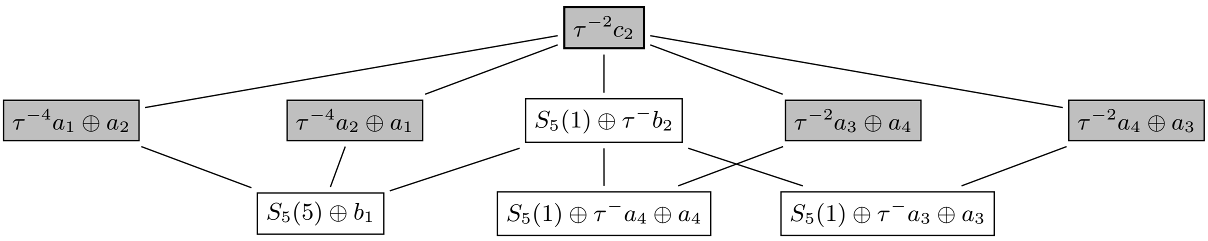


fig. A.48: Degeneration diagram of $c_1 \oplus S_5(6)$ in Period 6

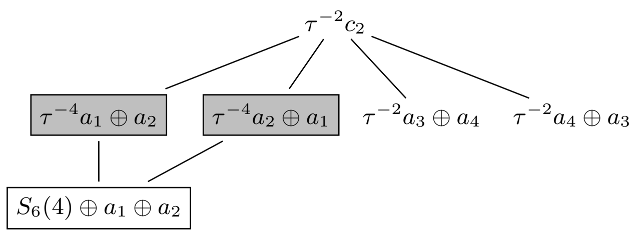


fig. A.49: Degeneration diagram of $c_1 \oplus S_6(6)$ in Period 6

Period 2:	$S_1(1) = \begin{bmatrix} 1 & & 0 \\ & 1\,1\,1\,1\,1 & \\ 0 & & 1 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 1 \\ & 1\,1\,1\,1\,1 & \\ 1 & & 0 \end{bmatrix}$

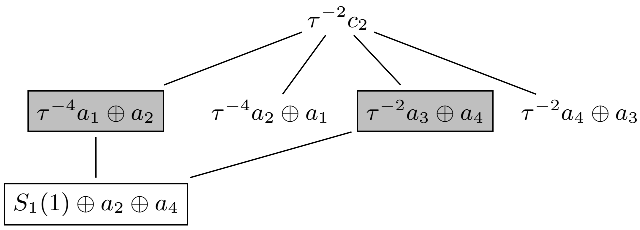


fig. A.50: Degeneration diagram of $c_1 \oplus S_1(2)$ in Period 2

A.8.4 $\mathbf{U = d}$

$$\mathbf{Codim}(U \oplus V, \tau^{-3}d) = 3$$

Period 6:	$S_1(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,0\,0\,1\,0 & \\ 0 & & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,0\,0\,0\,1 & \\ 0 & & 0 \end{bmatrix}$	$S_3(1) = \begin{bmatrix} 0 & & 1 \\ & 0\,0\,1\,1\,1 & \\ 0 & & 1 \end{bmatrix}$
	$S_4(1) = \begin{bmatrix} 0 & & 0 \\ & 0\,1\,0\,0\,0 & \\ 0 & & 0 \end{bmatrix}$	$S_5(1) = \begin{bmatrix} 0 & & 0 \\ & 1\,0\,0\,0\,0 & \\ 0 & & 0 \end{bmatrix}$	$S_6(1) = \begin{bmatrix} 1 & & 0 \\ & 1\,1\,1\,0\,0 & \\ 1 & & 0 \end{bmatrix}$

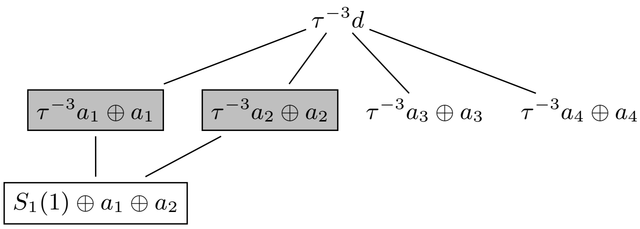


fig. A.51: Degeneration diagram of $d \oplus S_1(6)$ in Period 6

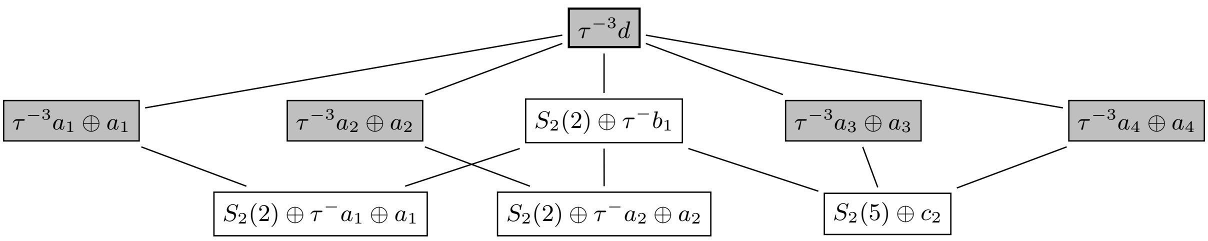


fig. A.52: Degeneration diagram of $d \oplus S_2(6)$ in Period 6

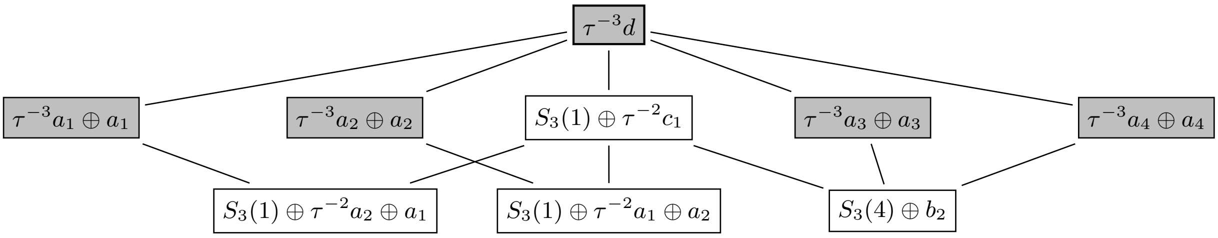


fig. A.53: Degeneration diagram of $d \oplus S_3(6)$ in Period 6

Period 2: $S_1(1) = \begin{bmatrix} 1 & & & & & & 0 \\ & 1 & 1 & 1 & 1 & 1 & \\ 0 & & & & & & 1 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} 0 & & & & & & 1 \\ & 1 & 1 & 1 & 1 & 1 & \\ 1 & & & & & & 0 \end{bmatrix}$

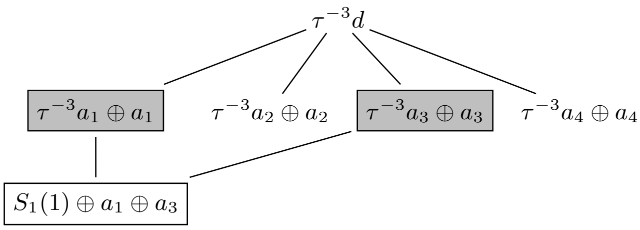
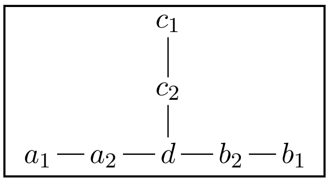


fig. A.54: Degeneration diagram of $d \oplus S_1(2)$ in Period 2

A.9 $\tilde{E}6$



A.9.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-6}a_1) = 2$$

Period 3: $S_1(1) = \begin{bmatrix} & & 0 & & \\ & & 0 & & \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} & & 0 & & \\ & & 1 & & \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$ $S_3(1) = \begin{bmatrix} & & 1 & & \\ & & 1 & & \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

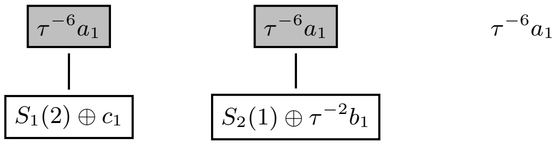


fig. A.55: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2, 3\}$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 0 & & \\ & & 1 & & \\ 0 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & \\ & & 1 & & \\ 1 & 1 & 2 & 1 & 1 \end{smallmatrix}\right]$

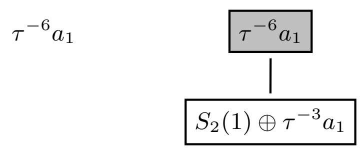


fig. A.56: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period

A.9.2 $\mathbf{U = a_2}$

$$\mathbf{Codim}(U \oplus V, \tau^{-3}a_2) = 3$$

Period 3:	$S_1(1) = \left[\begin{smallmatrix} & & 0 & & \\ & & 0 & & \\ 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 0 & & \\ & & 1 & & \\ 0 & 0 & 1 & 0 & 0 \end{smallmatrix}\right]$	$S_3(1) = \left[\begin{smallmatrix} & & 1 & & \\ & & 1 & & \\ 0 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$

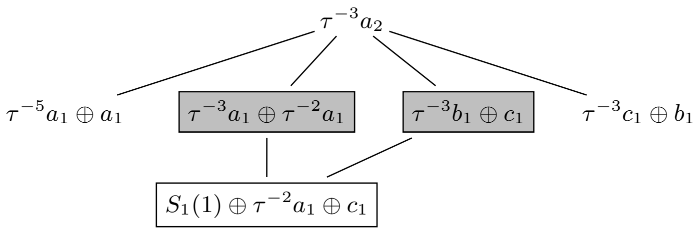


fig. A.57: Degeneration diagram of $a_2 \oplus S_1(3)$ in Period 3

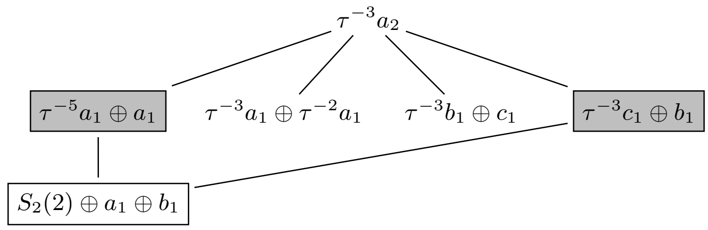


fig. A.58: Degeneration diagram of $a_2 \oplus S_2(3)$ in Period 3

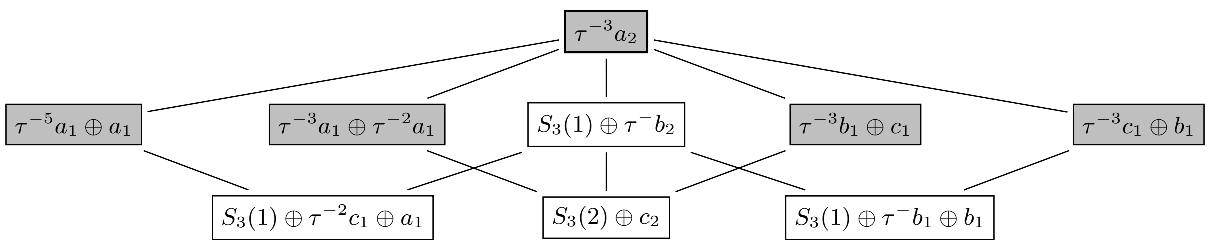


fig. A.59: Degeneration diagram of $a_2 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \begin{bmatrix} & & 0 & & \\ & & 1 & & \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} & & 1 & & \\ & & 1 & & \\ 0 & 1 & 2 & 1 & 1 \end{bmatrix}$

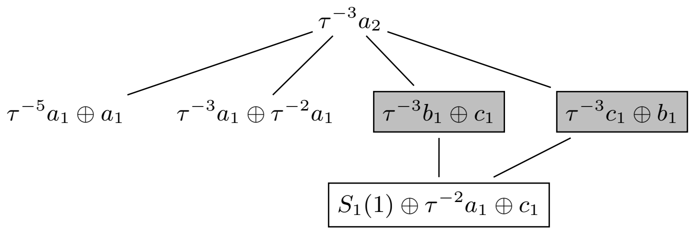


fig. A.60: Degeneration diagram of $a_2 \oplus S_1(2)$ in Period 2

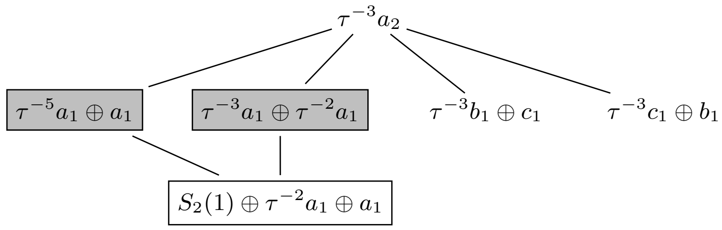


fig. A.61: Degeneration diagram of $a_2 \oplus S_2(2)$ in Period 2

A.9.3 $\mathbf{U = d}$

$$\mathbf{Codim}(U \oplus V, \tau^{-2}a_2) = 4$$

Now the diagrams are a bit on the large side to give an explicit description in it. We give in table 3 the preprojective deformations for $d \oplus V$, where V is indecomposable, regular with $\underline{\dim}\, V = \underline{\delta}$.

Period 3:	$S_1(1) = \begin{bmatrix} & & 0 & & \\ & & 0 & & \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$	$S_2(1) = \begin{bmatrix} & & 0 & & \\ & & 1 & & \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	$S_3(1) = \begin{bmatrix} & & 1 & & \\ & & 1 & & \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

Codimension	Dot	Preprojective deformations
4	1	$\tau^{-2}d$
3	2	$\tau^{-2}a_1 \oplus \tau^{-}a_2$
	3	$\tau^{-2}a_2 \oplus a_1$
	4	$\tau^{-2}b_1 \oplus \tau^{-}b_2$
	5	$\tau^{-2}b_2 \oplus b_1$
	6	$\tau^{-2}c_1 \oplus \tau^{-}c_2$
	7	$\tau^{-2}c_2 \oplus c_1$
2	8	$\tau^{-2}a_1 \oplus \tau^{-}a_1 \oplus a_1$
	9	$\tau^{-4}c_1 \oplus c_2$
	10	$\tau^{-3}b_1 \oplus a_1 \oplus c_1$
	11	$\tau^{-2}b_1 \oplus \tau^{-}b_1 \oplus b_1$
	12	$\tau^{-4}a_1 \oplus a_2$
	13	$\tau^{-3}c_1 \oplus a_1 \oplus b_1$
	14	$\tau^{-2}c_1 \oplus \tau^{-}c_1 \oplus c_1$
	15	$\tau^{-4}b_1 \oplus b_2$
	16	$\tau^{-3}a_1 \oplus b_1 \oplus c_1$

Table 3: Preprojective deformations of $d \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

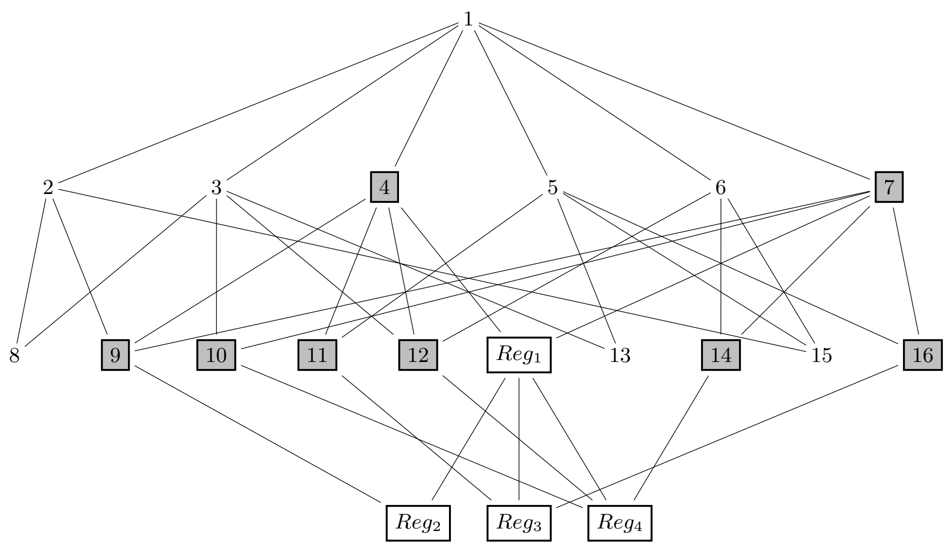


fig. A.62: Degeneration diagram of $d \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_1(1) \oplus \tau^{-}b_2 \oplus c_1$
1	Reg_2	$S_1(1) \oplus \tau^{-2}a_1 \oplus c_2$
	Reg_3	$S_1(1) \oplus \tau^{-}b_1 \oplus b_1 \oplus c_1$
	Reg_4	$S_1(2) \oplus a_2 \oplus c_1$

Table 4: Deformations with nonzero regular part of $d \oplus S_1(3)$ in Period 3

Period 2: $S_1(1) = \begin{bmatrix} & & 0 & & \\ & & 1 & & \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$ $S_2(1) = \begin{bmatrix} & & 1 & & \\ & & 1 & & \\ 1 & 1 & 2 & 1 & 1 \end{bmatrix}$

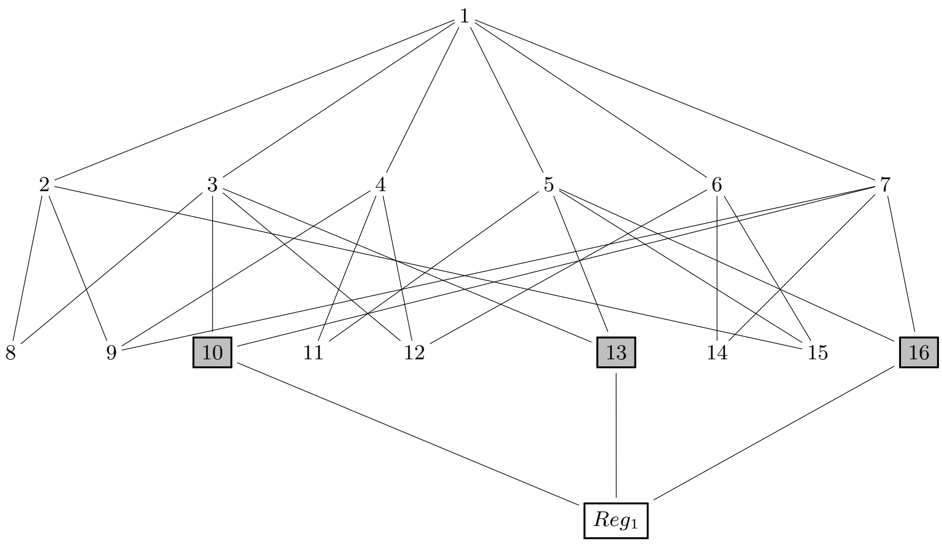


fig. A.63: Degeneration diagram of $d \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part
1	Reg_1	$S_1(1) \oplus a_1 \oplus b_1 \oplus c_1$

Table 5: Deformations with nonzero regular part of $d \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_2(1) \oplus \tau^- d$
1	Reg_2	$S_2(1) \oplus \tau^- a_1 \oplus a_2$
	Reg_3	$S_2(1) \oplus \tau^- b_1 \oplus b_2$
	Reg_4	$S_2(1) \oplus \tau^- c_1 \oplus c_2$

Table 6: Deformations with nonzero regular part of $d \oplus S_2(2)$ in Period 2

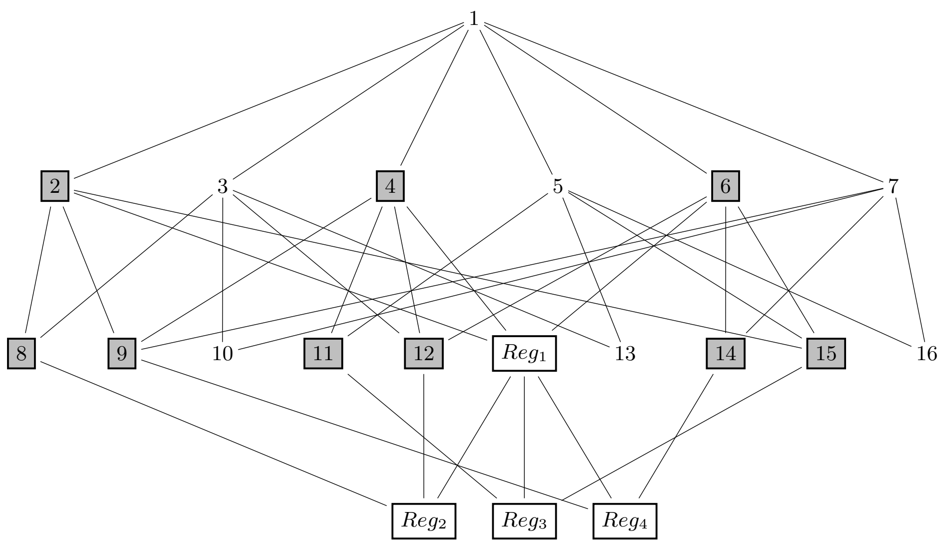


fig. A.64: Degeneration diagram of $d \oplus S_2(2)$ in Period 2

A.10 $\tilde{E}7$

$$\begin{array}{ccccccccccccc} & & & & & & c & & & & & & \\ & & & & & & | & & & & & & \\ a_1 & — & a_2 & — & a_3 & — & d & — & b_3 & — & b_2 & — & b_1 \end{array}$$

A.10.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-12} a_1) = 2$$

Period 4: $S_1(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$

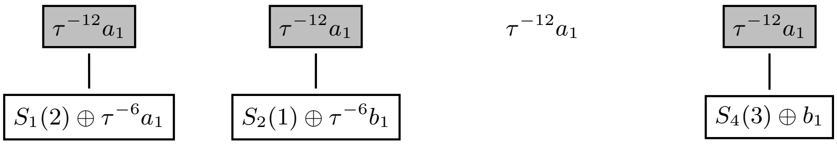


fig. A.65: Degeneration diagrams of $a_1 \oplus S_i(4)$, $i \in \{1,2,3,4\}$ in Period 4

Period 3: $S_1(1) = \left[\begin{smallmatrix} &&&0&&& \\ 0&0&1&1&1&1&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} &&&1&&& \\ 1&1&1&2&1&1&1 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} &&&1&&& \\ 0&1&1&1&1&0&0 \end{smallmatrix}\right]$

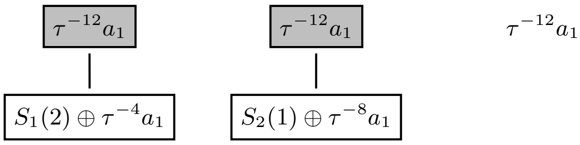


fig. A.66: Degeneration diagrams of $a_1 \oplus S_i(3)$, $i \in \{1,2,3\}$ in Period 3

Period 2: $S_1(1) = \left[\begin{smallmatrix} &&&1&&& \\ 0&1&1&2&1&1&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} &&&1&&& \\ 1&1&2&2&2&1&1 \end{smallmatrix}\right]$

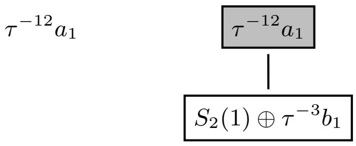


fig. A.67: Degeneration diagrams of $a_1 \oplus S_i(2)$, $i \in \{1,2\}$ in Period 2

A.10.2 $\mathbf{U = a_2}$

$$\mathbf{Codim}(U \oplus V, \tau^{-4}b_2) = 3$$

Period 4:

$S_1(1) = \left[\begin{smallmatrix} &&&0&&& \\ 0&0&0&1&1&0&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} &&&1&&& \\ 0&1&1&1&1&1&0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} &&&0&&& \\ 1&1&1&1&1&1&1 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} &&&1&&& \\ 0&0&1&1&0&0&0 \end{smallmatrix}\right]$

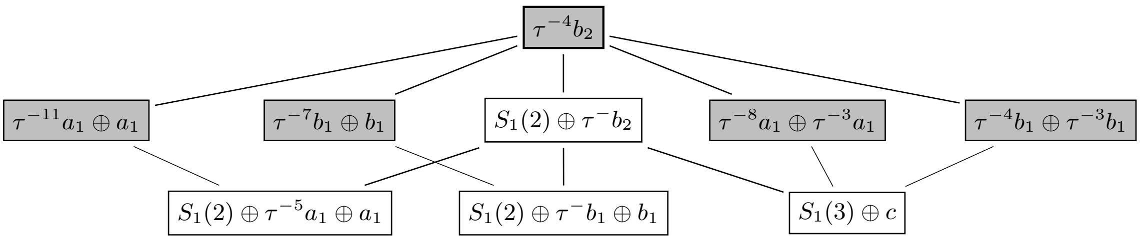


fig. A.68: Degeneration diagram of $a_2 \oplus S_1(4)$ in Period 4

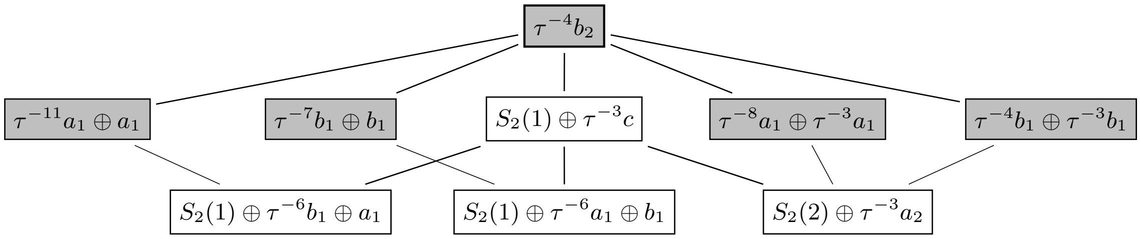


fig. A.69: Degeneration diagram of $a_2 \oplus S_2(4)$ in Period 4

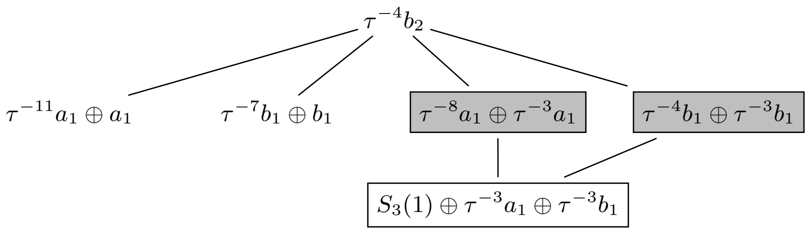


fig. A.70: Degeneration diagram of $a_2 \oplus S_3(4)$ in Period 4

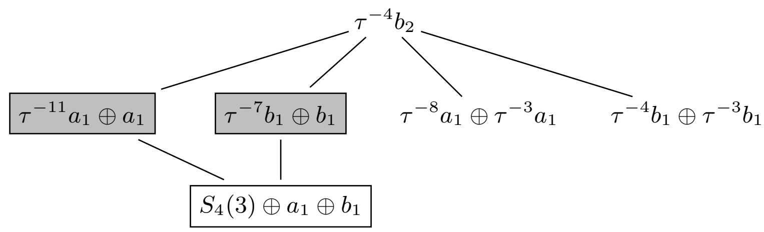


fig. A.71: Degeneration diagram of $a_2 \oplus S_4(4)$ in Period 4

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 1 & 2 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$

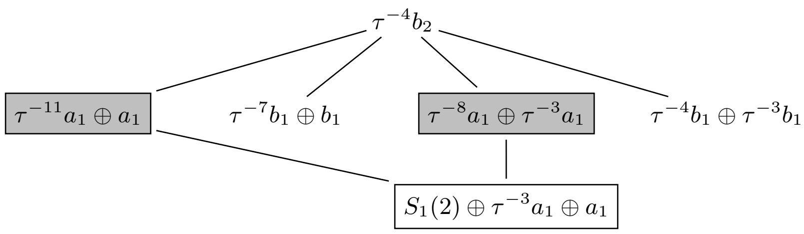


fig. A.72: Degeneration diagram of $a_2 \oplus S_1(3)$ in Period 3

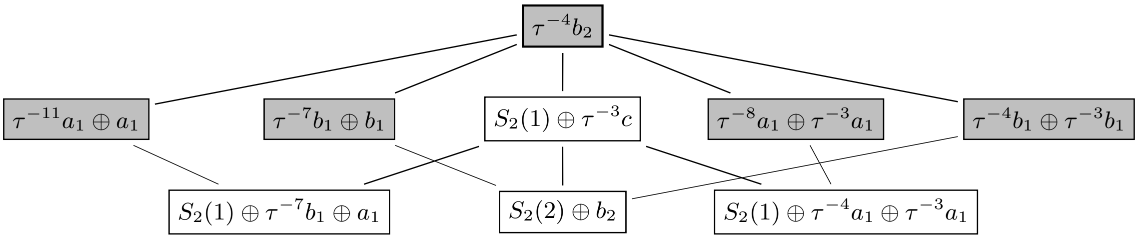


fig. A.73: Degeneration diagram of $a_2 \oplus S_2(3)$ in Period 3

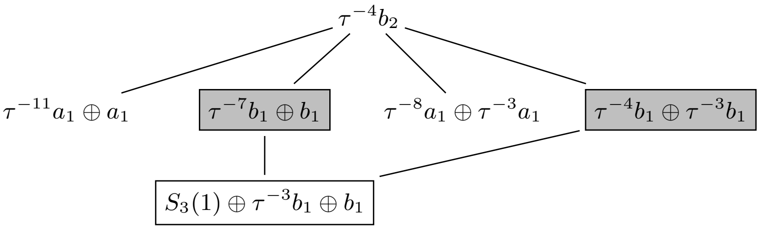


fig. A.74: Degeneration diagram of $a_2 \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 2 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

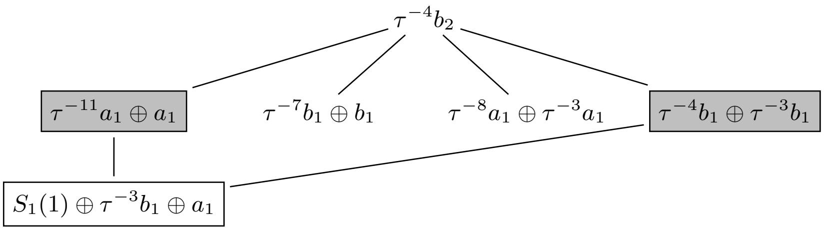


fig. A.75: Degeneration diagram of $a_2 \oplus S_1(2)$ in Period 2

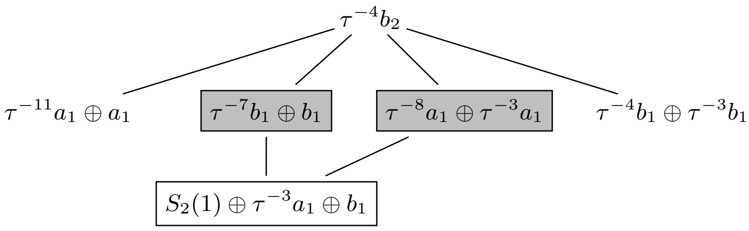


fig. A.76: Degeneration diagram of $a_2 \oplus S_2(2)$ in Period 2

A.10.3 $\mathbf{U = a_3}$

$$\mathbf{Codim}(U \oplus V, \tau^{-4}a_3) = 4$$

Codimension	Dot	Preprojective deformations				
4	1					$\tau^{-4}a_3$
3	2			$\tau^{-10}a_1$	$\oplus$	$\tau^{-}a_2$
	3			$\tau^{-4}a_2$	$\oplus$	$\tau^{-3}a_1$
	4			$\tau^{-6}a_1$	$\oplus$	$\tau^{-}b_2$
	5			$\tau^{-4}b_2$	$\oplus$	a_1
	6			$\tau^{-4}b_1$	$\oplus$	$\tau^{-2}c$
	7			$\tau^{-4}c$	$\oplus$	b_1
2	8	$\tau^{-4}a_1$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-2}a_1$
	9	$\tau^{-7}a_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	a_1
	10			$\tau^{-8}b_1$	$\oplus$	c
	11			$\tau^{-10}a_1$	$\oplus$	a_2
	12			$\tau^{-8}a_1$	$\oplus$	b_2
	13	$\tau^{-5}b_1$	$\oplus$	$\tau^{2}a_1$	$\oplus$	b_1
	14	$\tau^{-6}a_1$	$\oplus$	$\tau^{-}b_1$	$\oplus$	b_1
	15	$\tau^{-4}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_1
	16	$\tau^{-3}a_1$	$\oplus$	b_1	$\oplus$	c_1

Table 7: Preprojective deformations of $a_3 \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

Period 4:

$$S_1(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{smallmatrix}\right] \quad S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$$

$$S_3(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right] \quad S_4(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(2)$	$\oplus$	$\tau^{-}b_2$	$\oplus$	a_1
1	Reg_2			$S_1(2)$	$\oplus$	$\tau^{-4}a_1$	$\oplus$	a_2
	Reg_3			$S_1(3)$	$\oplus$	c	$\oplus$	a_1
	Reg_4	$S_1(2)$	$\oplus$	$\tau^{-}b_1$	$\oplus$	a_1	$\oplus$	b_1

Table 8: Deformations with nonzero regular part of $a_3 \oplus S_1(4)$ in Period 4

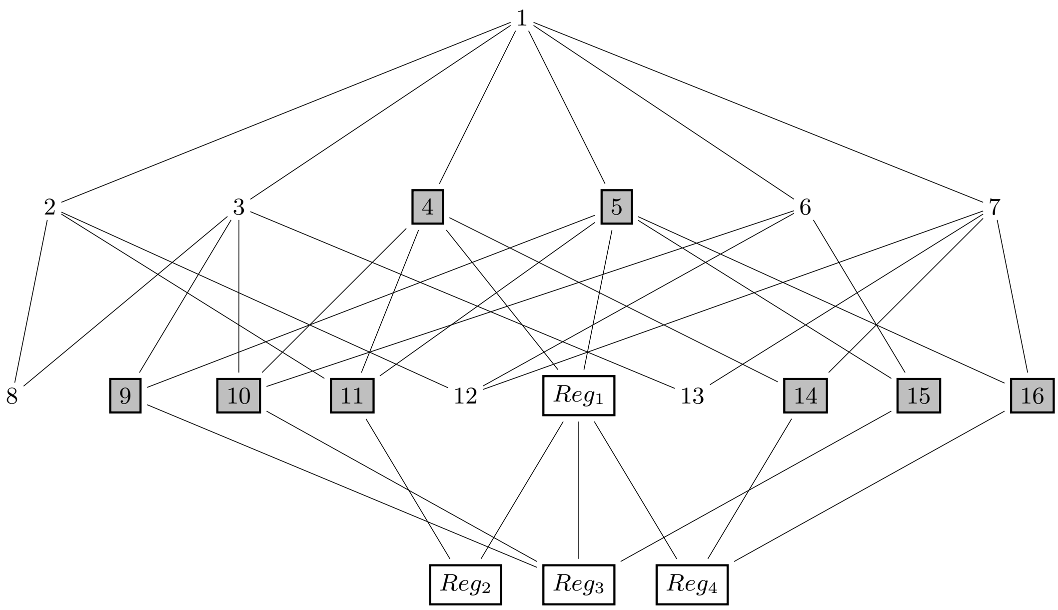


fig. A.77: Degeneration diagram of $a_3 \oplus S_1(4)$ in Period 4

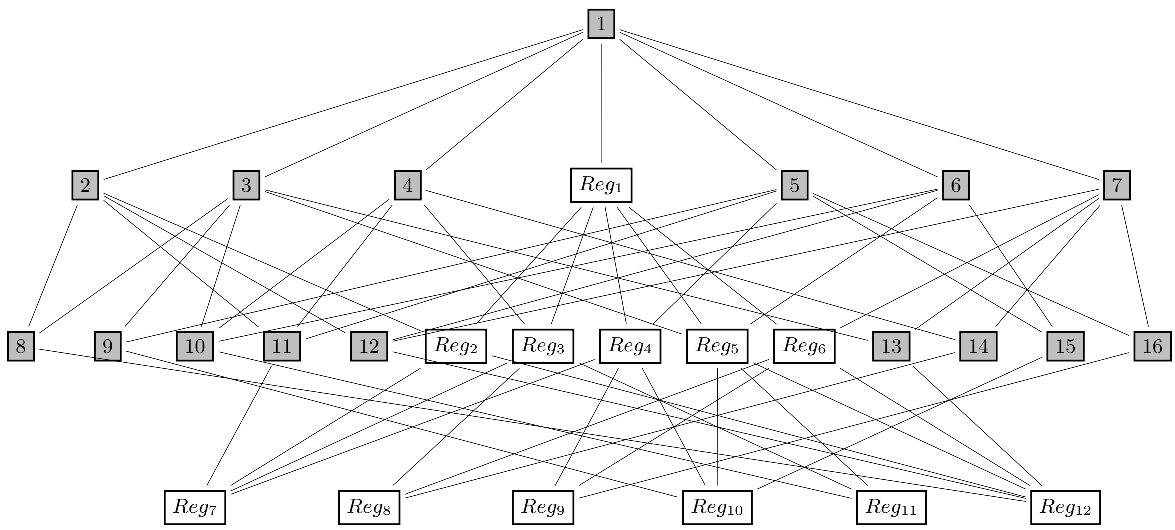


fig. A.78: Degeneration diagram of $a_3 \oplus S_2(4)$ in Period 4

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_2(1) \oplus \tau^{-2}b_2$
2	Reg_2	$S_2(1) \oplus \tau^{-4}a_1 \oplus \tau^{-}c$
	Reg_3	$S_2(1) \oplus \tau^{-2}b_1 \oplus \tau^{-}b_2$
	Reg_4	$S_2(1) \oplus \tau^{-3}c \oplus a_1$
	Reg_5	$S_2(2) \oplus \tau^{-2}a_3$
	Reg_6	$S_2(1) \oplus \tau^{-2}a_2 \oplus a_1$
1	Reg_7	$S_2(1) \oplus \tau^{-6}b_1 \oplus \tau^{-2}a_2$
	Reg_8	$S_2(1) \oplus \tau^{-2}b_1 \oplus b_1$
	Reg_9	$S_2(1) \oplus \tau^{-5}a_1 \oplus b_1 \oplus a_1$
	Reg_{10}	$S_2(2) \oplus \tau^{-2}a_2 \oplus a_1$
	Reg_{11}	$S_2(2) \oplus \tau^{-2}b_1 \oplus c$
	Reg_{12}	$S_2(3) \oplus b_3$

Table 9: Deformations with nonzero regular part of $a_3 \oplus S_2(4)$ in Period 4

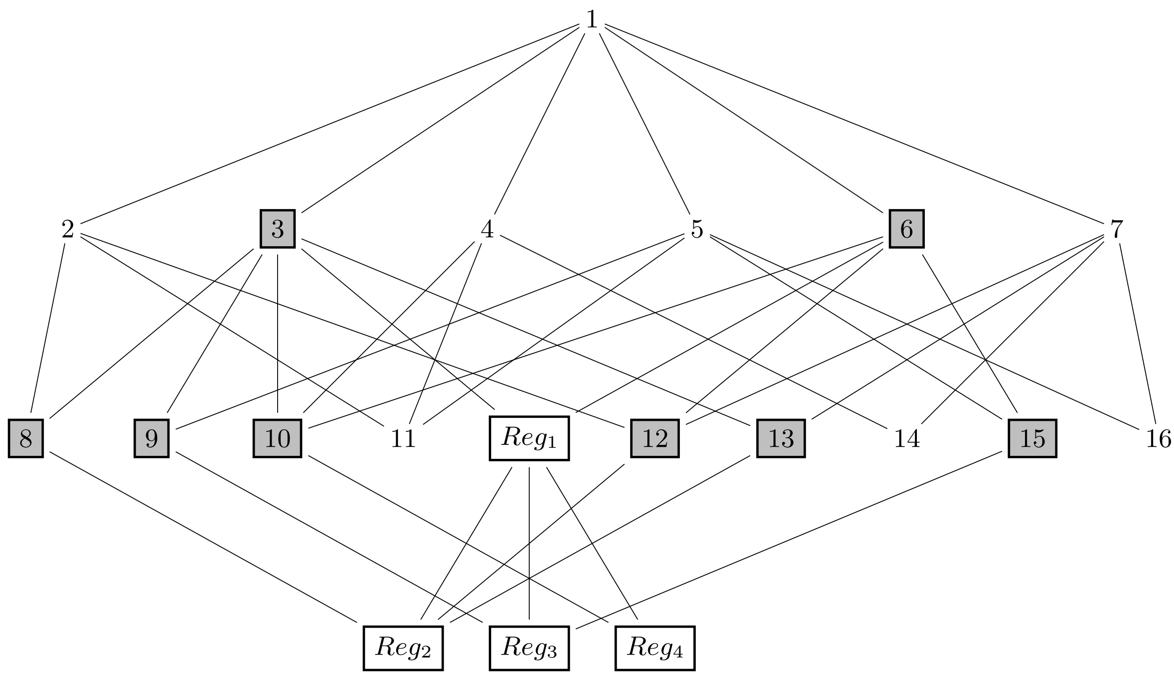


fig. A.79: Degeneration diagram of $a_3 \oplus S_3(4)$ in Period 4

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_4(1)$	$\oplus$	$\tau^{-3}a_2$	$\oplus$	b_1
1	Reg_2	$S_4(1)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1
	Reg_3			$S_4(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_2
	Reg_4			$S_4(3)$	$\oplus$	a_2	$\oplus$	b_1

Table 11: Deformations with nonzero regular part of $a_3 \oplus S_4(4)$ in Period 4

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_3(1)$	$\oplus$	$\tau^{-2}c$	$\oplus$	$\tau^{-2}a_1$
1	Reg_2			$S_3(2)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_2
	Reg_3	$S_3(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	a_1
	Reg_4			$S_3(1)$	$\oplus$	$\tau^{-6}a_1$	$\oplus$	c

Table 10: Deformations with nonzero regular part of $a_3 \oplus S_3(4)$ in Period 4

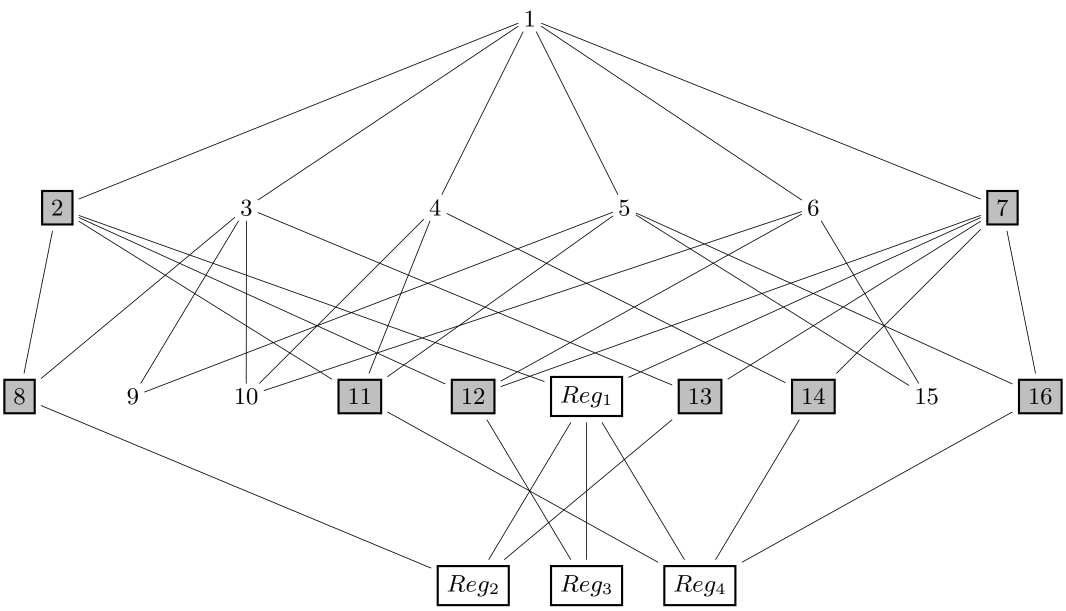


fig. A.80: Degeneration diagram of $a_3 \oplus S_4(4)$ in Period 4

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 1&1&1&1&1&1&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&0&1&2&1&1&1 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&1&1&1&1&0&0 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_1(1) \oplus \tau^{-2}a_1 \oplus \tau^{-}b_2$
1	Reg_2	$S_1(2) \oplus \tau^{-2}a_1 \oplus a_2$
	Reg_3	$S_1(1) \oplus \tau^{-4}b_1 \oplus c$
	Reg_4	$S_1(1) \oplus \tau^{-2}a_1 \oplus \tau^{-}b_1 \oplus b_1$

Table 12: Deformations with nonzero regular part of $a_3 \oplus S_1(3)$ in Period 3

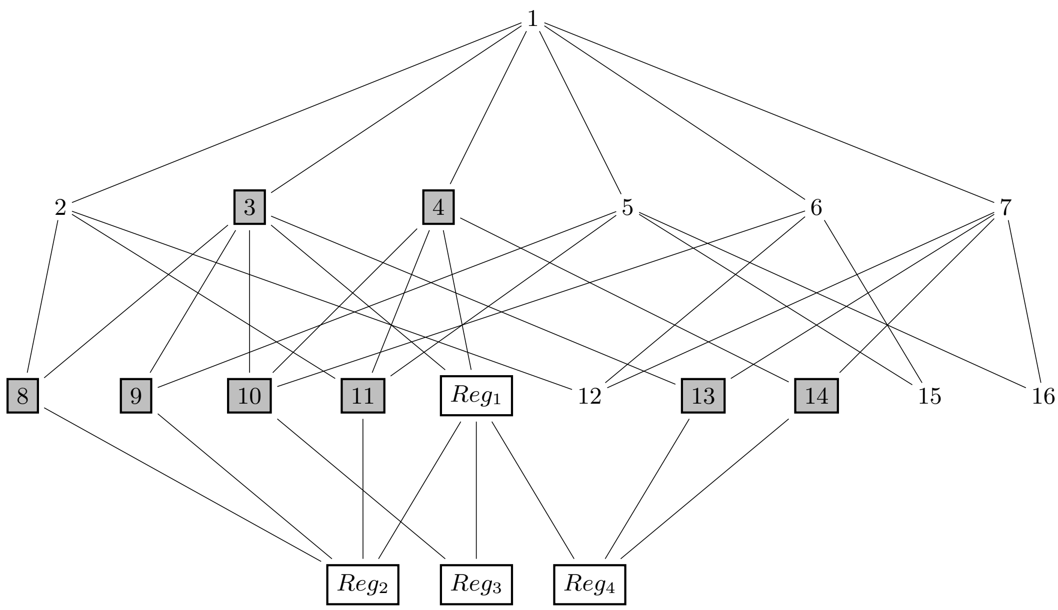


fig. A.81: Degeneration diagram of $a_3 \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_2(1) \oplus \tau^{-3}a_2 \oplus a_1$
1	Reg_2	$S_2(1) \oplus \tau^{-3}a_1 \oplus \tau^{-2}a_1 \oplus a_1$
	Reg_3	$S_2(1) \oplus \tau^{-6}a_1 \oplus a_2$
	Reg_4	$S_2(2) \oplus b_2 \oplus a_1$

Table 13: Deformations with nonzero regular part of $a_3 \oplus S_2(3)$ in Period 3

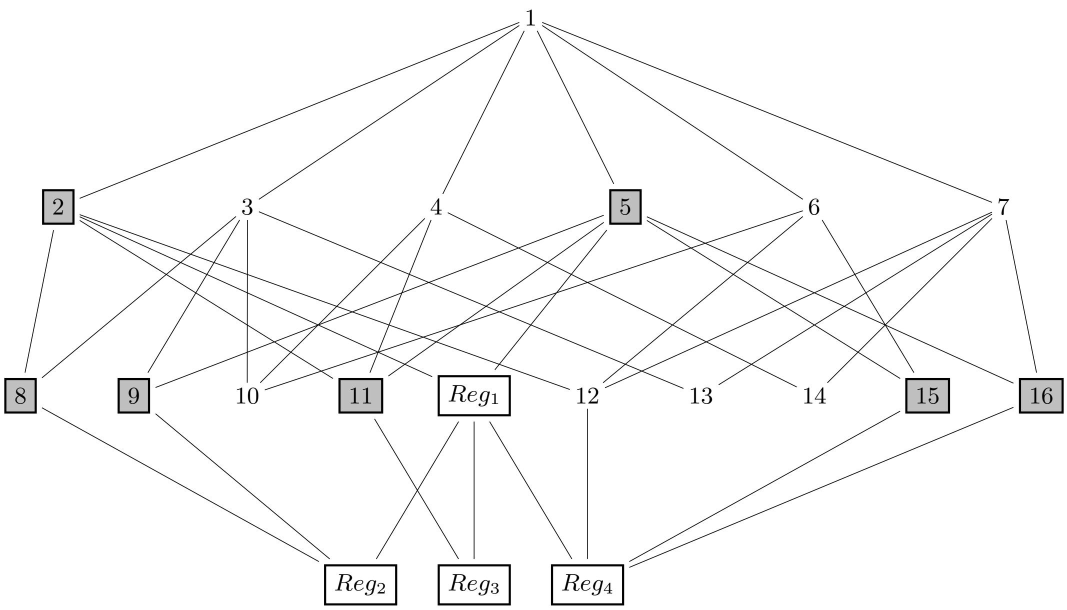


fig. A.82: Degeneration diagram of $a_3 \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_3(1) \oplus \tau^{-2}c \oplus b_1$
1	Reg_2	$S_3(2) \oplus c \oplus a_1$
	Reg_3	$S_3(1) \oplus \tau^{-6}a_1 \oplus b_1$
	Reg_4	$S_3(1) \oplus \tau^{-3}b_1 \oplus a_1 \oplus b_1$

Table 14: Deformations with nonzero regular part of $a_3 \oplus S_3(3)$ in Period 3

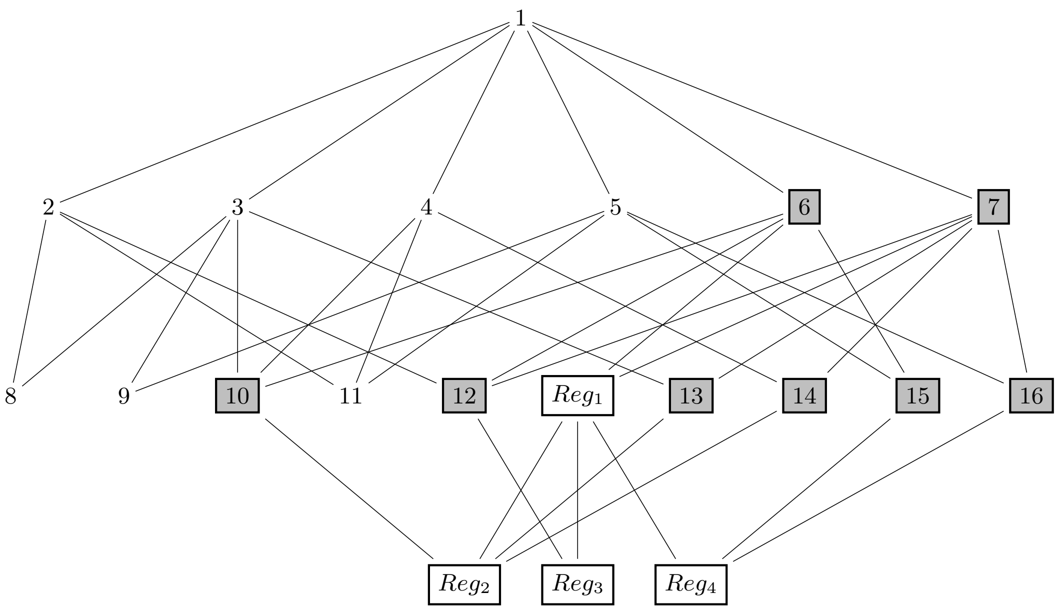


fig. A.83: Degeneration diagram of $a_3 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\,0\ 1\ 1\ {}^{1}_{2}\ 1\ 1\ 0\,\right]$	$S_2(1) = \left[\,1\ 1\ 2\ {}^{1}_{2}\ 2\ 1\ 1\,\right]$

Codimension	Dot	Deformations with nonzero regular part
1	Reg_2	$S_1(1) \quad \oplus \quad \tau^{-2}a_1 \quad \oplus \quad \tau^{-}a_1 \quad \oplus \quad b_1$

Table 15: Deformations with nonzero regular part of $a_3 \oplus S_1(2)$ in Period 2

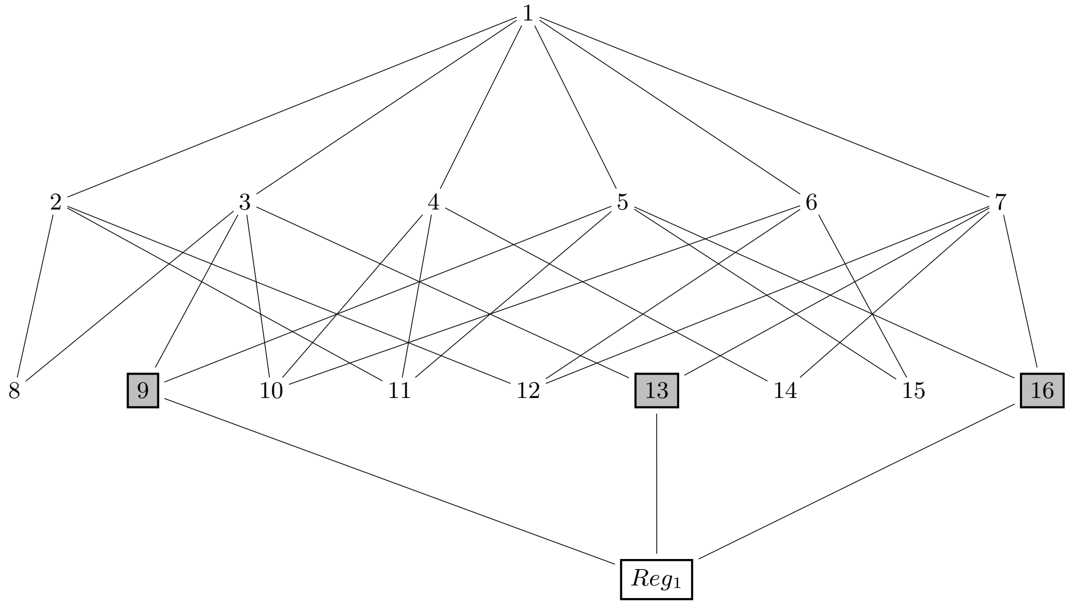


fig. A.84: Degeneration diagram of $a_3 \oplus S_1(2)$ in Period 2

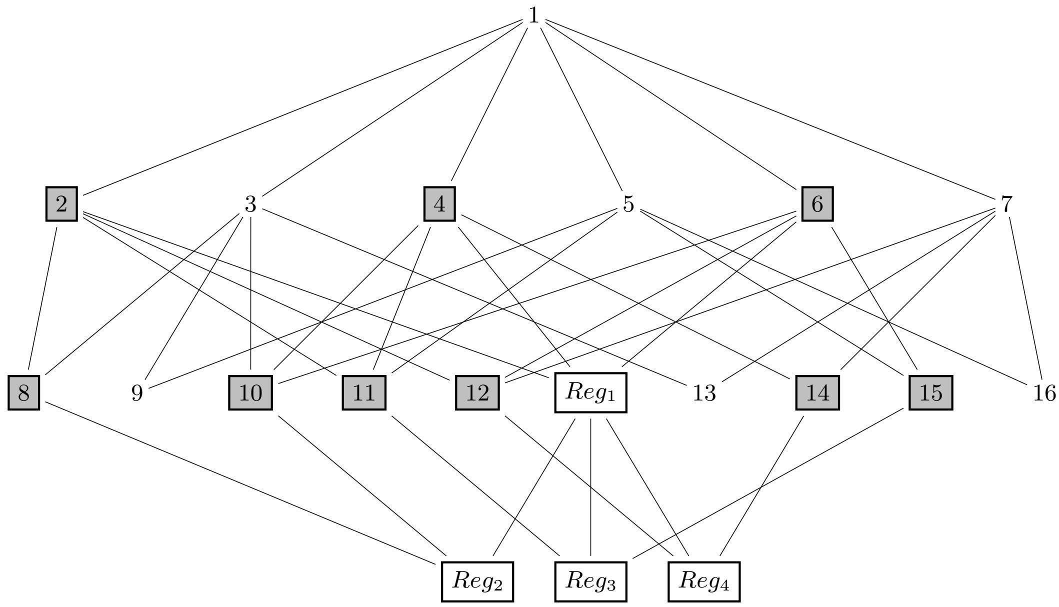


fig. A.85: Degeneration diagram of $a_3 \oplus S_2(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_2(1) \oplus \tau^{-}b_3$
1	Reg_2	$S_2(1) \oplus \tau^{-3}a_1 \oplus c$
	Reg_3	$S_2(1) \oplus \tau^{-3}b_1 \oplus a_2$
	Reg_4	$S_2(1) \oplus \tau^{-}b_1 \oplus b_2$

Table 16: Deformations with nonzero regular part of $a_3 \oplus S_2(2)$ in Period 2

A.10.4 $\mathbf{U = d}$

$$\mathbf{Codim}(U \oplus V, \tau^{-3}d) = 5$$

Period 4:

$S_1(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&0&0&1&1&1&0 \end{smallmatrix}\right]$ $\quad S_2(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0&0&1&1&1&1&1 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&1&1&1&0&0&0 \end{smallmatrix}\right]$ $\quad S_4(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 1&1&1&1&1&0&0 \end{smallmatrix}\right]$

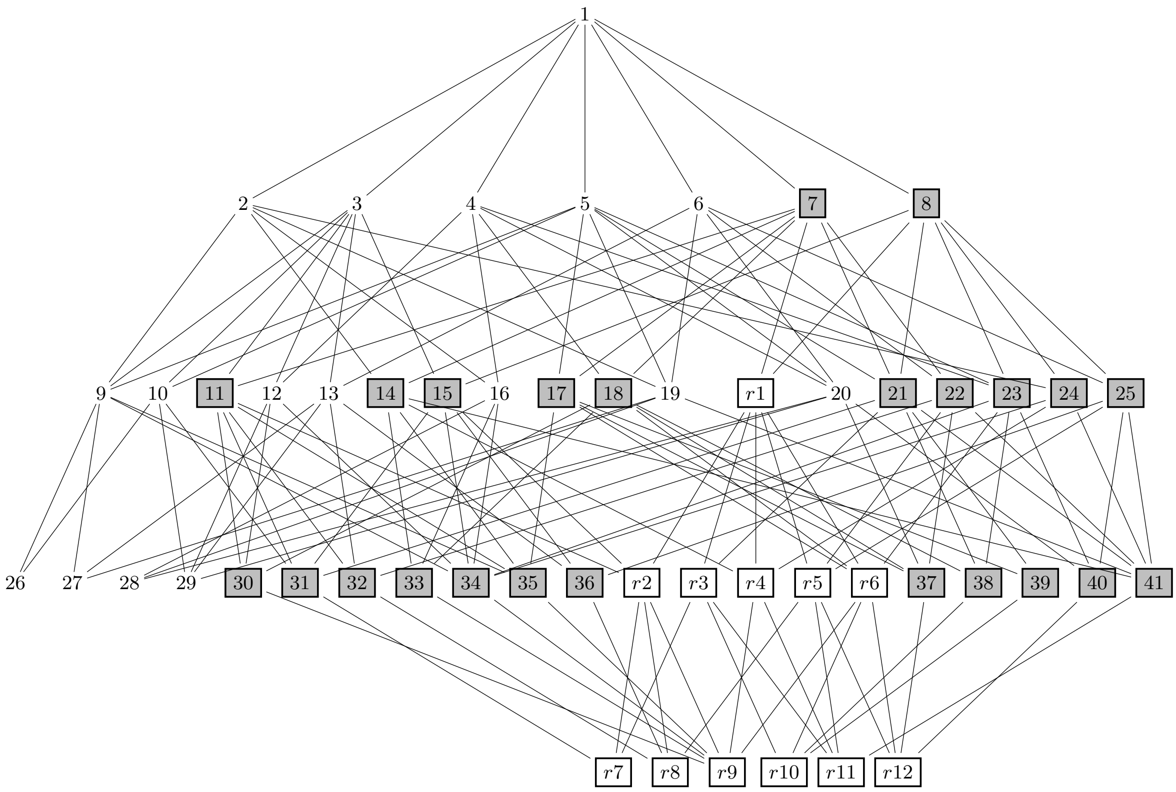


fig. A.86: Degeneration diagram of $d \oplus S_1(4)$ in Period 4

Codimension	Dot	Preprojective deformations
5	1	$\tau^{-3}d$
4	2	$\tau^{-3}a_1 \oplus \tau^{-2}a_3$
	3	$\tau^{-3}b_3 \oplus b_1$
	4	$\tau^{-3}a_2 \oplus \tau^{-1}a_2$
	5	$\tau^{-3}c \oplus \tau^{-2}c$
	6	$\tau^{-3}b_2 \oplus \tau^{-1}b_2$
	7	$\tau^{-3}a_3 \oplus a_1$
	8	$\tau^{-3}b_1 \oplus \tau^{-2}b_3$
3	9	$\tau^{-4}a_2 \oplus b_2$
	10	$\tau^{-4}a_1 \oplus \tau^{-2}c \oplus b_1$
	11	$\tau^{-4}c \oplus a_1 \oplus b_1$
	12	$\tau^{-5}b_1 \oplus \tau^{-1}a_2 \oplus b_1$
	13	$\tau^{-3}b_2 \oplus b_1 \oplus \tau^{-1}b_1$
	14	$\tau^{-3}a_1 \oplus \tau^{-2}a_2 \oplus a_1$
	15	$\tau^{-3}b_1 \oplus \tau^{-2}b_2 \oplus b_1$
	16	$\tau^{-3}a_1 \oplus \tau^{-1}a_2 \oplus \tau^{-2}a_1$
	17	$\tau^{-4}b_1 \oplus \tau^{-2}c \oplus a_1$
	18	$\tau^{-3}a_2 \oplus a_1 \oplus \tau^{-1}a_1$
	19	$\tau^{-6}b_1 \oplus \tau^{-1}b_3$
	20	$\tau^{-5}c \oplus c$
	21	$\tau^{-4}b_2 \oplus a_2$
	22	$\tau^{-5}a_1 \oplus \tau^{-1}b_2 \oplus a_1$
	23	$\tau^{-6}a_1 \oplus \tau^{-1}a_3$
	24	$\tau^{-3}a_1 \oplus \tau^{-1}c \oplus \tau^{-3}b_1$
	25	$\tau^{-3}b_1 \oplus \tau^{-1}b_2 \oplus \tau^{-2}b_1$
2	26	$\tau^{-4}a_1 \oplus b_2 \oplus \tau^{-3}a_1$
	27	$\tau^{-6}b_1 \oplus b_2 \oplus \tau^{-1}b_1$
	28	$\tau^{-6}b_1 \oplus c \oplus \tau^{-2}a_1$
	29	$\tau^{-8}a_1 \oplus c \oplus b_1$
	30	$\tau^{-5}b_1 \oplus a_1 \oplus \tau^{-1}a_1 \oplus b_1$
	31	$\tau^{-7}b_1 \oplus a_2 \oplus b_1$
	32	$\tau^{-5}a_1 \oplus a_1 \oplus b_1 \oplus \tau^{-1}b_1$
	33	$\tau^{-3}a_1 \oplus a_1 \oplus \tau^{-1}a_1 \oplus \tau^{-2}a_1$
	34	$\tau^{-9}b_1 \oplus b_3$
	35	$\tau^{-7}a_1 \oplus b_2 \oplus a_1$
	36	$\tau^{-3}b_1 \oplus b_1 \oplus \tau^{-1}b_1 \oplus \tau^{-2}b_1$
	37	$\tau^{-8}b_1 \oplus c \oplus a_1$
	38	$\tau^{-6}a_1 \oplus a_2 \oplus \tau^{-1}a_1$
	39	$\tau^{-4}b_1 \oplus a_2 \oplus \tau^{-3}b_1$
	40	$\tau^{-6}a_1 \oplus c \oplus \tau^{-2}b_1$
	41	$\tau^{-9}a_1 \oplus a_3$

Table 17: Preprojective deformations of $d \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_1(1) \oplus \tau^{-2}b_3 \oplus a_1$
2	Reg_2	$S_1(1) \oplus \tau^{-2}b_2 \oplus a_1 \oplus b_1$
	Reg_3	$S_1(1) \oplus \tau^{-3}c \oplus a_2$
	Reg_4	$S_1(1) \oplus \tau^{-3}a_1 \oplus \tau^{-1}c \oplus a_1$
	Reg_5	$S_1(1) \oplus \tau^{-2}b_1 \oplus \tau^{-1}b_2 \oplus a_1$
	Reg_6	$S_1(2) \oplus \tau^{-1}a_3 \oplus a_1$
1	Reg_7	$S_1(1) \oplus \tau^{-4}a_1 \oplus a_2 \oplus b_1$
	Reg_8	$S_1(1) \oplus \tau^{-2}b_1 \oplus a_1 \oplus b_1 \oplus \tau^{-1}b_1$
	Reg_9	$S_1(3) \oplus b_3 \oplus a_1$
	Reg_{10}	$S_1(2) \oplus \tau^{-1}a_2 \oplus a_2$
	Reg_{11}	$S_1(1) \oplus \tau^{-6}b_1 \oplus a_3$
	Reg_{12}	$S_1(2) \oplus \tau^{-2}b_1 \oplus c \oplus a_1$

Table 18: Deformations with nonzero regular part of $d \oplus S_1(4)$ in Period 4

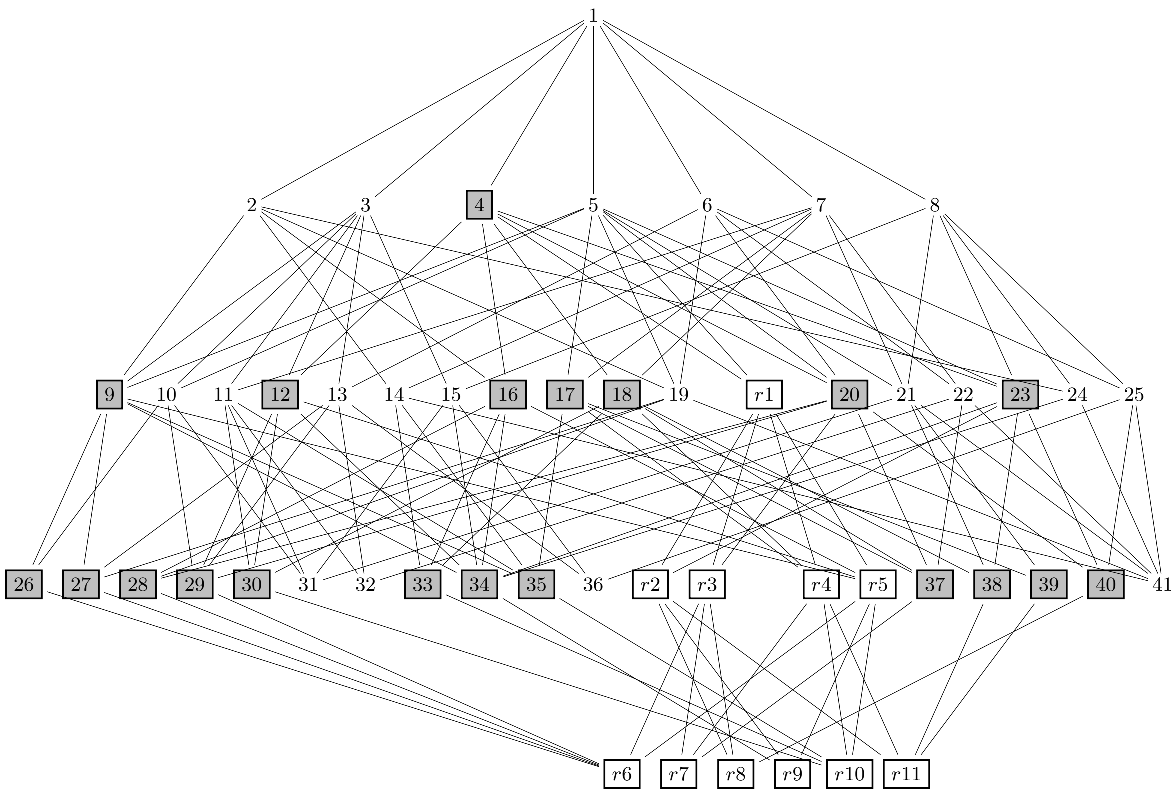


fig. A.87: Degeneration diagram of $d \oplus S_2(4)$ in Period 4

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_2(1) \oplus \tau^{-2}c \oplus \tau^{-1}a_2$
2	Reg_2	$S_2(1) \oplus \tau^{-3}b_1 \oplus \tau^{-1}a_3$
	Reg_3	$S_2(1) \oplus \tau^{-3}b_2 \oplus c$
	Reg_4	$S_2(1) \oplus \tau^{-2}c \oplus a_1 \oplus \tau^{-1}a_1$
	Reg_5	$S_2(2) \oplus \tau^{-1}a_2 \oplus b_2$
1	Reg_6	$S_2(3) \oplus b_2 \oplus c$
	Reg_7	$S_2(1) \oplus \tau^{-5}a_1 \oplus c \oplus a_1$
	Reg_8	$S_2(1) \oplus \tau^{-3}b_1 \oplus c \oplus \tau^{-2}b_1$
	Reg_9	$S_2(2) \oplus \tau^{-3}b_1 \oplus b_3$
	Reg_{10}	$S_2(2) \oplus \tau^{-1}a_1 \oplus b_2 \oplus a_1$
	Reg_{11}	$S_2(1) \oplus \tau^{-3}b_1 \oplus a_2 \oplus \tau^{-1}a_1$

Table 19: Deformations with nonzero regular part of $d \oplus S_2(4)$ in Period 4

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&0&1&1&1&0&0\end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0&1&1&1&1&1&0\end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&1&1&2&1&1&1\end{smallmatrix}\right]$

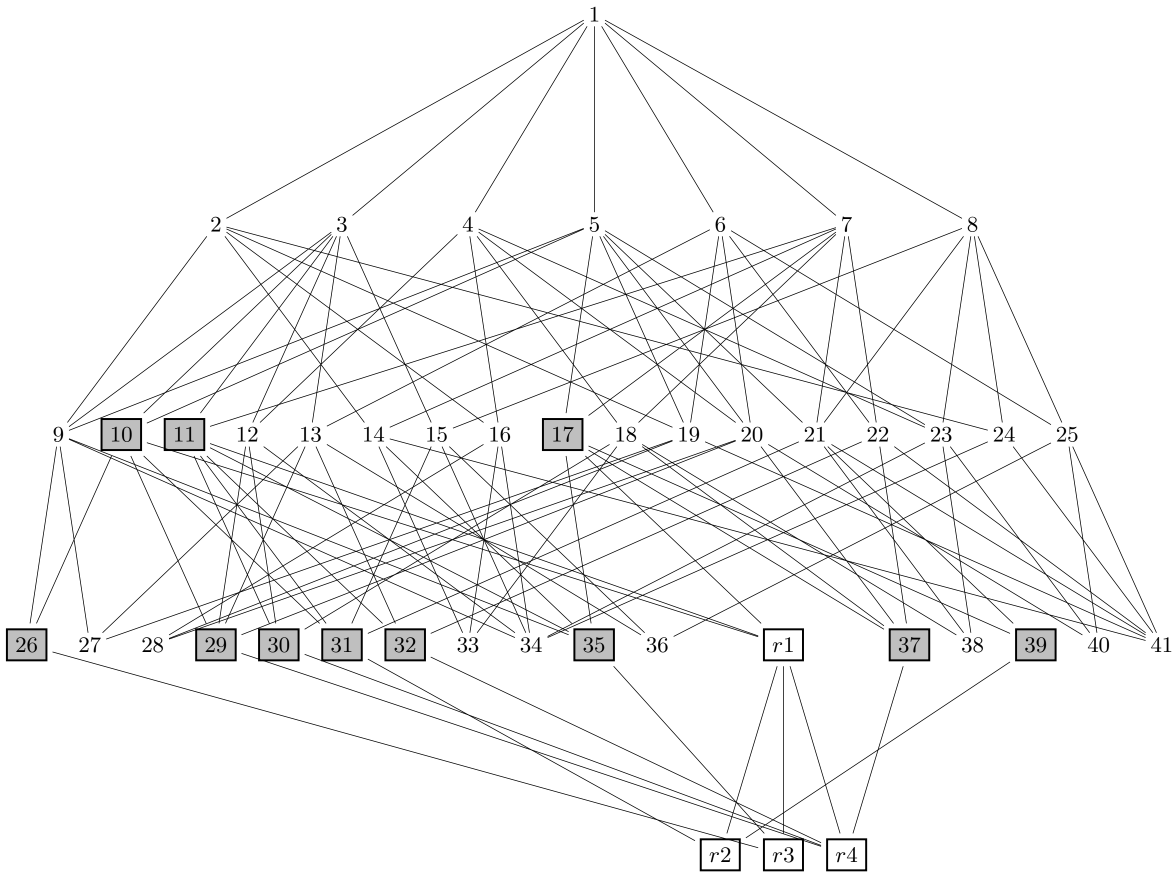


fig. A.88: Degeneration diagram of $d \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1	$S_1(1)$	$\oplus$	$\tau^{-2}c$	$\oplus$	a_1	$\oplus$	b_1
1	Reg_2	$S_1(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_2	$\oplus$	b_1
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	b_2	$\oplus$	a_1
	Reg_4	$S_1(2)$	$\oplus$	c	$\oplus$	a_1	$\oplus$	b_1

Table 20: Deformations with nonzero regular part of $d \oplus S_1(3)$ in Period 3

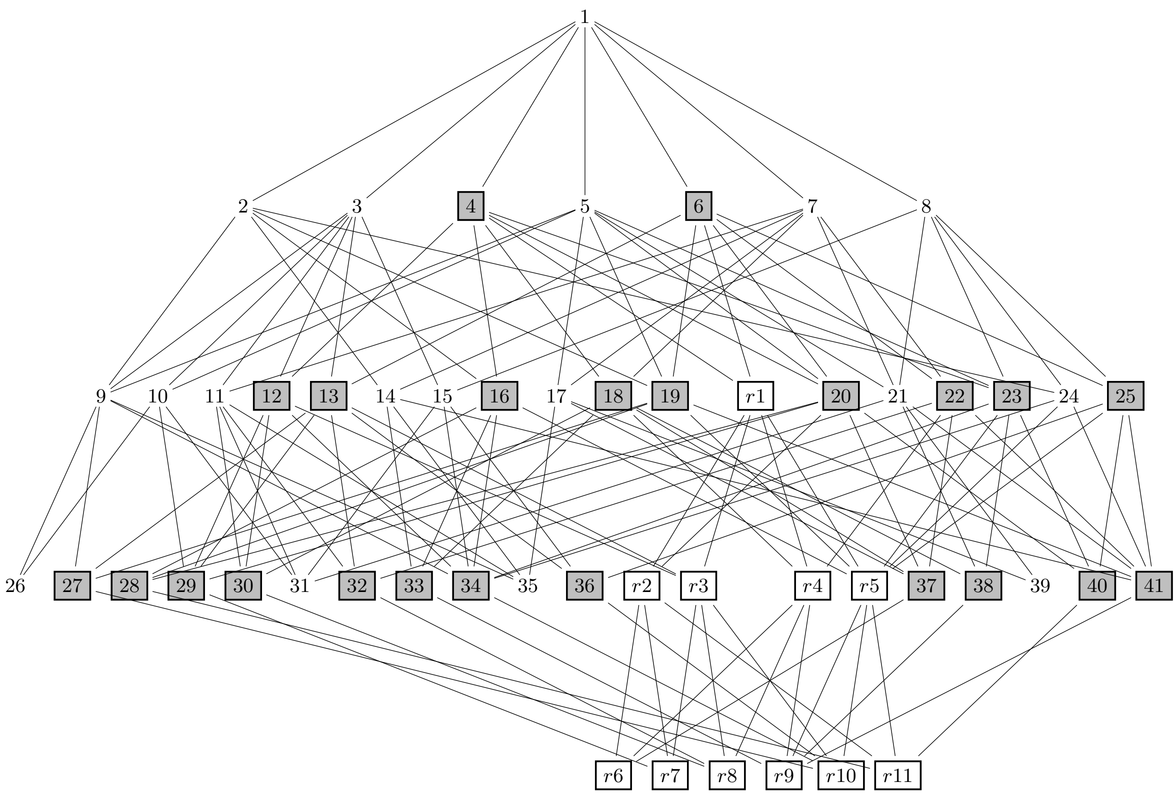


fig. A.89: Degeneration diagram of $d \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_2(1) \oplus \tau^{-1}a_2 \oplus \tau^{-1}b_2$
2	Reg_2	$S_2(1) \oplus \tau^{-3}c \oplus c$
	Reg_3	$S_2(1) \oplus \tau^{-1}a_2 \oplus b_1 \oplus \tau^{-1}b_1$
	Reg_4	$S_2(1) \oplus \tau^{-1}b_2 \oplus a_1 \oplus \tau^{-1}a_1$
	Reg_5	$S_2(2) \oplus \tau^{-1}d$
1	Reg_6	$S_2(1) \oplus \tau^{-4}b_1 \oplus c \oplus a_1$
	Reg_7	$S_2(1) \oplus \tau^{-4}a_1 \oplus c \oplus b_1$
	Reg_8	$S_2(1) \oplus \tau^{-1}a_1 \oplus a_1 \oplus b_1 \oplus \tau^{-1}b_1$
	Reg_9	$S_2(2) \oplus \tau^{-1}a_1 \oplus a_3$
	Reg_{10}	$S_2(2) \oplus \tau^{-1}b_1 \oplus b_3$
	Reg_{11}	$S_2(2) \oplus \tau^{-1}c \oplus c$

Table 21: Deformations with nonzero regular part of $d \oplus S_2(3)$ in Period 3

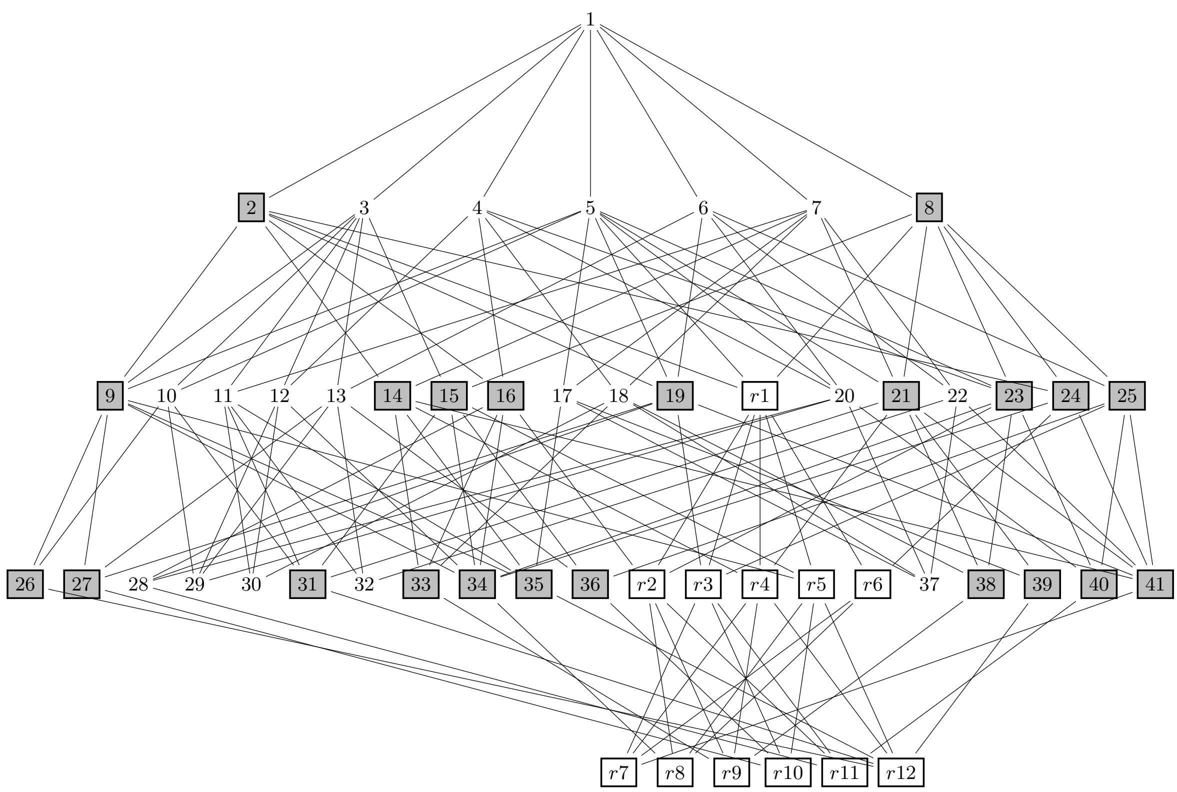


fig. A.90: Degeneration diagram of $d \oplus S_3(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part						
3	Reg_1					$S_3(1)$	$\oplus$	$\tau^{-2}d$
2	Reg_2			$S_3(1)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-1}a_3$
	Reg_3			$S_3(1)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-1}b_3$
	Reg_4			$S_3(1)$	$\oplus$	$\tau^{-2}a_2$	$\oplus$	a_2
	Reg_5			$S_3(1)$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_2
	Reg_6			$S_3(1)$	$\oplus$	$\tau^{-2}c$	$\oplus$	$\tau^{-1}c$
1	Reg_7			$S_3(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	a_3
	Reg_8			$S_3(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_3
	Reg_9	$S_3(1)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	a_2	$\oplus$	$\tau^{-1}a_1$
	Reg_{10}	$S_3(1)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$
	Reg_{11}	$S_3(1)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$
	Reg_{12}			$S_3(2)$	$\oplus$	a_2	$\oplus$	b_2

Table 22: Deformations with nonzero regular part of $d \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0 & 1 & 1 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(1)$	$\oplus$	$\tau^{-1}b_3$	$\oplus$	a_1
1	Reg_2			$S_1(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_3
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-1}b_1$	$\oplus$	b_2	$\oplus$	a_1
	Reg_4	$S_1(1)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	c	$\oplus$	a_1

Table 23: Deformations with nonzero regular part of $d \oplus S_1(2)$ in Period 2

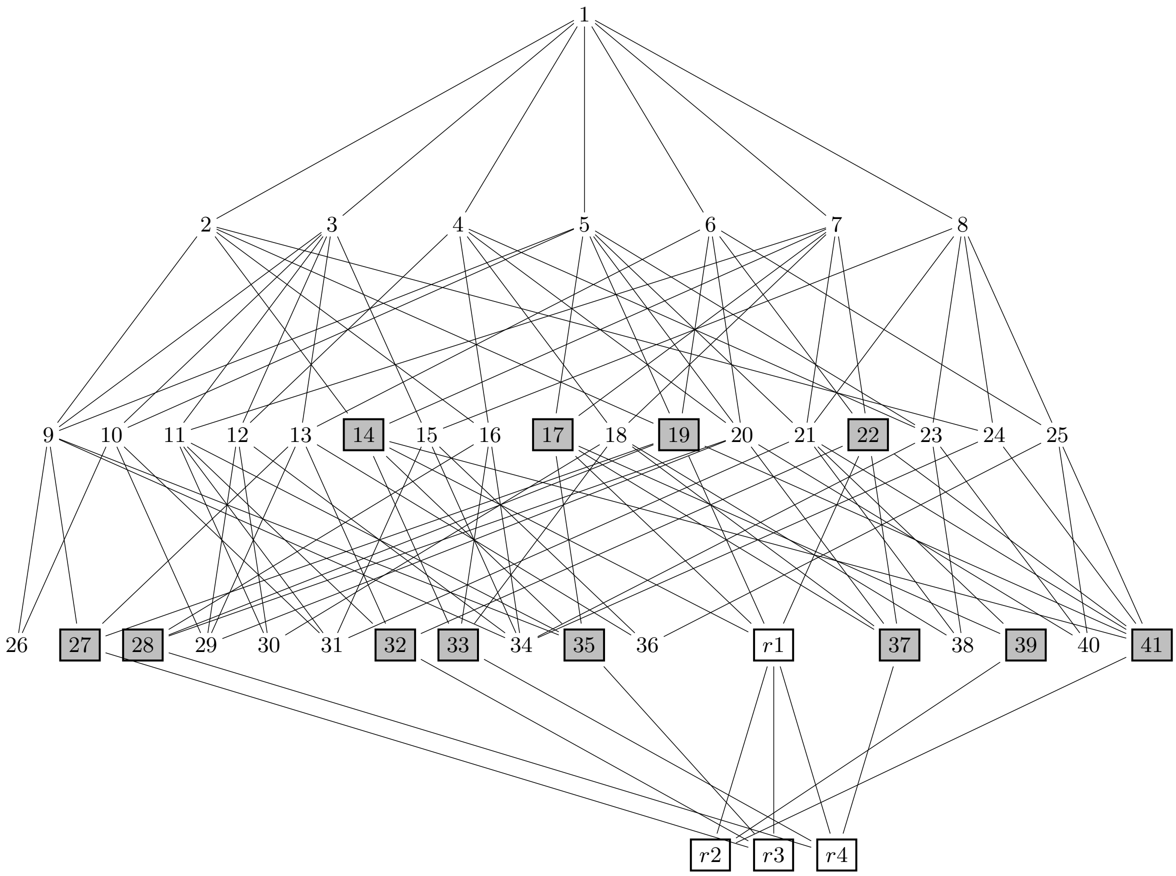


fig. A.91: Degeneration diagram of $d \oplus S_1(2)$ in Period 2

A.10.5 $\mathbf{U = c}$

Period 4:

$S_1(1) = [\,0\;0\;0\;{}^{0}_{1}\;1\;1\;0\,]$ $S_2(1) = [\,0\;0\;1\;{}^{1}_{1}\;1\;1\;1\,]$

$S_3(1) = [\,0\;1\;1\;{}^{0}_{1}\;0\;0\;0\,]$ $S_4(1) = [\,1\;1\;1\;{}^{1}_{1}\;1\;0\;0\,]$

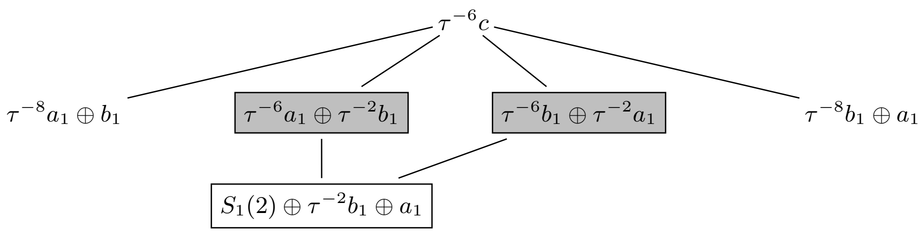


fig. A.92: Degeneration diagram of $c \oplus S_1(4)$ in Period 4

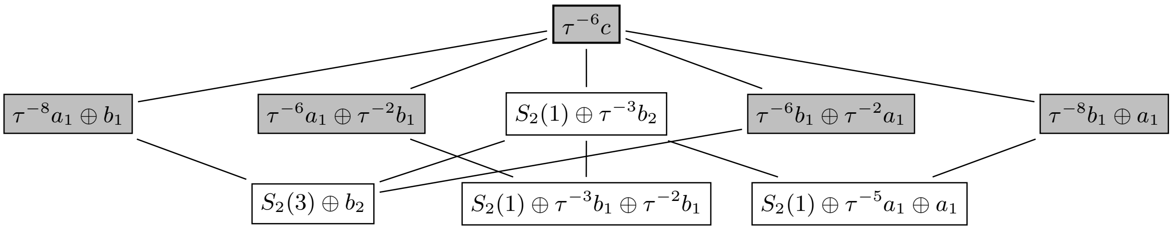


fig. A.93: Degeneration diagram of $c \oplus S_2(4)$ in Period 4

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & & 0 & & & \\ 0&0&1&1&1&0&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&1&1&1&1&1&0 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&1&1&2&1&1&1 \end{smallmatrix}\right]$

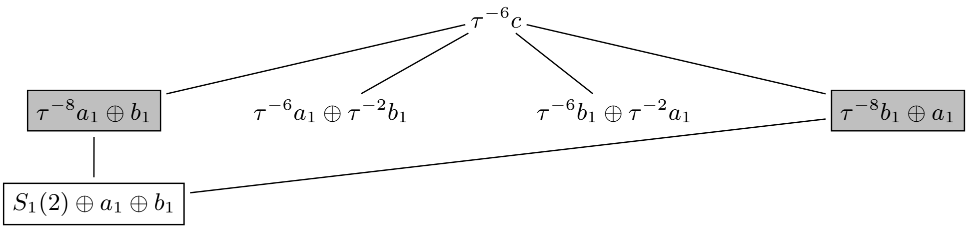


fig. A.94: Degeneration diagram of $c \oplus S_1(3)$ in Period 3

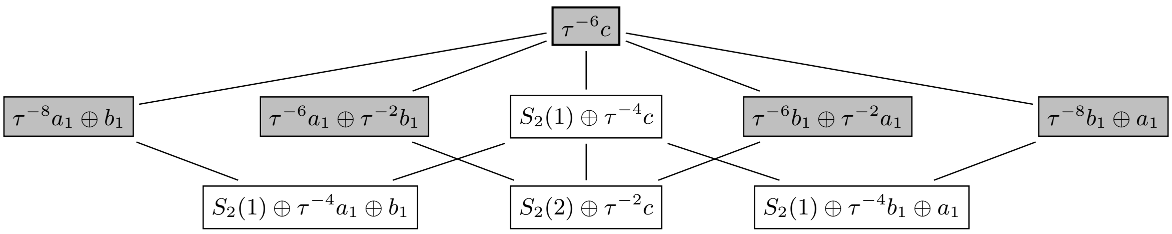


fig. A.95: Degeneration diagram of $c \oplus S_2(3)$ in Period 3

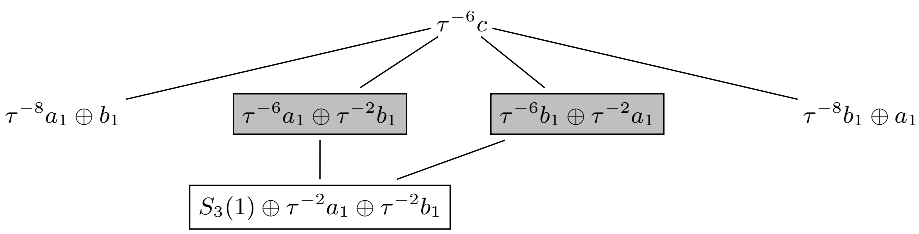


fig. A.96: Degeneration diagram of $c \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 0&1&1&2&2&1&1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & & 1 & & & \\ 1&1&2&2&1&1&0 \end{smallmatrix}\right]$

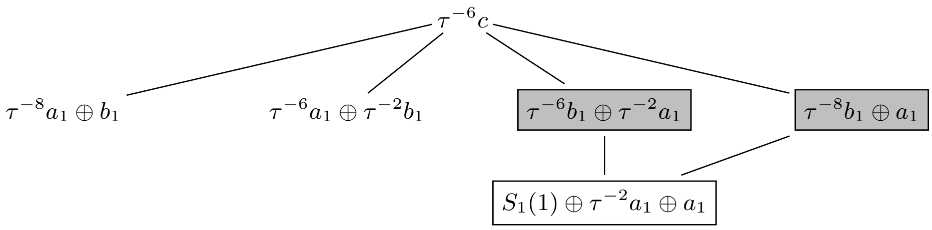
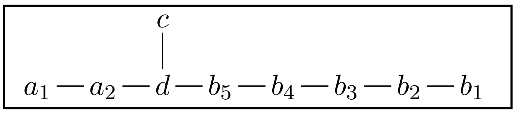


fig. A.97: Degeneration diagram of $c \oplus S_1(2)$ in Period 2

A.11 $\tilde{E}8$



A.11.1 $\mathbf{U = a_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-15}a_1) = 3$$

Period 5:

$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0&0&1&1&1&0&0&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&1&1&1&1&0&0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0&1&1&1&1&1&1&0 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&2&1&1&1&1&1 \end{smallmatrix}\right]$ $S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0&1&1&1&0&0&0&0 \end{smallmatrix}\right]$

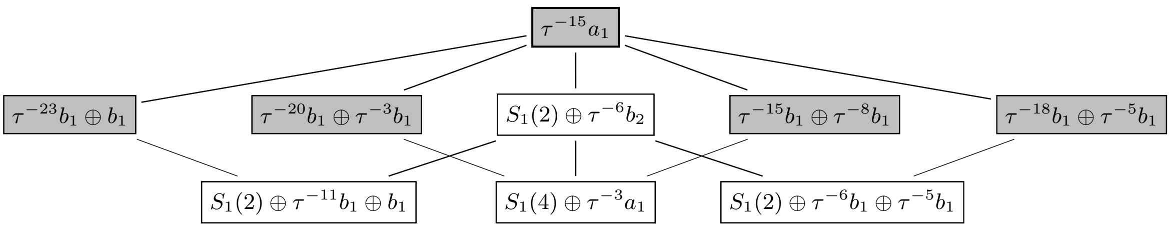


fig. A.98: Degeneration diagram of $a_1 \oplus S_1(5)$ in Period 5

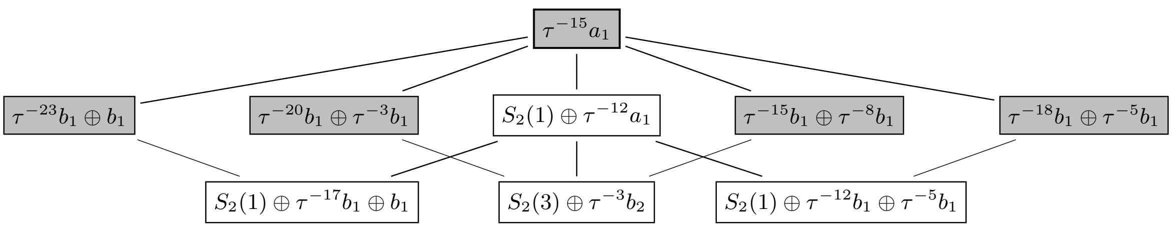


fig. A.99: Degeneration diagram of $a_1 \oplus S_2(5)$ in Period 5

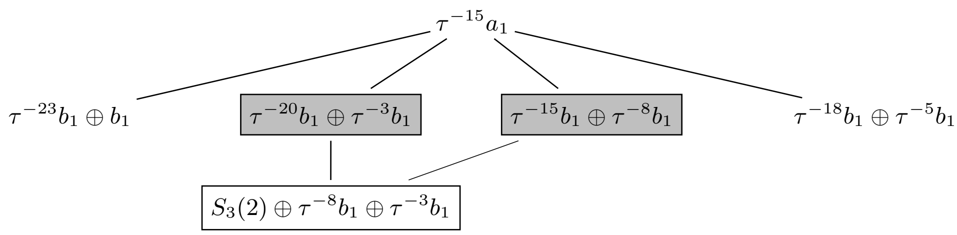


fig. A.100: Degeneration diagram of $a_1 \oplus S_3(5)$ in Period 5

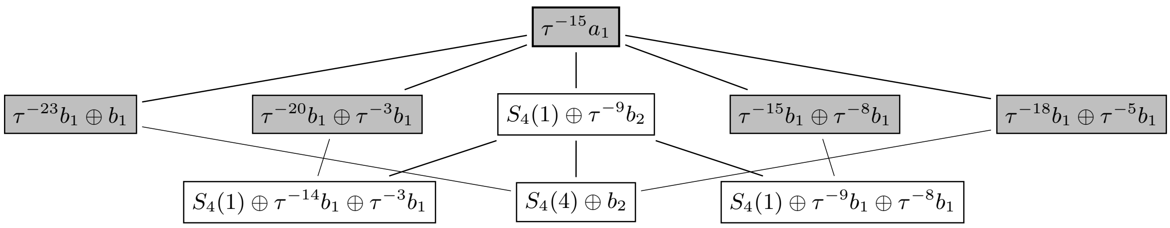


fig. A.101: Degeneration diagram of $a_1 \oplus S_4(5)$ in Period 5

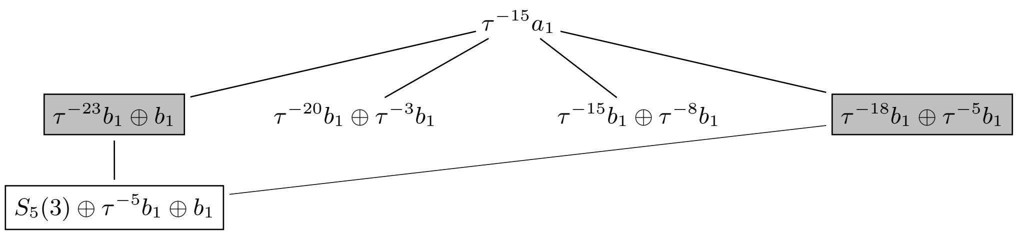


fig. A.102: Degeneration diagram of $a_1 \oplus S_5(5)$ in Period 5

Period 3:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$	$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 2 & 2 & 1 & 1 & 1 \end{smallmatrix}\right]$

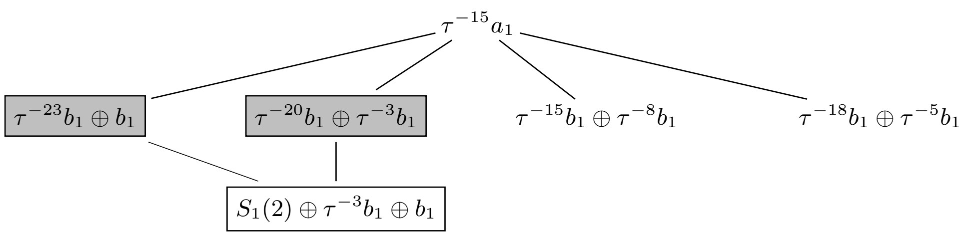


fig. A.103: Degeneration diagram of $a_1 \oplus S_1(3)$ in Period 3

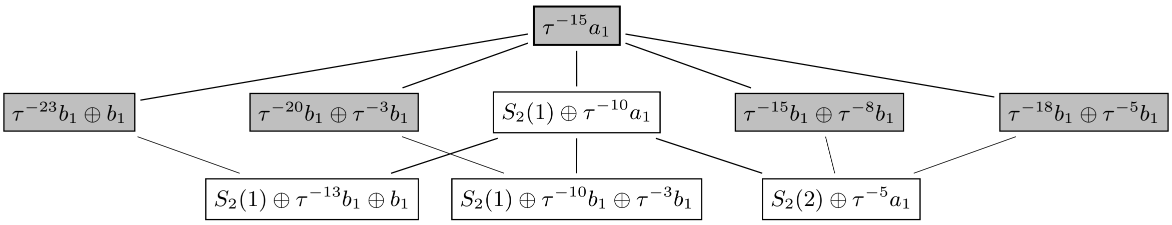


fig. A.104: Degeneration diagram of $a_1 \oplus S_2(3)$ in Period 3

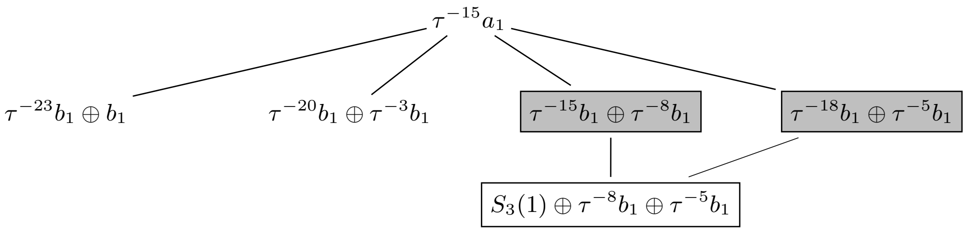


fig. A.105: Degeneration diagram of $a_1 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 3 & 3 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1 & 2 & 3 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

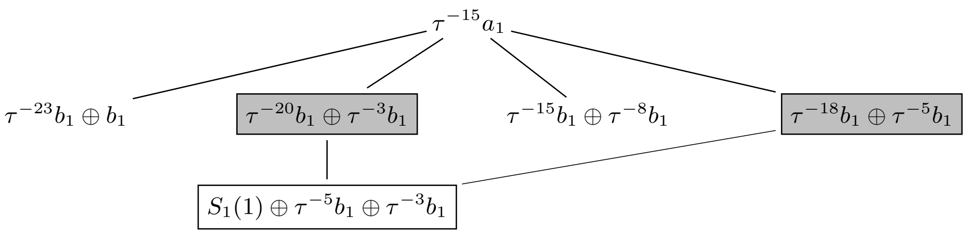


fig. A.106: Degeneration diagram of $a_1 \oplus S_1(2)$ in Period 2

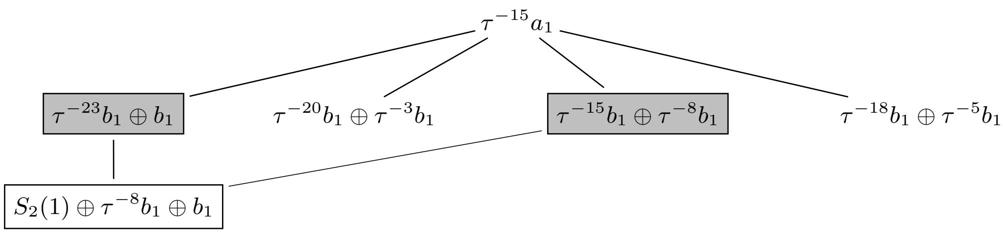


fig. A.107: Degeneration diagram of $a_1 \oplus S_2(2)$ in Period 2

A.11.2 $\mathbf{U = a_2}$

$$\mathbf{Codim}(U \oplus V, \tau^{-6}b_4) = 5$$

Period 5:	$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{smallmatrix}\right]$

Codimension	Dot	Preprojective deformations						
5	1							$\tau^{-6}b_4$
4	2					$\tau^{-6}b_1$	$\oplus$	$\tau^{-5}b_3$
	3					$\tau^{-6}b_3$	$\oplus$	$\tau^{-3}b_1$
	4					$\tau^{-9}b_1$	$\oplus$	$\tau^{-5}c$
	5					$\tau^{-8}c$	$\oplus$	b_1
	6					$\tau^{-9}a_1$	$\oplus$	$\tau^{-5}a_1$
	7					$\tau^{-9}b_2$	$\oplus$	$\tau^{-1}b_2$
	8					$\tau^{-6}b_2$	$\oplus$	$\tau^{-4}b_2$
3	9					$\tau^{-18}b_1$	$\oplus$	$\tau^{-1}b_3$
	10					$\tau^{-15}b_1$	$\oplus$	$\tau^{-3}c$
	11					$\tau^{-14}a_1$	$\oplus$	a_1
	12					$\tau^{-10}b_2$	$\oplus$	$\tau^{-2}a_1$
	13					$\tau^{-12}a_1$	$\oplus$	b_2
	14			$\tau^{-9}b_1$	$\oplus$	$\tau^{-4}a_1$	$\oplus$	$\tau^{-6}b_1$
	15			$\tau^{-13}b_1$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	b_1
	16			$\tau^{-10}b_1$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-3}b_1$
	17			$\tau^{-9}b_1$	$\oplus$	$\tau^{-7}a_1$	$\oplus$	b_1
	18			$\tau^{-10}a_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	19			$\tau^{-14}b_1$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	$\tau^{-3}b_1$
	20			$\tau^{-9}b_1$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	$\tau^{-8}b_1$
	21			$\tau^{-11}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1
	22			$\tau^{-6}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	$\tau^{-5}b_1$
	23			$\tau^{-6}b_1$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	$\tau^{-3}b_1$
	24			$\tau^{-6}b_2$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	25			$\tau^{-9}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$
2	26					$\tau^{-21}b_1$	$\oplus$	b_3
	27					$\tau^{-24}b_1$	$\oplus$	c
	28			$\tau^{-23}b_1$	$\oplus$	a_1	$\oplus$	b_1
	29			$\tau^{-20}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	30			$\tau^{-18}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-5}b_1$
	31			$\tau^{-7}b_1$	$\oplus$	a_2	$\oplus$	b_1
	32			$\tau^{-15}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-8}b_1$
	33			$\tau^{-19}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1
	34			$\tau^{-15}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-4}b_1$
	35			$\tau^{-10}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-9}b_1$
	36			$\tau^{-18}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$
	37			$\tau^{-16}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	38			$\tau^{-13}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-6}b_1$
	39	$\tau^{-14}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	40	$\tau^{-11}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	41	$\tau^{-6}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$

Table 24: Preprojective deformations of $a_2 \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

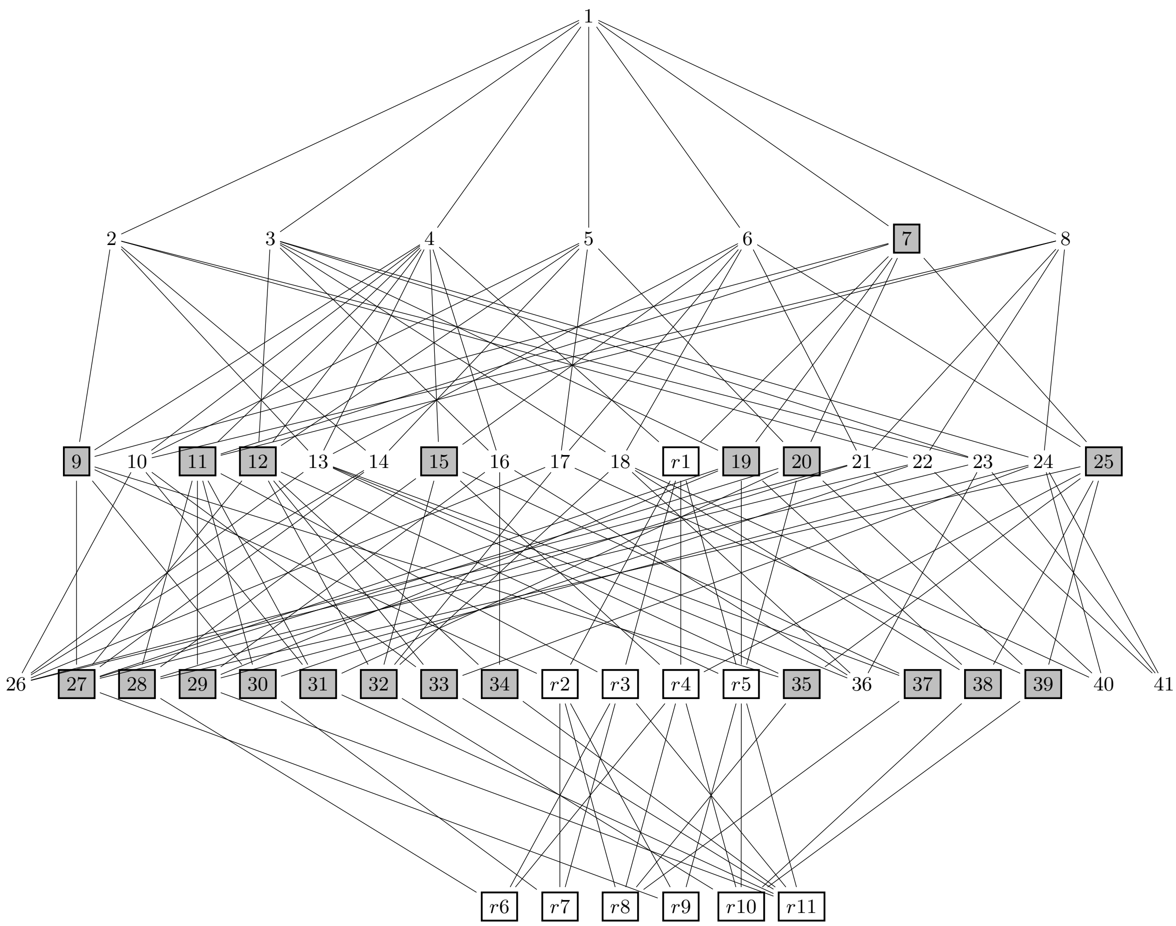


fig. A.108: Degeneration diagram of $a_2 \oplus S_1(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_1(2) \oplus \tau^{-5}a_1 \oplus \tau^{-1}b_2$
2	Reg_2	$S_1(2) \oplus \tau^{-6}b_1 \oplus \tau^{-1}b_3$
	Reg_3	$S_1(2) \oplus \tau^{-6}b_2 \oplus a_1$
	Reg_4	$S_1(2) \oplus \tau^{-5}a_1 \oplus b_1 \oplus \tau^{-1}b_1$
	Reg_5	$S_1(3) \oplus \tau^{-2}a_1 \oplus \tau^{-1}b_2$
1	Reg_6	$S_1(2) \oplus \tau^{-11}b_1 \oplus a_1 \oplus b_1$
	Reg_7	$S_1(2) \oplus \tau^{-6}b_1 \oplus a_1 \oplus \tau^{-5}b_1$
	Reg_8	$S_1(2) \oplus \tau^{-6}b_1 \oplus b_2 \oplus \tau^{-1}b_1$
	Reg_9	$S_1(3) \oplus \tau^{-6}b_1 \oplus c$
	Reg_{10}	$S_1(3) \oplus \tau^{-2}a_1 \oplus b_1 \oplus \tau^{-1}b_1$
	Reg_{11}	$S_1(4) \oplus \tau^{-2}a_1 \oplus a_1$

Table 25: Deformations with nonzero regular part of $a_2 \oplus S_1(5)$ in Period 5

The case of $U \oplus V = a_2 \oplus S_2(5)$ is a little bit too large to draw the degeneration diagram, because there are 70's deformations. All of the preprojective deformations are not extensions of $U \oplus V$ for this case. So we only give a table with the minimal deformations in table 26.

Codimension	Minimal deformations							
1	1	$S_2(1)$	$\oplus$	$\tau^{-17}b_1$	$\oplus$	a_1	$\oplus$	b_1
	2	$S_2(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-5}b_1$
	3	$S_2(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$
	4	$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-6}b_1$
	5			$S_2(2)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	c
	6	$S_2(2)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1
	7	$S_2(2)$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$
	8			$S_2(3)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-2}a_1$
	9			$S_2(3)$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	a_1
	10					$S_2(4)$	$\oplus$	b_4

Table 26: Minimal deformations $a_2 \oplus S_2(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part								
3	Reg_1					$S_3(1)$	$\oplus$	$\tau^{-5}c$	$\oplus$	$\tau^{-3}b_1$
2	Reg_2					$S_3(1)$	$\oplus$	$\tau^{-9}a_1$	$\oplus$	$\tau^{-2}a_1$
	Reg_3			$S_3(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	$\tau^{-4}a_1$	$\oplus$	$\tau^{-3}b_1$
	Reg_4			$S_3(1)$	$\oplus$	$\tau^{-7}a_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	Reg_5			$S_3(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	$\tau^{-3}b_1$
	Reg_6					$S_3(2)$	$\oplus$	$\tau^{-3}c$	$\oplus$	$\tau^{-3}b_1$
1	Reg_7					$S_3(1)$	$\oplus$	$\tau^{-18}b_1$	$\oplus$	c
	Reg_8			$S_3(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1
	Reg_9	$S_3(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	Reg_{10}					$S_3(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	$\tau^{-2}a_1$
	Reg_{11}			$S_3(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_{12}					$S_3(3)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_3

Table 27: Deformations with nonzero regular part of $a_2 \oplus S_3(5)$ in Period 5

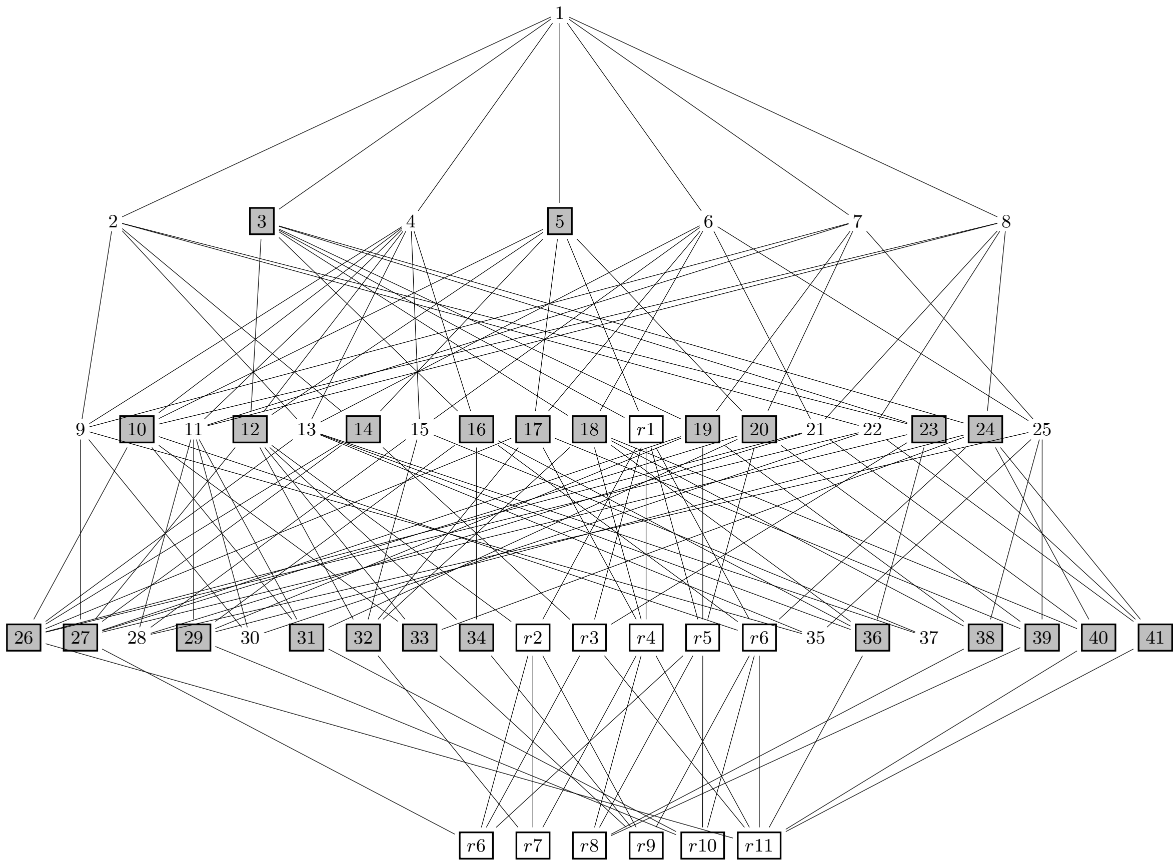


fig. A.109: Degeneration diagram of $a_2 \oplus S_3(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part						
3	Reg_1			$S_4(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-4}b_2$
2	Reg_2			$S_4(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-3}c$
	Reg_3			$S_4(1)$	$\oplus$	$\tau^{-9}b_2$	$\oplus$	a_1
	Reg_4	$S_4(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_5			$S_4(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_2
1	Reg_6	$S_4(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_7	$S_4(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-8}b_1$
	Reg_8	$S_4(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_9			$S_4(2)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	b_3
	Reg_{10}	$S_4(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	Reg_{11}			$S_4(4)$	$\oplus$	a_1	$\oplus$	b_2

Table 28: Deformations with nonzero regular part of $a_2 \oplus S_4(5)$ in Period 5

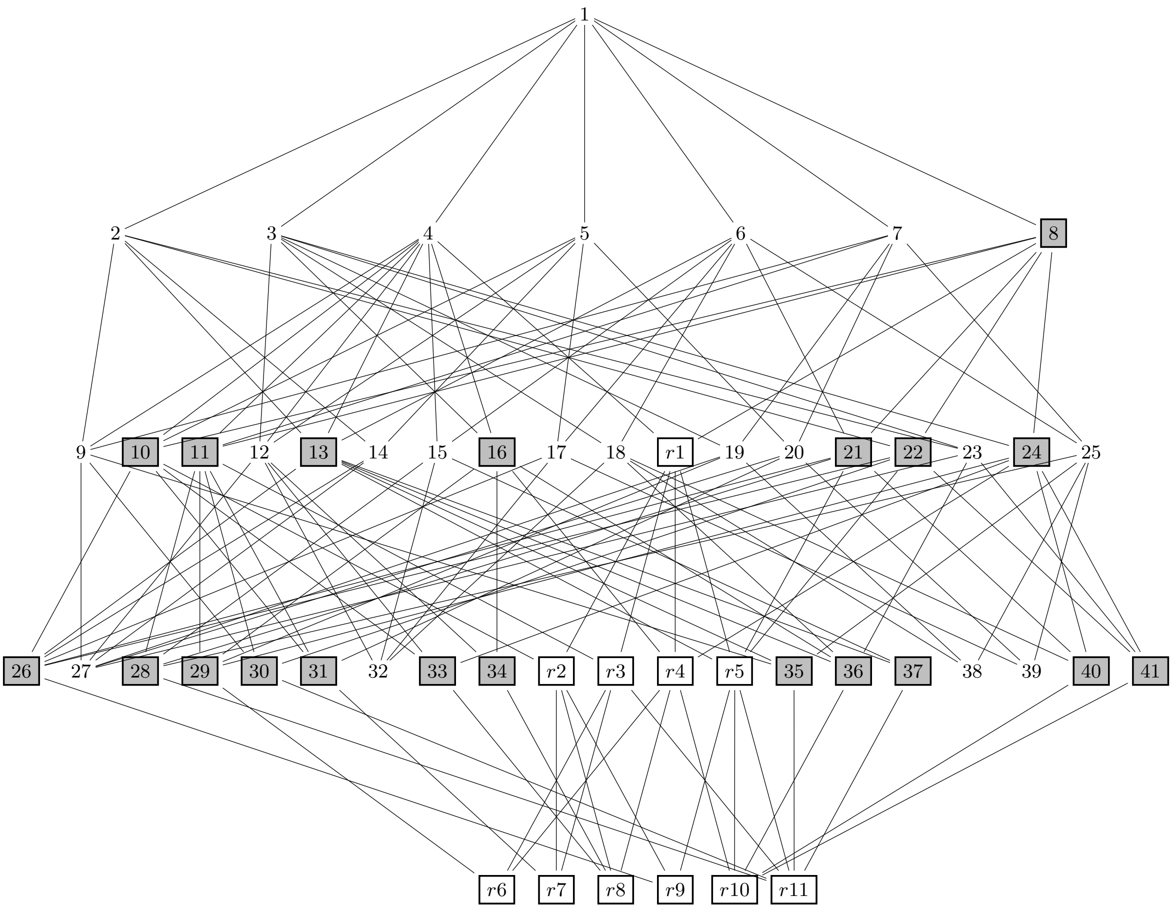


fig. A.110: Degeneration diagram of $a_2 \oplus S_4(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_5(1) \oplus \tau^{-5}b_3 \oplus b_1$
2	Reg_2	$S_5(1) \oplus \tau^{-9}a_1 \oplus b_2$
	Reg_3	$S_5(1) \oplus \tau^{-9}b_1 \oplus \tau^{-4}a_1 \oplus b_1$
	Reg_4	$S_5(1) \oplus \tau^{-5}b_1 \oplus \tau^{-4}b_2 \oplus b_1$
	Reg_5	$S_5(1) \oplus \tau^{-5}b_2 \oplus b_1 \oplus \tau^{-3}b_1$
	Reg_6	$S_5(3) \oplus \tau^{-1}b_3 \oplus b_1$
1	Reg_7	$S_5(1) \oplus \tau^{-15}b_1 \oplus b_3$
	Reg_8	$S_5(1) \oplus \tau^{-10}b_1 \oplus b_2 \oplus \tau^{-3}b_1$
	Reg_9	$S_5(1) \oplus \tau^{-5}b_1 \oplus b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1$
	Reg_{10}	$S_5(3) \oplus \tau^{-1}b_2 \oplus b_2$
	Reg_{11}	$S_5(3) \oplus \tau^{-5}b_1 \oplus a_1 \oplus b_1$
	Reg_{12}	$S_5(4) \oplus c \oplus b_1$

Table 29: Deformations with nonzero regular part of $a_2 \oplus S_5(5)$ in Period 5

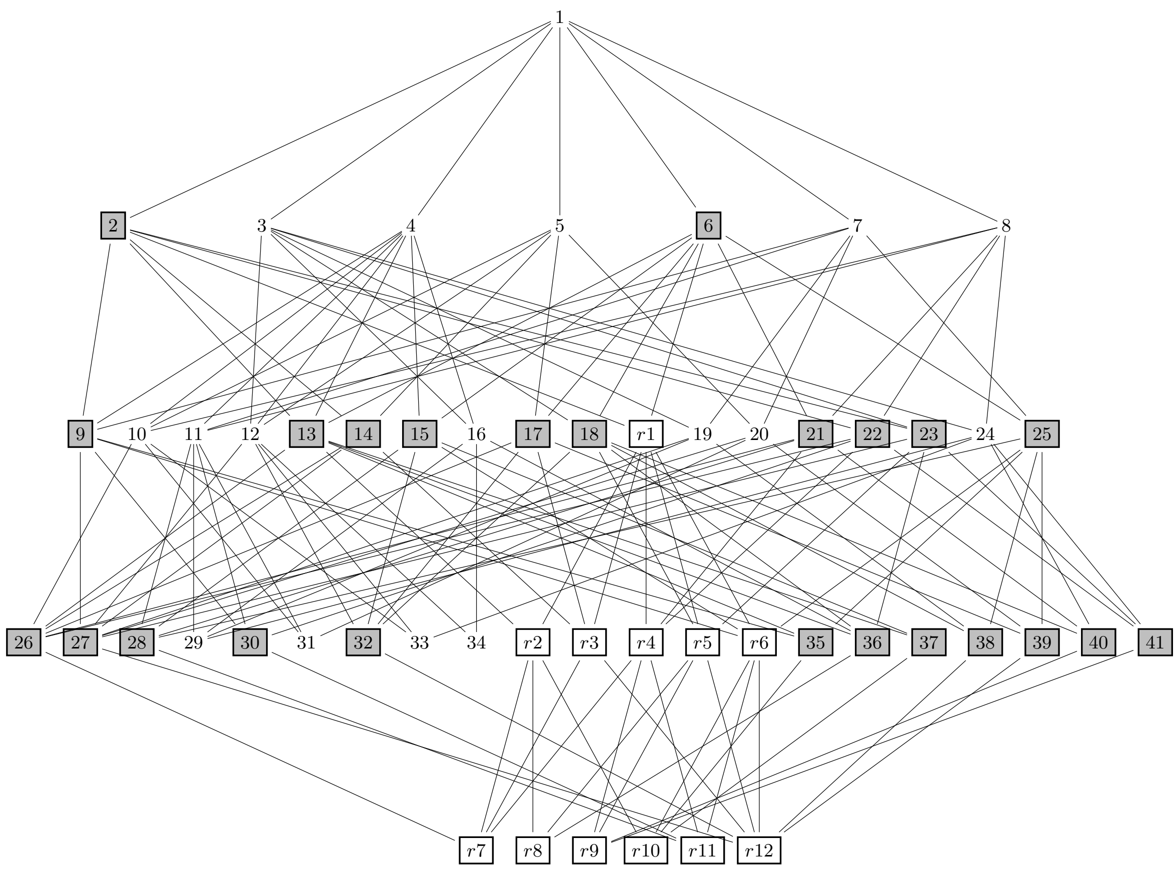


fig. A.111: Degeneration diagram of $a_2 \oplus S_5(5)$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&2&2&2&2&1&1&1\end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&2&1&1&1&0&0\end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0&1&2&2&1&1&1&0\end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
3	Reg_1					$S_1(1)$	$\oplus$	$\tau^{-5}a_2$
2	Reg_2			$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	$\tau^{-1}b_3$
	Reg_3			$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-3}c$
	Reg_4			$S_1(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-4}a_1$
	Reg_5			$S_1(1)$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	$\tau^{-2}a_1$
	Reg_6			$S_1(1)$	$\oplus$	$\tau^{-7}a_1$	$\oplus$	b_2
1	Reg_7			$S_1(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	b_3
	Reg_8			$S_1(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	c
	Reg_9	$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-5}b_1$
	Reg_{10}	$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_{11}	$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$
	Reg_{12}			$S_1(2)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_2

Table 30: Deformations with nonzero regular part of $a_2 \oplus S_1(3)$ in Period 3

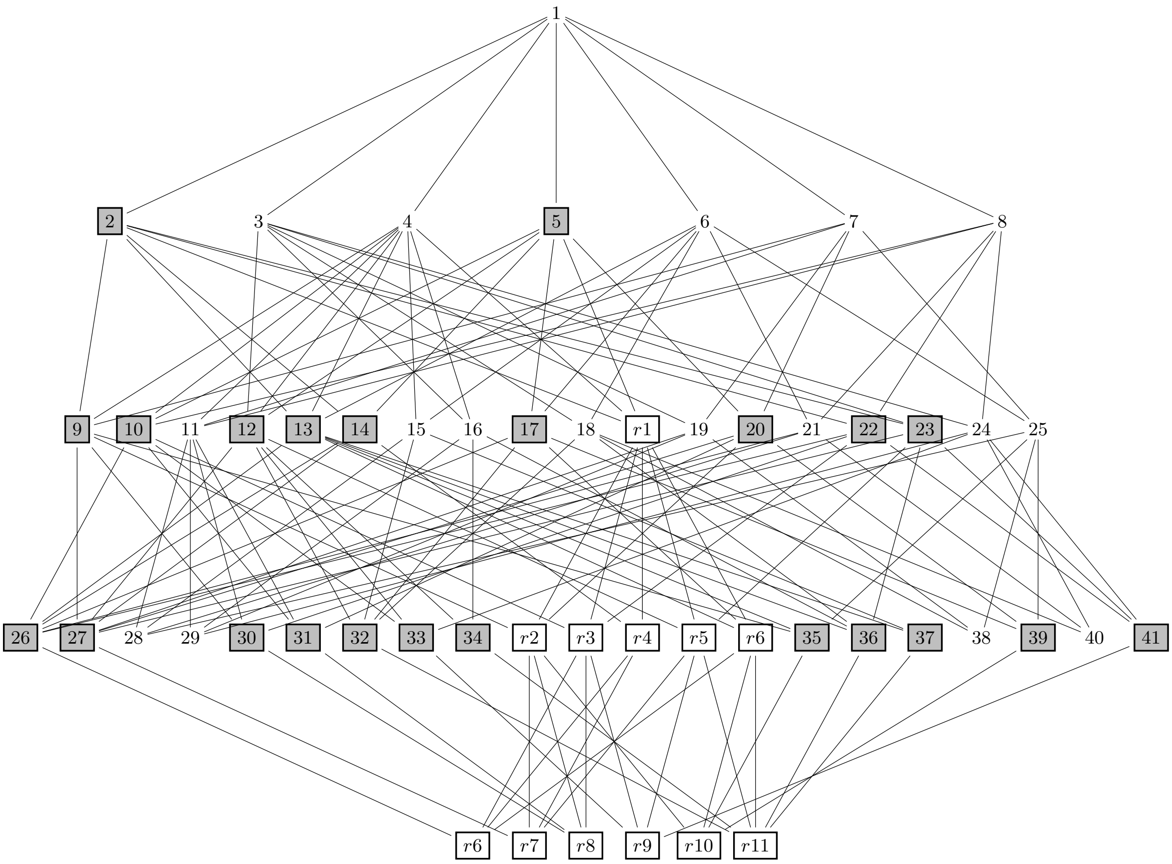


fig. A.112: Degeneration diagram of $a_2 \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1	$S_2(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
1	Reg_2	$S_2(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1
	Reg_3	$S_2(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	Reg_4	$S_2(2)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_1	$\oplus$	b_1

Table 31: Deformations with nonzero regular part of $a_2 \oplus S_2(3)$ in Period 3

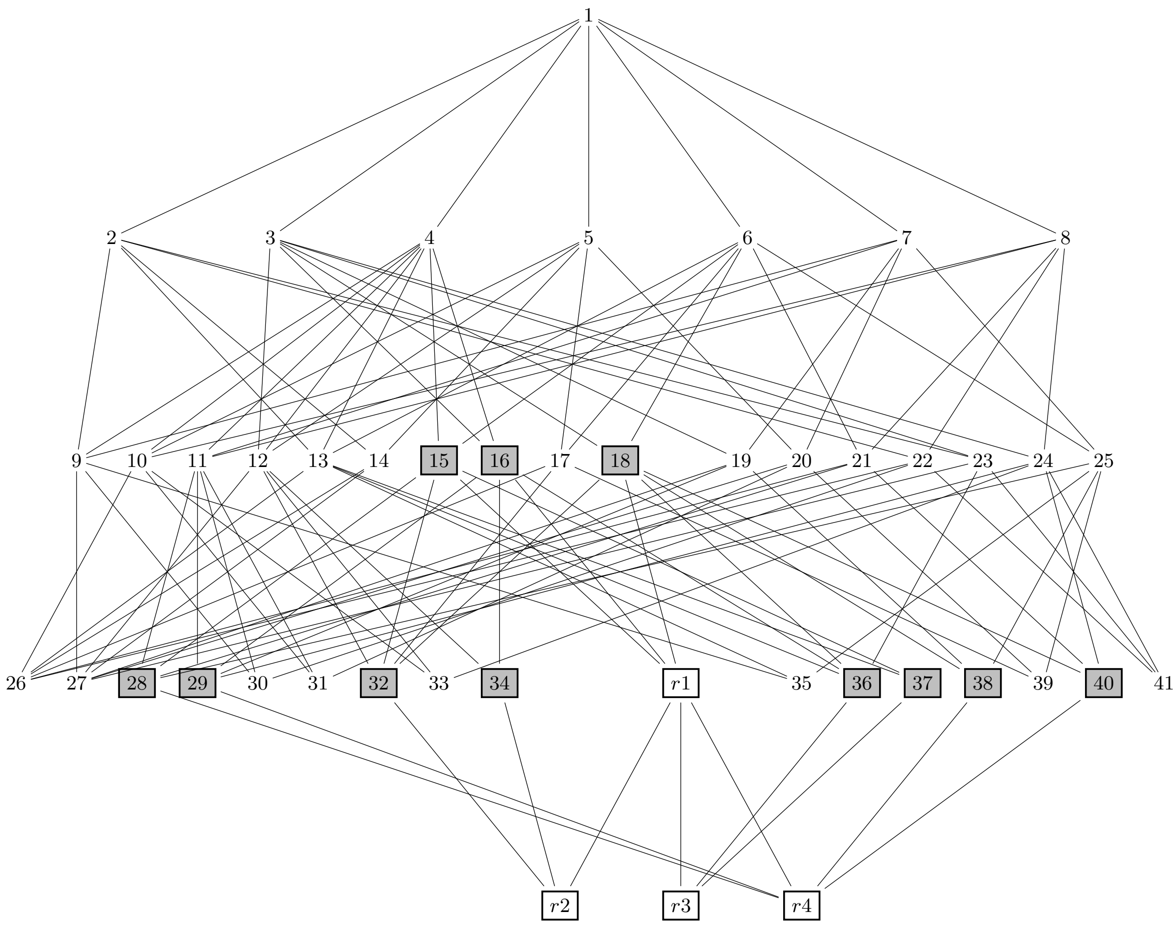


fig. A.113: Degeneration diagram of $a_2 \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part								
3	Reg_1					$S_3(1)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	$\tau^{-1}b_2$
2	Reg_2					$S_3(1)$	$\oplus$	$\tau^{-9}a_1$	$\oplus$	a_1
	Reg_3			$S_3(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	$\tau^{-3}b_1$
	Reg_4			$S_3(1)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$
	Reg_5							$S_3(2)$	$\oplus$	$\tau^{-1}b_4$
1	Reg_6			$S_3(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	a_1	$\oplus$	b_1
	Reg_7			$S_3(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_8	$S_3(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	Reg_9					$S_3(2)$	$\oplus$	$\tau^{-1}b_1$	$\oplus$	b_3
	Reg_{10}					$S_3(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	c
	Reg_{11}					$S_3(2)$	$\oplus$	$\tau^{-4}a_1$	$\oplus$	a_1

Table 32: Deformations with nonzero regular part of $a_2 \oplus S_3(3)$ in Period 3

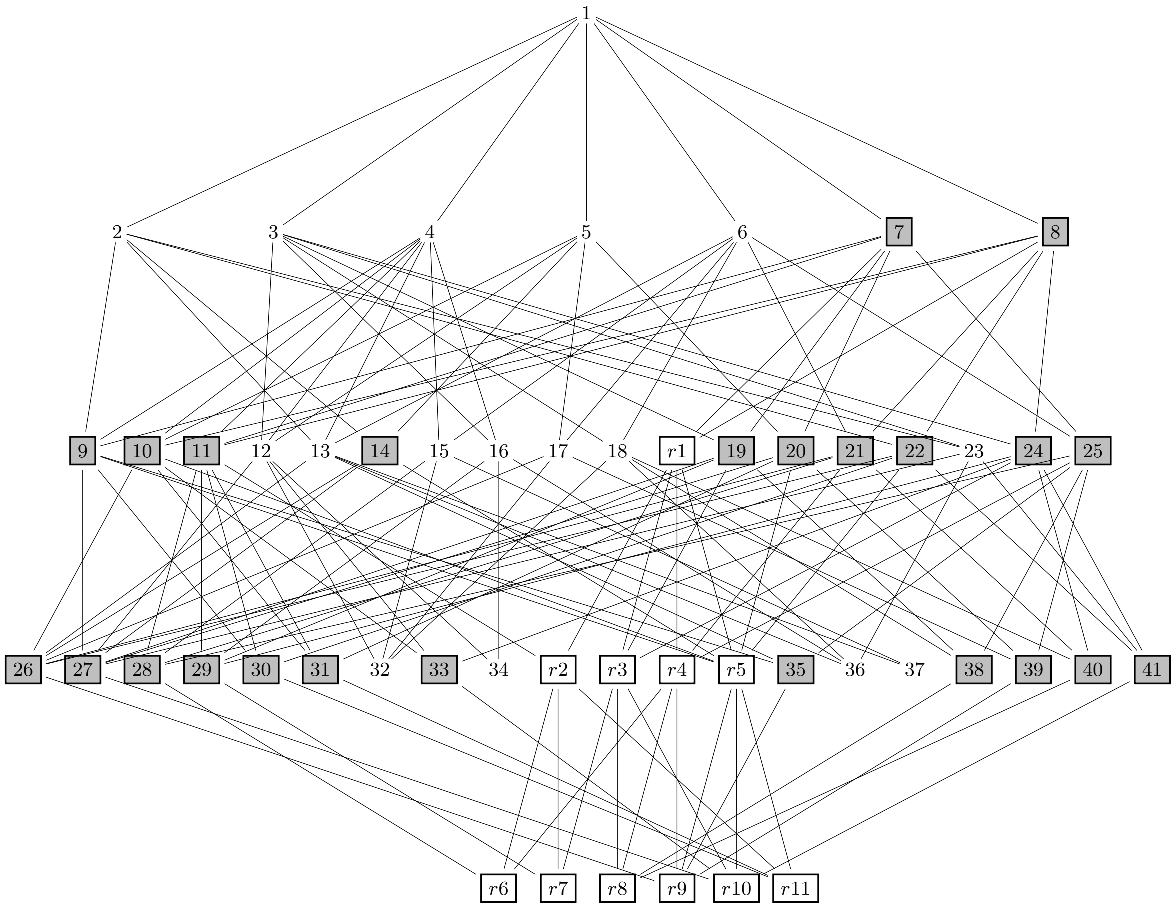


fig. A.114: Degeneration diagram of $a_2 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 3 & 3 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1 & 2 & 3 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-1}b_3$
1	Reg_2			$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_4	$S_1(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$

Table 33: Deformations with nonzero regular part of $a_2 \oplus S_1(2)$ in Period 2

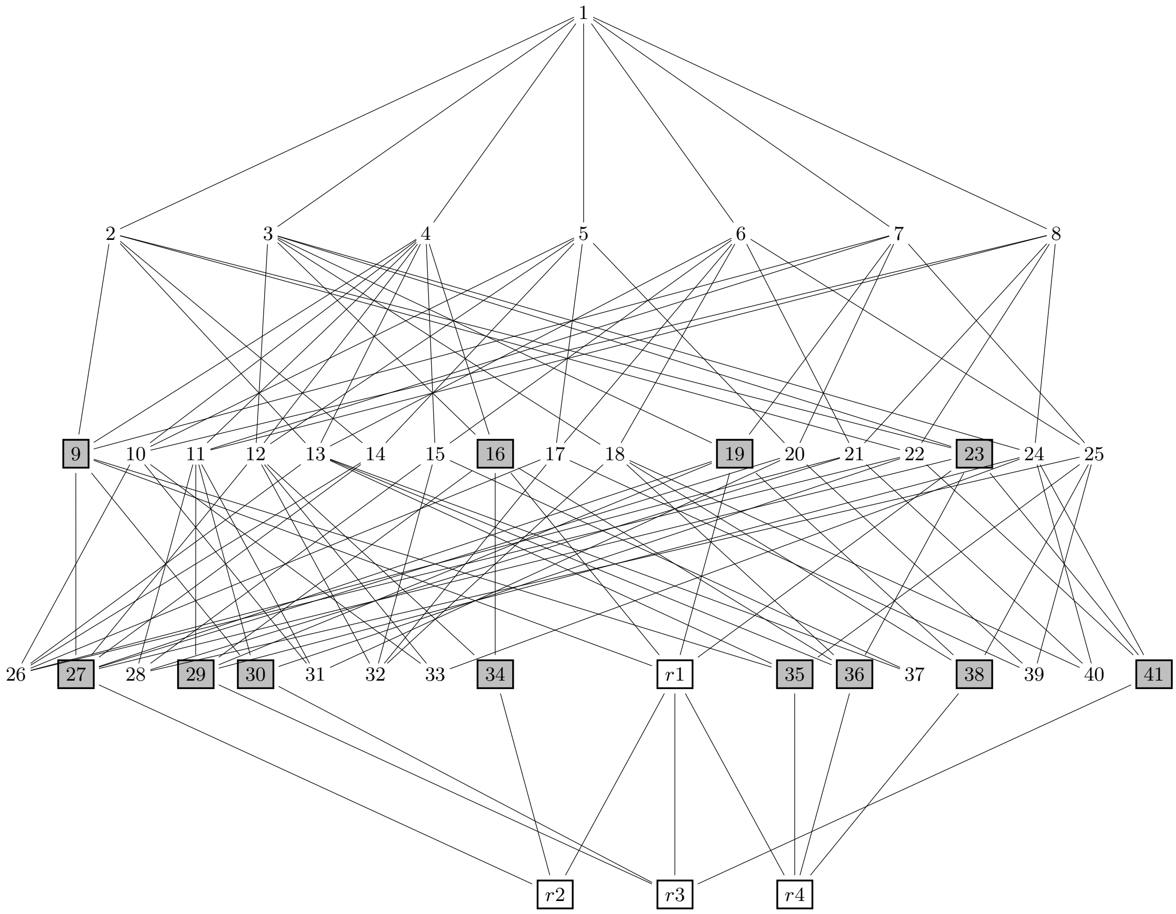


fig. A.115: Degeneration diagram of $a_2 \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_2(1)$	$\oplus$	$\tau^{-3}c$	$\oplus$	b_1
1	Reg_2			$S_2(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_3
	Reg_3	$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	b_1
	Reg_4	$S_2(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	b_1

Table 34: Deformations with nonzero regular part of $a_2 \oplus S_2(2)$ in Period 2

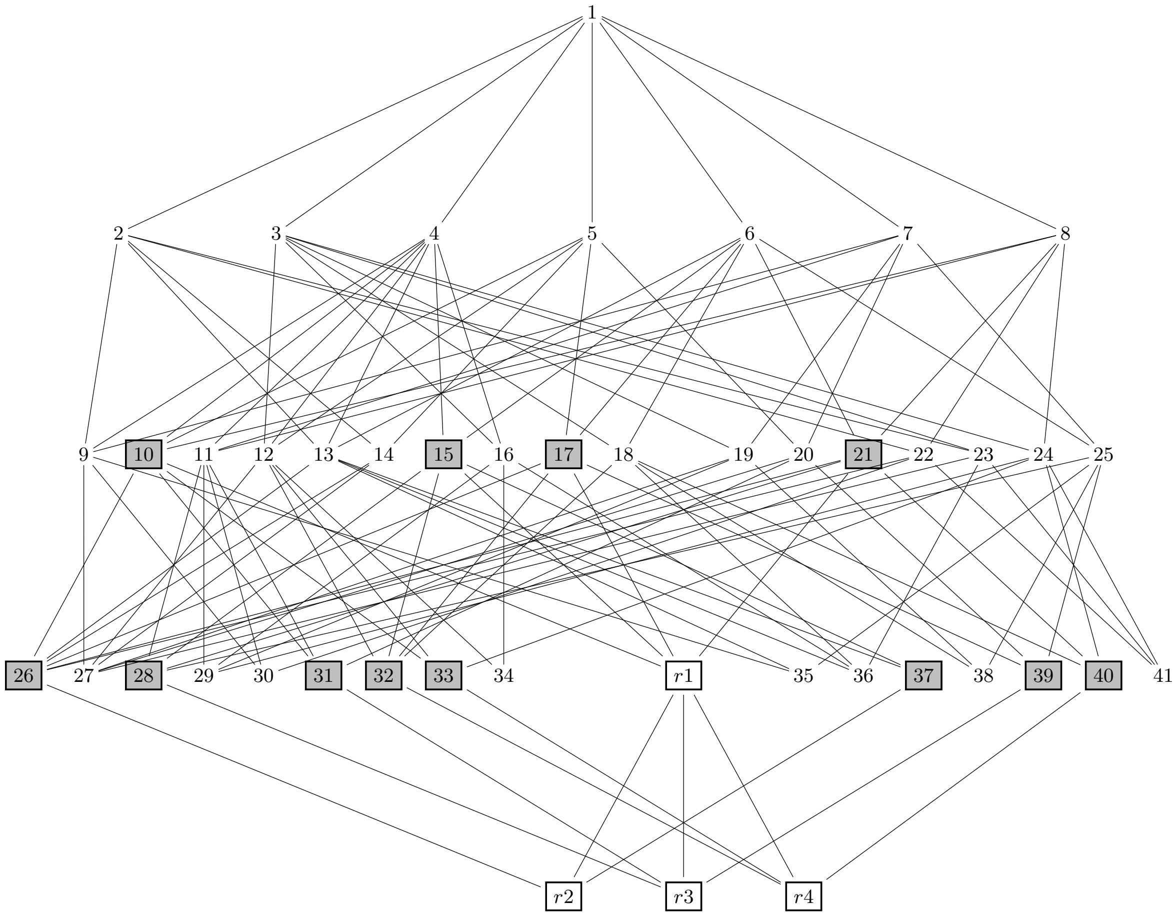


fig. A.116: Degeneration diagram of $a_2 \oplus S_2(2)$ in Period 2

A.11.3 $\mathbf{U = d}$

Now we are in the case of $\partial(U) = -6$, thus the degeneration diagrams are too large to put information in it in a useful way. So we only write the minimal deformations down.

Period 5:

$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{smallmatrix}\right]$ $S_5(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$

Codimension	Minimal deformations									
2	1					$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	2					$\tau^{-22}b_1$	$\oplus$	c	$\oplus$	b_2
	3			$\tau^{-12}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-5}b_1$
	4			$\tau^{-7}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-6}b_1$
	5			$\tau^{-17}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	6			$\tau^{-12}b_1$	$\oplus$	c	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	7	$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	8	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
1	9			$S_1(1)$	$\oplus$	$\tau^{-17}b_1$	$\oplus$	a_2	$\oplus$	b_1
	10			$S_1(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-5}b_1$
	11			$S_1(2)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	c	$\oplus$	a_1
	12	$S_1(2)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	13					$S_1(3)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_5
	14					$S_1(3)$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	a_2
	15			$S_1(3)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
	16			$S_1(3)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$
	17			$S_1(3)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	18					$S_1(4)$	$\oplus$	b_4	$\oplus$	a_1

Table 35: Minimal deformations $d \oplus S_1(5)$ in Period 5

Codimension	Minimal deformations											
2	1							$\tau^{-23}b_1$	$\oplus$	a_2	$\oplus$	b_1
	2							$\tau^{-18}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-5}b_1$
1	3					$S_2(1)$	$\oplus$	$\tau^{-18}b_1$	$\oplus$	c	$\oplus$	a_1
	4			$S_2(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	5			$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	6	$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	7							$S_2(2)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	b_5
	8					$S_2(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$
	9					$S_2(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
	10					$S_2(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$
	11					$S_2(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$
	12					$S_2(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	13			$S_2(2)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$
	14							$S_2(3)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	b_4
	15					$S_2(3)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_3	$\oplus$	a_1
	16							$S_2(4)$	$\oplus$	b_3	$\oplus$	c

Table 36: Minimal deformations $d \oplus S_2(5)$ in Period 5

Codimension	Minimal deformations											
2	1							$\tau^{-24}b_1$	$\oplus$	c	$\oplus$	a_1
	2					$\tau^{-19}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	3					$\tau^{-14}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	4			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	5			$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
	6			$\tau^{-11}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	7			$\tau^{-6}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
1	8							$S_3(1)$	$\oplus$	$\tau^{-19}b_1$	$\oplus$	b_5
	9					$S_3(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$
	10					$S_3(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-8}b_1$
	11					$S_3(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
	12					$S_3(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$
	13			$S_3(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$
	14			$S_3(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	15	$S_3(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	16							$S_3(2)$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	b_4
	17					$S_3(2)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	b_3	$\oplus$	a_1
	18							$S_3(3)$	$\oplus$	$\tau^{-1}b_3$	$\oplus$	b_3
	19					$S_3(3)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	c	$\oplus$	b_2
	20							$S_3(4)$	$\oplus$	a_2	$\oplus$	b_2

Table 37: Minimal deformations $d \oplus S_3(5)$ in Period 5

Codimension	Minimal deformations											
2	1								$\tau^{-25}b_1$	$\oplus$	b_5	
	2						$\tau^{-20}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$	
	3						$\tau^{-15}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-8}b_1$	
	4						$\tau^{-20}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$	
	5						$\tau^{-20}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$	
	6				$\tau^{-15}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	
	7				$\tau^{-15}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$	
	8				$\tau^{-10}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-9}b_1$	
	9			$\tau^{-10}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	10	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
1	11					$S_4(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-5}b_1$
	12					$S_4(1)$	$\oplus$	$\tau^{-15}b_1$	$\oplus$	b_3	$\oplus$	a_1
	13	$S_4(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	14					$S_4(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	15					$S_4(2)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	c	$\oplus$	b_2
	16			$S_4(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	17					$S_4(3)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_2	$\oplus$	b_1
	18					$S_4(3)$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	a_1	$\oplus$	b_2
	19					$S_4(4)$	$\oplus$	c	$\oplus$	a_1	$\oplus$	b_1

Table 38: Minimal deformations $d \oplus S_4(5)$ in Period 5

Codimension	Minimal deformations									
2	1					$\tau^{-16}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-5}b_1$
	2					$\tau^{-11}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-10}b_1$
	3					$\tau^{-21}b_1$	$\oplus$	b_3	$\oplus$	a_1
	4	$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	5	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
	6	$\tau^{-11}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	7	$\tau^{-6}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
1	8			$S_5(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	9			$S_5(1)$	$\oplus$	$\tau^{-16}b_1$	$\oplus$	c	$\oplus$	b_2
	10	$S_5(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-5}b_1$
	11	$S_5(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	12	$S_5(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	c	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	13			$S_5(2)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	a_2	$\oplus$	b_1
	14			$S_5(2)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-5}b_1$
	15			$S_5(2)$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_2$
	16			$S_5(3)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	c	$\oplus$	a_1
	17			$S_5(3)$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	c	$\oplus$	b_1
	18					$S_5(4)$	$\oplus$	$\tau^{-1}b_1$	$\oplus$	b_5
	19					$S_5(4)$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	a_2
	20					$S_5(4)$	$\oplus$	$\tau^{-1}c$	$\oplus$	c

Table 39: Minimal deformations $d \oplus S_5(5)$ in Period 5

Period 3:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$	$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 2 & 2 & 1 & 1 & 1 \end{smallmatrix}\right]$

Codimension	Minimal deformations									
	1					$\tau^{-18}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-5}b_1$
	2					$\tau^{-15}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-8}b_1$
	3					$\tau^{-21}b_1$	$\oplus$	b_3	$\oplus$	a_1
	4					$\tau^{-24}b_1$	$\oplus$	c	$\oplus$	a_1
	5			$\tau^{-15}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$
	6			$\tau^{-12}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-5}b_1$
2	7			$\tau^{-15}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	8			$\tau^{-12}b_1$	$\oplus$	c	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	9	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
	10	$\tau^{-6}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	11	$\tau^{-10}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	12					$S_1(1)$	$\oplus$	$\tau^{-15}b_1$	$\oplus$	b_5
	13			$S_1(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-5}b_1$
	14			$S_1(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	c	$\oplus$	b_2
	15	$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
1	16					$S_1(2)$	$\oplus$	$\tau^{-1}b_2$	$\oplus$	b_4
	17			$S_1(2)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_2	$\oplus$	b_1
	18			$S_1(2)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	c	$\oplus$	b_1

Table 40: Minimal deformations $d \oplus S_1(3)$ in Period 3

Codimension	Minimal deformations											
2	1									$\tau^{-25}b_1$	$\oplus$	b_5
	2							$\tau^{-16}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-5}b_1$
	3							$\tau^{-22}b_1$	$\oplus$	c	$\oplus$	b_2
	4					$\tau^{-7}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-6}b_1$
	5					$\tau^{-19}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	6					$\tau^{-10}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-9}b_1$
	7			$\tau^{-10}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
1	8					$S_2(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	a_2	$\oplus$	b_1
	9					$S_2(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$
	10					$S_2(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
	11					$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	12					$S_2(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$
	13			$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	14	$S_2(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	15							$S_2(2)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	a_2
	16					$S_2(2)$	$\oplus$	$\tau^{-1}b_1$	$\oplus$	b_3	$\oplus$	a_1
	17					$S_2(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	c	$\oplus$	a_1

Table 41: Minimal deformations $d \oplus S_2(3)$ in Period 3

Codimension	Minimal deformations											
	1							$\tau^{-23}b_1$	$\oplus$	a_2	$\oplus$	b_1
	2							$\tau^{-20}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$
	3							$\tau^{-20}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
	4							$\tau^{-11}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-10}b_1$
	5							$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	6							$\tau^{-20}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$
	7					$\tau^{-17}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
2	8					$\tau^{-14}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	9			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	10			$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
	11			$\tau^{-11}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	12			$\tau^{-6}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	13	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	14					$S_3(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-5}b_1$
	15					$S_3(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	b_3	$\oplus$	a_1
	16					$S_3(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	c	$\oplus$	a_1
	17			$S_3(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$
1	18			$S_3(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	19							$S_3(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_5
	20					$S_3(2)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	c	$\oplus$	b_2
	21					$S_3(2)$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	a_1	$\oplus$	b_2

Table 42: Minimal deformations $d \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 3 & 3 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1 & 2 & 3 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

Codimension	Minimal deformations											
	1									$\tau^{-25}b_1$	$\oplus$	b_5
	2							$\tau^{-23}b_1$	$\oplus$	a_2	$\oplus$	b_1
	3							$\tau^{-15}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-8}b_1$
	4							$\tau^{-11}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-10}b_1$
	5							$\tau^{-21}b_1$	$\oplus$	b_3	$\oplus$	a_1
	6							$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_2$
	7					$\tau^{-15}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$
	8					$\tau^{-7}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-6}b_1$
	9					$\tau^{-19}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
2	10					$\tau^{-17}b_1$	$\oplus$	c	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	11					$\tau^{-15}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	12			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-3}b_1$
	13			$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-8}b_1$
	14			$\tau^{-11}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	15			$\tau^{-10}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	16	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-1}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	17					$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_2	$\oplus$	$\tau^{-3}b_1$
	18					$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_4	$\oplus$	$\tau^{-1}b_1$
1	19					$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c	$\oplus$	a_1
	20					$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	c	$\oplus$	$\tau^{-2}a_1$
	21					$S_1(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	c	$\oplus$	b_2

Table 43: Minimal deformations $d \oplus S_1(2)$ in Period 2

Codimension		Minimal deformations
2	1	$\tau^{-20}b_1 \oplus a_2 \oplus \tau^{-3}b_1$
	2	$\tau^{-18}b_1 \oplus a_2 \oplus \tau^{-5}b_1$
	3	$\tau^{-20}b_1 \oplus b_4 \oplus \tau^{-1}b_1$
	4	$\tau^{-16}b_1 \oplus b_4 \oplus \tau^{-5}b_1$
	5	$\tau^{-24}b_1 \oplus c \oplus a_1$
	6	$\tau^{-20}b_1 \oplus c \oplus \tau^{-2}a_1$
	7	$\tau^{-22}b_1 \oplus c \oplus b_2$
	8	$\tau^{-12}b_1 \oplus b_3 \oplus \tau^{-1}b_1 \oplus \tau^{-5}b_1$
	9	$\tau^{-14}b_1 \oplus c \oplus b_1 \oplus \tau^{-7}b_1$
	10	$\tau^{-10}b_1 \oplus c \oplus \tau^{-2}b_1 \oplus \tau^{-9}b_1$
	11	$\tau^{-12}b_1 \oplus c \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	12	$\tau^{-14}b_1 \oplus a_1 \oplus b_1 \oplus \tau^{-1}b_1 \oplus \tau^{-3}b_1$
	13	$\tau^{-11}b_1 \oplus a_1 \oplus b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1$
	14	$\tau^{-6}b_1 \oplus a_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	15	$\tau^{-10}b_1 \oplus b_2 \oplus \tau^{-1}b_1 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1$
1	16	$S_2(1) \oplus \tau^{-10}b_1 \oplus b_5$
	17	$S_2(1) \oplus \tau^{-8}b_1 \oplus a_2 \oplus b_1$
	18	$S_2(1) \oplus \tau^{-6}b_1 \oplus b_3 \oplus a_1$
	19	$S_2(1) \oplus \tau^{-2}b_1 \oplus b_3 \oplus \tau^{-1}b_2$
	20	$S_2(1) \oplus \tau^{-4}b_1 \oplus c \oplus b_1 \oplus \tau^{-2}b_1$

Table 44: Minimal deformations $d \oplus S_2(2)$ in Period 2

A.11.4 $\mathbf{U = b_5}$

Again the case of $\partial(U) = -5$ yields degeneration diagrams, which are too large to put information in it in a useful way, so we only write the minimal deformations down.

Period 5: $S_1(1) = [\,0\ 0\ \genfrac{}{}{0pt}{}{1}{1}\ 1\ 1\ 1\ 1\ 0\,]$ $\quad S_2(1) = [\,0\ 1\ \genfrac{}{}{0pt}{}{0}{1}\ 1\ 1\ 1\ 1\ 1\,]$

$S_3(1) = [\,1\ 1\ \genfrac{}{}{0pt}{}{1}{2}\ 1\ 0\ 0\ 0\ 0\,]$ $\quad S_4(1) = [\,0\ 1\ \genfrac{}{}{0pt}{}{1}{1}\ 1\ 1\ 0\ 0\ 0\,]$ $\quad S_5(1) = [\,1\ 1\ \genfrac{}{}{0pt}{}{0}{1}\ 1\ 1\ 1\ 0\ 0\,]$

Codimension	Minimal deformations									
1	1					$\tau^{-18}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	2					$\tau^{-23}b_1$	$\oplus$	c	$\oplus$	b_1
	3					$\tau^{-18}b_1$	$\oplus$	c	$\oplus$	$\tau^{-5}b_1$
	4			$\tau^{-13}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-6}b_1$
	5			$\tau^{-8}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-6}b_1$	$\oplus$	$\tau^{-7}b_1$
2	6					$S_1(1)$	$\oplus$	$\tau^{-18}b_1$	$\oplus$	a_2
	7	$S_1(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	8	$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	9	$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	10			$S_1(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	11			$S_1(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$
	12			$S_1(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$
	13					$S_1(3)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_4
	14			$S_1(3)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	15					$S_1(4)$	$\oplus$	b_3	$\oplus$	a_1

Table 45: Minimal deformations $b_5 \oplus S_1(5)$ in Period 5

Codimension	Minimal deformations									
1	1							$\tau^{-24}b_1$	$\oplus$	a_2
	2			$\tau^{-19}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	3			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	4			$\tau^{-14}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	5			$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-8}b_1$
2	6			$S_2(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	7			$S_2(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$
	8			$S_2(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c	$\oplus$	$\tau^{-8}b_1$
	9	$S_2(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	10					$S_2(2)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	b_4
	11			$S_2(2)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	12	$S_2(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	13					$S_2(3)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	b_3
	14			$S_2(3)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	a_1	$\oplus$	b_2
	15					$S_2(4)$	$\oplus$	c	$\oplus$	b_2

Table 46: Minimal deformations $b_5 \oplus S_2(5)$ in Period 5

Codimension		Minimal deformations
1	1	$\tau^{-20}b_1 \oplus a_1 \oplus \tau^{-1}a_1$
	2	$\tau^{-20}b_1 \oplus c \oplus \tau^{-3}b_1$
	3	$\tau^{-15}b_1 \oplus c \oplus \tau^{-8}b_1$
	4	$\tau^{-15}b_1 \oplus a_1 \oplus \tau^{-2}b_1 \oplus \tau^{-4}b_1$
	5	$\tau^{-10}b_1 \oplus a_1 \oplus \tau^{-2}b_1 \oplus \tau^{-9}b_1$
2	6	$S_3(1) \oplus \tau^{-20}b_1 \oplus b_4$
	7	$S_3(1) \oplus \tau^{-15}b_1 \oplus b_3 \oplus \tau^{-2}b_1$
	8	$S_3(1) \oplus \tau^{-10}b_1 \oplus b_2 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1$
	9	$S_3(1) \oplus \tau^{-5}b_1 \oplus b_1 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1$
	10	$S_3(2) \oplus \tau^{-6}b_2 \oplus b_3$
	11	$S_3(2) \oplus \tau^{-10}b_1 \oplus a_1 \oplus b_2$
	12	$S_3(2) \oplus \tau^{-5}b_1 \oplus a_1 \oplus b_1 \oplus \tau^{-4}b_1$
	13	$S_3(3) \oplus \tau^{-2}b_3 \oplus b_2$
	14	$S_3(3) \oplus \tau^{-5}b_1 \oplus c \oplus b_1$
	15	$S_3(4) \oplus a_2 \oplus b_1$

Table 47: Minimal deformations $b_5 \oplus S_3(5)$ in Period 5

Codimension		Minimal deformations
1	1	$\tau^{-26}b_1 \oplus b_4$
	2	$\tau^{-21}b_1 \oplus b_3 \oplus \tau^{-2}b_1$
	3	$\tau^{-16}b_1 \oplus b_2 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1$
	4	$\tau^{-11}b_1 \oplus b_1 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1$
	5	$\tau^{-6}b_1 \oplus \tau^{-2}b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
2	6	$S_4(1) \oplus \tau^{-16}b_1 \oplus a_1 \oplus b_2$
	7	$S_4(1) \oplus \tau^{-11}b_1 \oplus b_3 \oplus \tau^{-6}b_1$
	8	$S_4(1) \oplus \tau^{-11}b_1 \oplus a_1 \oplus b_1 \oplus \tau^{-4}b_1$
	9	$S_4(1) \oplus \tau^{-6}b_1 \oplus a_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	10	$S_4(2) \oplus \tau^{-6}b_1 \oplus b_2 \oplus \tau^{-2}b_2$
	11	$S_4(2) \oplus \tau^{-11}b_1 \oplus c \oplus b_1$
	12	$S_4(2) \oplus \tau^{-6}b_1 \oplus c \oplus \tau^{-5}b_1$
	13	$S_4(3) \oplus \tau^{-6}b_1 \oplus a_2$
	14	$S_4(3) \oplus \tau^{-2}b_2 \oplus a_1 \oplus b_1$
	15	$S_4(4) \oplus c \oplus a_1$

Table 48: Minimal deformations $b_5 \oplus S_4(5)$ in Period 5

Codimension	Minimal deformations									
1	1					$\tau^{-22}b_1$	$\oplus$	a_1	$\oplus$	b_2
	2					$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-6}b_1$
	3					$\tau^{-12}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-11}b_1$
	4			$\tau^{-17}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	5			$\tau^{-12}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
2	6			$S_5(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	7			$S_5(1)$	$\oplus$	$\tau^{-17}b_1$	$\oplus$	c	$\oplus$	b_1
	8			$S_5(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	c	$\oplus$	$\tau^{-5}b_1$
	9	$S_5(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-6}b_1$
	10					$S_5(2)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_2
	11			$S_5(2)$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1
	12	$S_5(2)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	13					$S_5(3)$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	c
	14			$S_5(3)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	15					$S_5(4)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	b_4

Table 49: Minimal deformations $b_5 \oplus S_5(5)$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 2 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 2 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$

Codimension	Minimal deformations									
1	1							$\tau^{-26}b_1$	$\oplus$	b_4
	2					$\tau^{-20}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	3					$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-6}b_1$
	4					$\tau^{-23}b_1$	$\oplus$	c	$\oplus$	b_1
	5					$\tau^{-20}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$
	6			$\tau^{-17}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	7			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	8			$\tau^{-8}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-6}b_1$	$\oplus$	$\tau^{-7}b_1$
	9			$\tau^{-14}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	10	$\tau^{-11}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
2	11					$S_1(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	a_2
	12			$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	13			$S_1(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	14			$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	c	$\oplus$	$\tau^{-5}b_1$
	15	$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	16			$S_1(2)$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	a_1	$\oplus$	b_2

Table 50: Minimal deformations $b_5 \oplus S_1(3)$ in Period 3

Codimension	Minimal deformations									
1	1							$\tau^{-24}b_1$	$\oplus$	a_2
	2					$\tau^{-18}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	3					$\tau^{-21}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	4					$\tau^{-12}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-11}b_1$
	5					$\tau^{-18}b_1$	$\oplus$	c	$\oplus$	$\tau^{-5}b_1$
	6					$\tau^{-15}b_1$	$\oplus$	c	$\oplus$	$\tau^{-8}b_1$
	7			$\tau^{-15}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	8			$\tau^{-12}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	9			$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-8}b_1$
	10	$\tau^{-6}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
2	11			$S_2(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_1	$\oplus$	b_2
	12	$S_2(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	13	$S_2(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	14					$S_2(2)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_4
	15			$S_2(2)$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	a_1	$\oplus$	b_1
	16			$S_2(2)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	c	$\oplus$	b_1

Table 51: Minimal deformations $b_5 \oplus S_2(3)$ in Period 3

Codimension	Minimal deformations									
1	1					$\tau^{-22}b_1$	$\oplus$	a_1	$\oplus$	b_2
	2			$\tau^{-19}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	3			$\tau^{-10}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-9}b_1$
	4			$\tau^{-16}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	5			$\tau^{-13}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-6}b_1$
2	6					$S_3(1)$	$\oplus$	$\tau^{-16}b_1$	$\oplus$	b_4
	7			$S_3(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	8			$S_3(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-6}b_1$
	9			$S_3(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	c	$\oplus$	b_1
	10			$S_3(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$
	11	$S_3(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	12	$S_3(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	13					$S_3(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	a_2
	14					$S_3(2)$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_3
	15					$S_3(2)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	c

Table 52: Minimal deformations $b_5 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1 & 2 & 3 & 3 & 2 & 2 & 1 & 1 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 3 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$

Codimension	Minimal deformations									
1	1					$\tau^{-21}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	2					$\tau^{-17}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-6}b_1$
	3					$\tau^{-23}b_1$	$\oplus$	c	$\oplus$	b_1
	4					$\tau^{-15}b_1$	$\oplus$	c	$\oplus$	$\tau^{-8}b_1$
	5			$\tau^{-19}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	6			$\tau^{-17}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	7			$\tau^{-15}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	8			$\tau^{-13}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-6}b_1$
	9			$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-8}b_1$
	10	$\tau^{-11}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
2	11					$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_2
	12					$S_1(1)$	$\oplus$	$\tau^{-11}b_1$	$\oplus$	b_4
	13			$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	14			$S_1(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	a_1	$\oplus$	b_2
	15			$S_1(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	16			$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$

Table 53: Minimal deformations $b_5 \oplus S_1(2)$ in Period 2

Codimension	Minimal deformations									
1	1							$\tau^{-24}b_1$	$\oplus$	a_2
	2							$\tau^{-26}b_1$	$\oplus$	b_4
	3					$\tau^{-20}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-1}a_1$
	4					$\tau^{-22}b_1$	$\oplus$	a_1	$\oplus$	b_2
	5					$\tau^{-18}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_2$
	6					$\tau^{-12}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-11}b_1$
	7					$\tau^{-20}b_1$	$\oplus$	c	$\oplus$	$\tau^{-3}b_1$
	8					$\tau^{-18}b_1$	$\oplus$	c	$\oplus$	$\tau^{-5}b_1$
	9			$\tau^{-14}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	10			$\tau^{-10}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-9}b_1$
	11			$\tau^{-12}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	12			$\tau^{-16}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$
	13			$\tau^{-8}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-6}b_1$	$\oplus$	$\tau^{-7}b_1$
	14			$\tau^{-14}b_1$	$\oplus$	$\tau^{-2}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	15	$\tau^{-6}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
2	16			$S_2(1)$	$\oplus$	$\tau^{-6}b_1$	$\oplus$	b_3	$\oplus$	$\tau^{-2}b_1$
	17			$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	c	$\oplus$	b_1
	18	$S_2(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	a_1	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$

Table 54: Minimal deformations $b_5 \oplus S_2(2)$ in Period 2

A.11.5 $\mathbf{U} = \mathbf{b}_4$

$$\mathbf{Codim}(U \oplus V, \tau^{-6}a_2) = 5$$

Period 5:

$$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right] \quad S_2(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{smallmatrix}\right]$$

$$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right] \quad S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right] \quad S_5(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$$

Codimension	Dot	Preprojective deformations						
5	1							$\tau^{-6}a_2$
	2					$\tau^{-12}b_1$	$\oplus$	$\tau^{-5}b_3$
	3					$\tau^{-9}b_3$	$\oplus$	b_1
	4					$\tau^{-9}b_1$	$\oplus$	$\tau^{-5}c$
4	5					$\tau^{-7}c$	$\oplus$	$\tau^{-3}b_1$
	6					$\tau^{-6}a_1$	$\oplus$	$\tau^{-5}a_1$
	7					$\tau^{-9}b_2$	$\oplus$	$\tau^{-3}a_1$
	8					$\tau^{-8}a_1$	$\oplus$	$\tau^{-4}b_2$
	9					$\tau^{-15}b_1$	$\oplus$	$\tau^{-4}b_3$
	10					$\tau^{-18}b_1$	$\oplus$	$\tau^{-2}c$
	11					$\tau^{-11}a_1$	$\oplus$	a_1
	12					$\tau^{-13}b_2$	$\oplus$	b_2
	13					$\tau^{-10}b_2$	$\oplus$	$\tau^{-3}b_2$
	14			$\tau^{-12}b_1$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	$\tau^{-9}b_1$
	15			$\tau^{-14}b_1$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-3}b_1$
	16			$\tau^{-9}b_1$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-8}b_1$
	17			$\tau^{-13}b_1$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	b_1
3	18			$\tau^{-10}b_1$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-3}b_1$
	19			$\tau^{-8}a_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	20			$\tau^{-10}a_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	21			$\tau^{-17}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1
	22			$\tau^{-12}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	$\tau^{-5}b_1$
	23			$\tau^{-12}b_1$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	$\tau^{-3}b_1$
	24			$\tau^{-9}b_1$	$\oplus$	$\tau^{-8}b_2$	$\oplus$	b_1
	25			$\tau^{-9}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	26					$\tau^{-27}b_1$	$\oplus$	b_3
	27					$\tau^{-24}b_1$	$\oplus$	c
	28			$\tau^{-23}b_1$	$\oplus$	a_1	$\oplus$	b_1
	29			$\tau^{-20}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	30			$\tau^{-18}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-5}b_1$
	31			$\tau^{-15}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-8}b_1$
	32			$\tau^{-22}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	33			$\tau^{-18}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-7}b_1$
2	34			$\tau^{-13}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-12}b_1$
	35			$\tau^{-19}b_1$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	b_1
	36			$\tau^{-15}b_1$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	$\tau^{-4}b_1$
	37			$\tau^{-10}b_1$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	$\tau^{-9}b_1$
	38	$\tau^{-17}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	39	$\tau^{-14}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-7}b_1$
	40	$\tau^{-9}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$	$\oplus$	$\tau^{-8}b_1$
	41	$\tau^{-12}b_1$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$

Table 55: Preprojective deformations of $b_4 \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_1(1) \oplus \tau^{-5}a_1 \oplus \tau^{-4}b_2$
2	Reg_2	$S_1(1) \oplus \tau^{-9}b_1 \oplus \tau^{-4}b_3$
	Reg_3	$S_1(1) \oplus \tau^{-9}b_2 \oplus a_1$
	Reg_4	$S_1(1) \oplus \tau^{-5}a_1 \oplus \tau^{-3}b_1 \oplus \tau^{-4}b_1$
	Reg_5	$S_1(3) \oplus \tau^{-4}b_2 \oplus b_2$
1	Reg_6	$S_1(1) \oplus \tau^{-14}b_1 \oplus a_1 \oplus \tau^{-3}b_1$
	Reg_7	$S_1(1) \oplus \tau^{-9}b_1 \oplus a_1 \oplus \tau^{-8}b_1$
	Reg_8	$S_1(1) \oplus \tau^{-9}b_1 \oplus \tau^{-3}b_2 \oplus \tau^{-4}b_1$
	Reg_9	$S_1(3) \oplus \tau^{-9}b_1 \oplus b_3$
	Reg_{10}	$S_1(3) \oplus \tau^{-4}b_1 \oplus b_2 \oplus \tau^{-3}b_1$
	Reg_{11}	$S_1(4) \oplus a_1 \oplus b_2$

Table 56: Deformations with nonzero regular part of $b_4 \oplus S_1(5)$ in Period 5

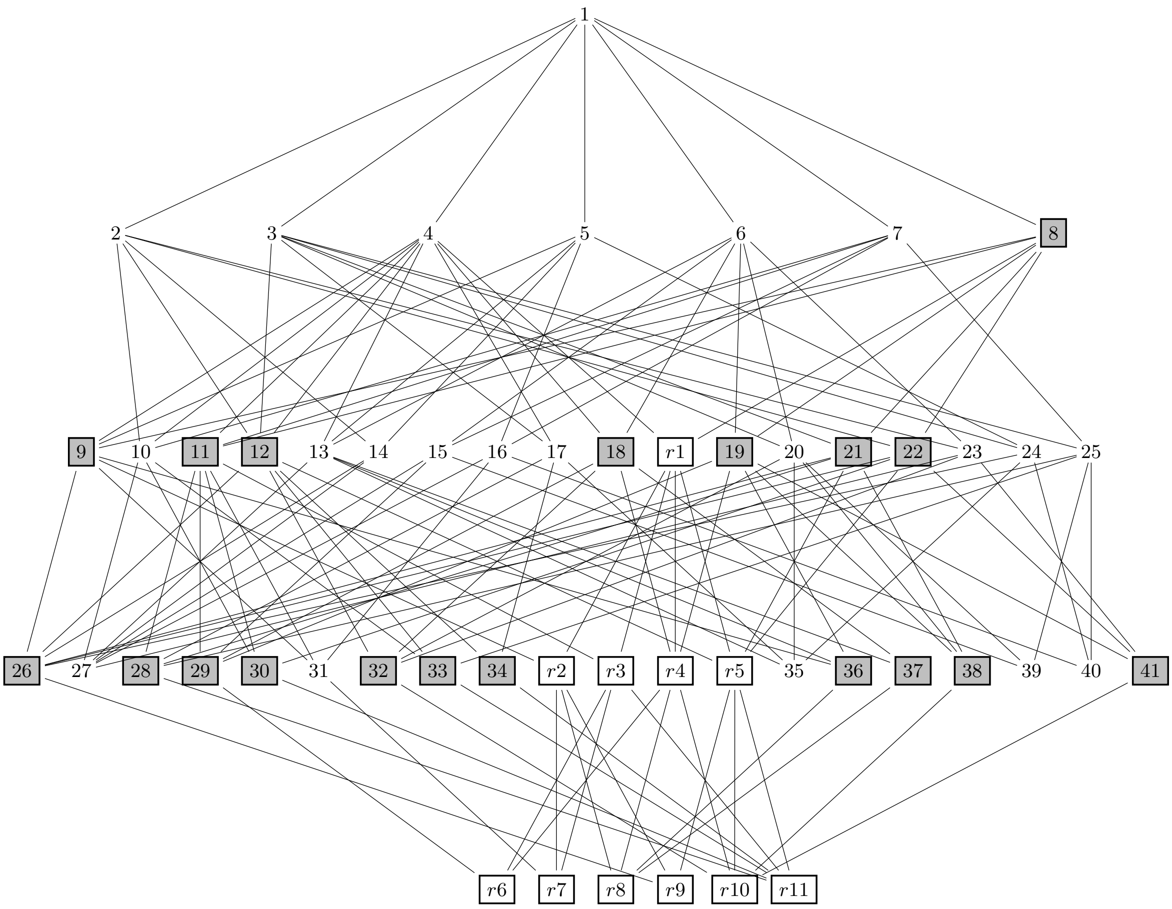


fig. A.117: Degeneration diagram of $b_4 \oplus S_1(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part								
3	Reg_1					$S_2(2)$	$\oplus$	$\tau^{-5}b_3$	$\oplus$	b_1
2	Reg_2					$S_2(2)$	$\oplus$	$\tau^{-6}a_1$	$\oplus$	b_2
	Reg_3			$S_2(2)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	b_1
	Reg_4			$S_2(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1
	Reg_5			$S_2(2)$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
	Reg_6					$S_2(3)$	$\oplus$	$\tau^{-2}c$	$\oplus$	b_1
1	Reg_7					$S_2(2)$	$\oplus$	$\tau^{-15}b_1$	$\oplus$	b_3
	Reg_8			$S_2(2)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	Reg_9	$S_2(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_{10}					$S_2(3)$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	b_2
	Reg_{11}			$S_2(3)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	b_1
	Reg_{12}					$S_2(4)$	$\oplus$	c	$\oplus$	b_1

Table 57: Deformations with nonzero regular part of $b_4 \oplus S_2(5)$ in Period 5

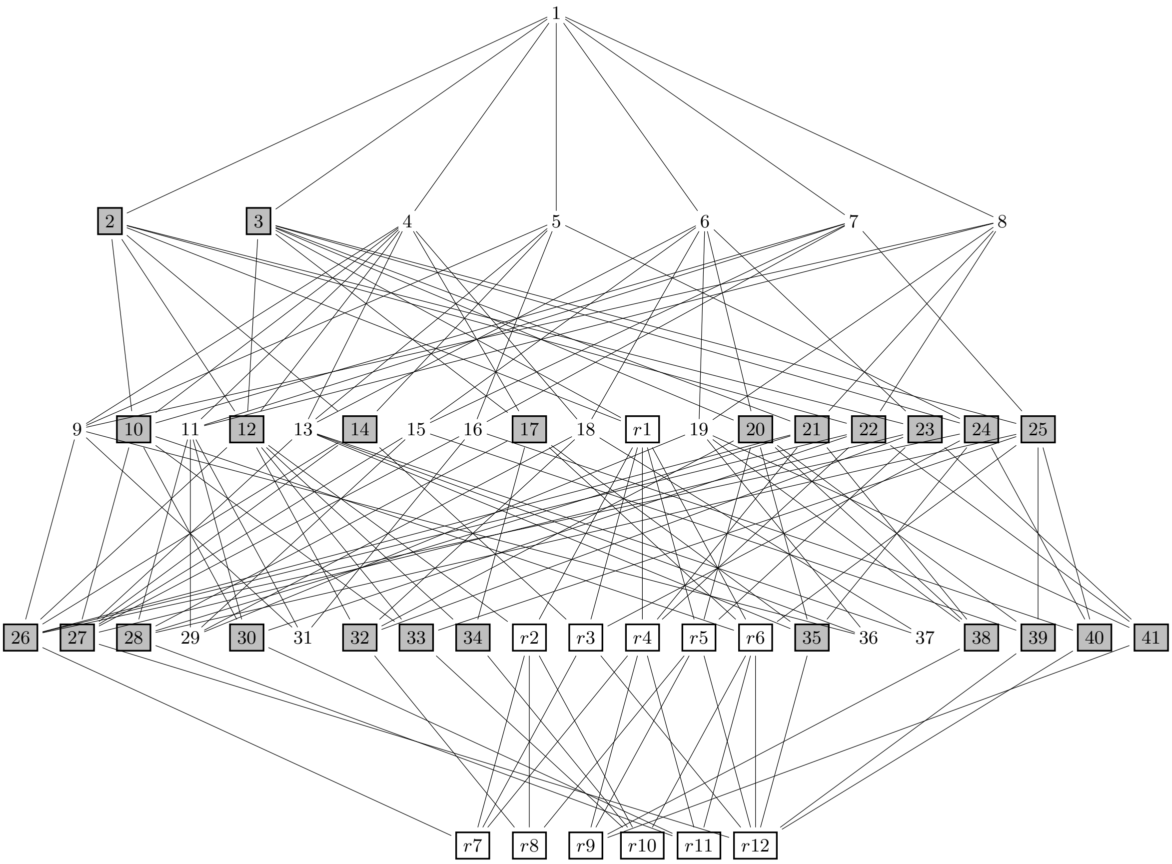


fig. A.118: Degeneration diagram of $b_4 \oplus S_2(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_4(1) \oplus \tau^{-5}a_1 \oplus \tau^{-3}a_1$
2	Reg_2	$S_4(1) \oplus \tau^{-12}b_1 \oplus \tau^{-2}c$
	Reg_3	$S_4(1) \oplus \tau^{-8}a_1 \oplus a_1$
	Reg_4	$S_4(1) \oplus \tau^{-7}b_1 \oplus \tau^{-5}a_1 \oplus b_1$
	Reg_5	$S_4(2) \oplus \tau^{-3}a_1 \oplus \tau^{-3}b_2$
1	Reg_6	$S_4(1) \oplus \tau^{-17}b_1 \oplus a_1 \oplus b_1$
	Reg_7	$S_4(1) \oplus \tau^{-12}b_1 \oplus a_1 \oplus \tau^{-5}b_1$
	Reg_8	$S_4(1) \oplus \tau^{-12}b_1 \oplus b_2 \oplus \tau^{-7}b_1$
	Reg_9	$S_4(2) \oplus \tau^{-12}b_1 \oplus c$
	Reg_{10}	$S_4(2) \oplus \tau^{-7}b_1 \oplus \tau^{-3}b_2 \oplus b_1$
	Reg_{11}	$S_4(3) \oplus \tau^{-3}b_2 \oplus a_1$

Table 58: Deformations with nonzero regular part of $b_4 \oplus S_4(5)$ in Period 5

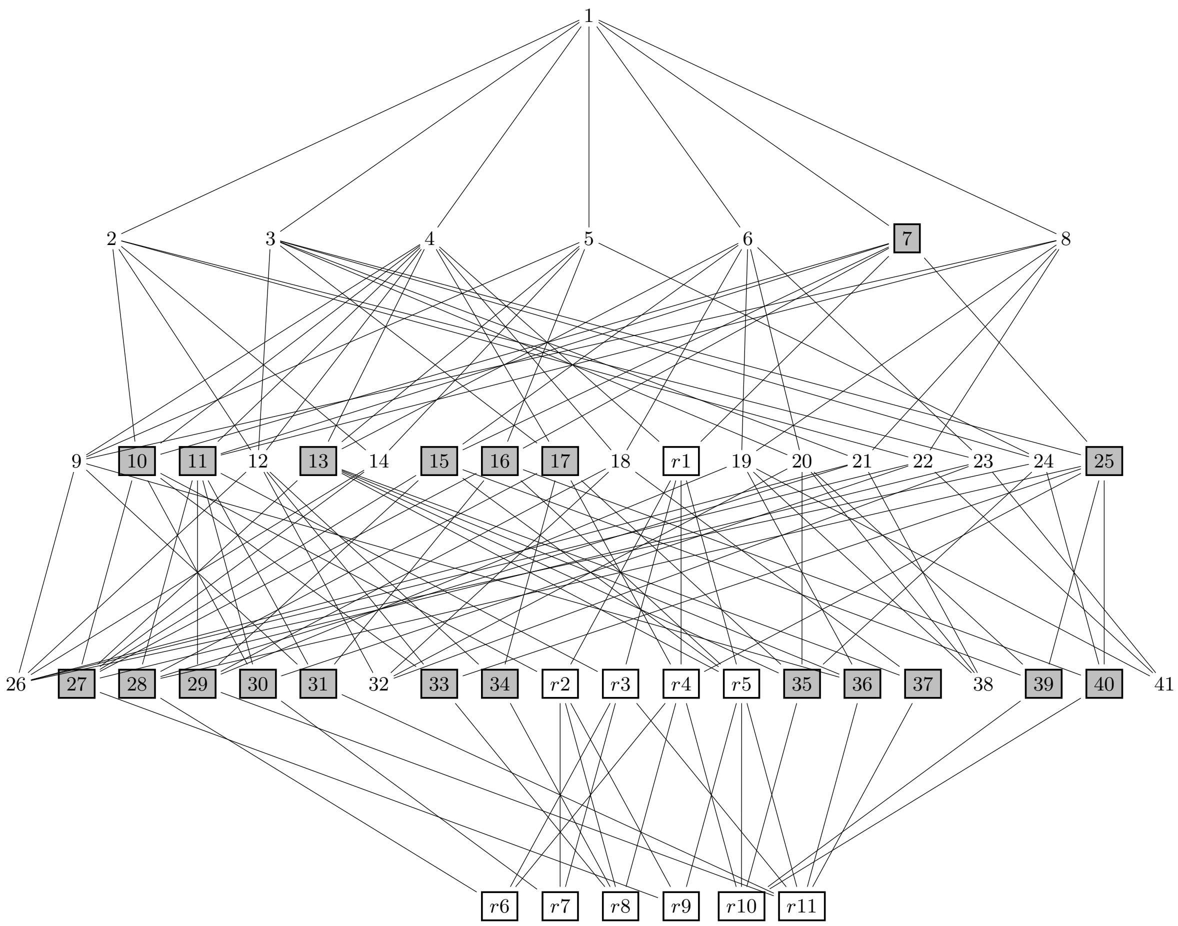


fig. A.119: Degeneration diagram of $b_4 \oplus S_4(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_5(1) \oplus \tau^{-5}c \oplus \tau^{-3}b_1$
2	Reg_2	$S_5(1) \oplus \tau^{-6}a_1 \oplus \tau^{-3}b_2$
	Reg_3	$S_5(1) \oplus \tau^{-12}b_1 \oplus \tau^{-1}a_1 \oplus \tau^{-3}b_1$
	Reg_4	$S_5(1) \oplus \tau^{-8}b_1 \oplus \tau^{-3}a_1 \oplus \tau^{-3}b_1$
	Reg_5	$S_5(1) \oplus \tau^{-8}b_2 \oplus b_1 \oplus \tau^{-3}b_1$
	Reg_6	$S_5(2) \oplus \tau^{-4}b_3 \oplus \tau^{-3}b_1$
1	Reg_7	$S_5(1) \oplus \tau^{-18}b_1 \oplus c$
	Reg_8	$S_5(1) \oplus \tau^{-13}b_1 \oplus \tau^{-3}b_2 \oplus b_1$
	Reg_9	$S_5(1) \oplus \tau^{-8}b_1 \oplus b_1 \oplus \tau^{-3}b_1 \oplus \tau^{-7}b_1$
	Reg_{10}	$S_5(2) \oplus \tau^{-4}b_2 \oplus \tau^{-3}b_2$
	Reg_{11}	$S_5(2) \oplus \tau^{-8}b_1 \oplus a_1 \oplus \tau^{-3}b_1$
	Reg_{12}	$S_5(4) \oplus \tau^{-3}b_1 \oplus b_3$

Table 59: Deformations with nonzero regular part of $b_4 \oplus S_5(5)$ in Period 5

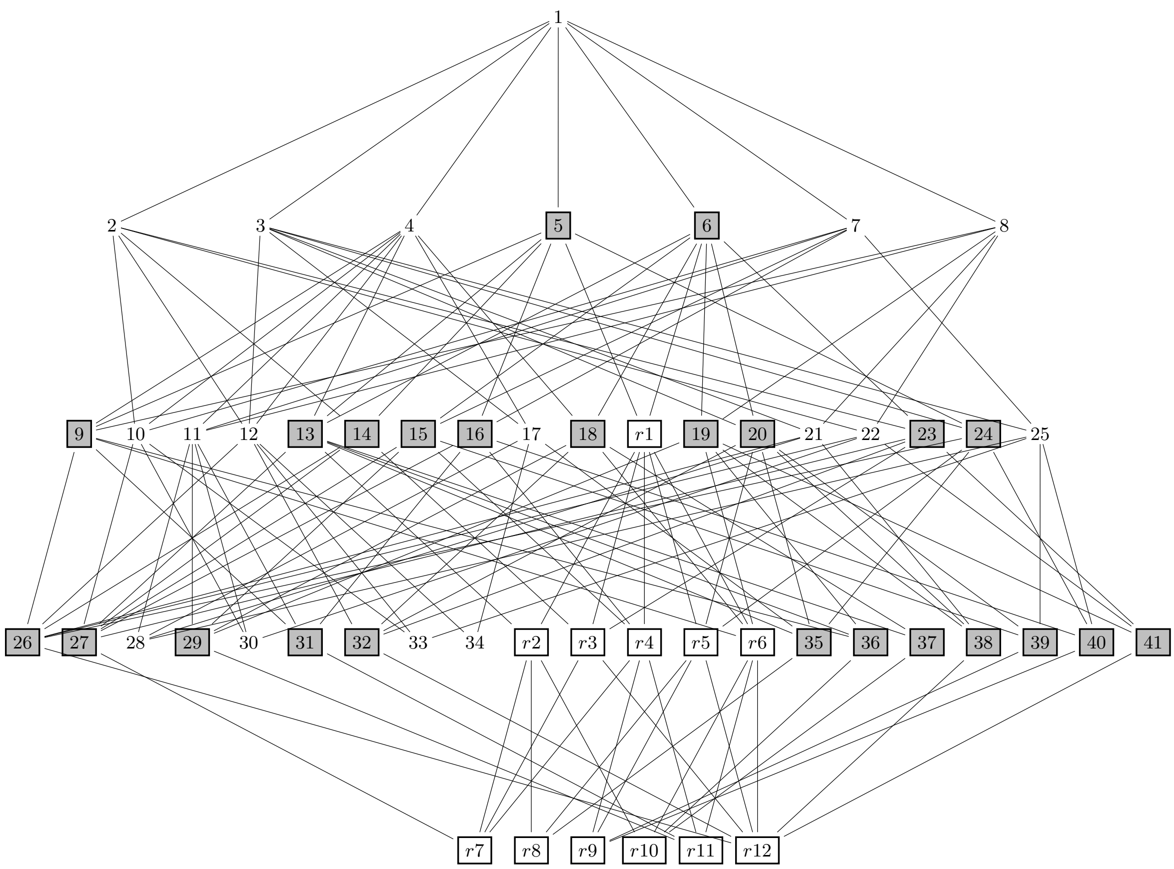


fig. A.120: Degeneration diagram of $b_4 \oplus S_5(5)$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 2 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1	$S_1(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$
1	Reg_2	$S_1(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	b_1
	Reg_4	$S_1(2)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	a_1	$\oplus$	b_1

Table 60: Deformations with nonzero regular part of $b_4 \oplus S_1(3)$ in Period 3

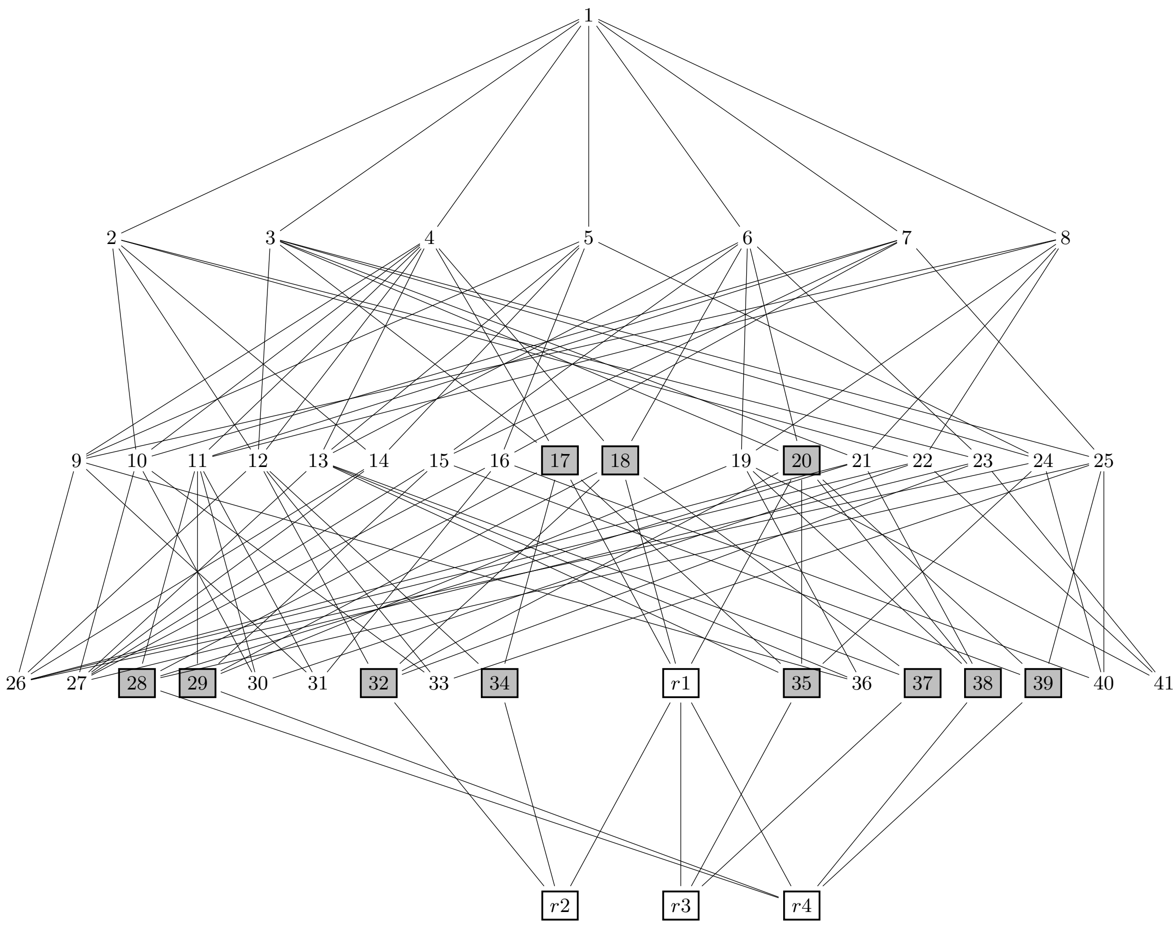


fig. A.121: Degeneration diagram of $b_4 \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part								
3	Reg_1					$S_2(1)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	$\tau^{-3}a_1$
2	Reg_2					$S_2(1)$	$\oplus$	$\tau^{-6}a_1$	$\oplus$	a_1
	Reg_3			$S_2(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-3}a_1$	$\oplus$	$\tau^{-3}b_1$
	Reg_4			$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1
	Reg_5							$S_2(2)$	$\oplus$	$\tau^{-1}a_2$
1	Reg_6			$S_2(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	a_1	$\oplus$	b_1
	Reg_7			$S_2(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_8	$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_9					$S_2(2)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_3
	Reg_{10}					$S_2(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	c
	Reg_{11}					$S_2(2)$	$\oplus$	$\tau^{-1}a_1$	$\oplus$	a_1

Table 61: Deformations with nonzero regular part of $b_4 \oplus S_2(3)$ in Period 3

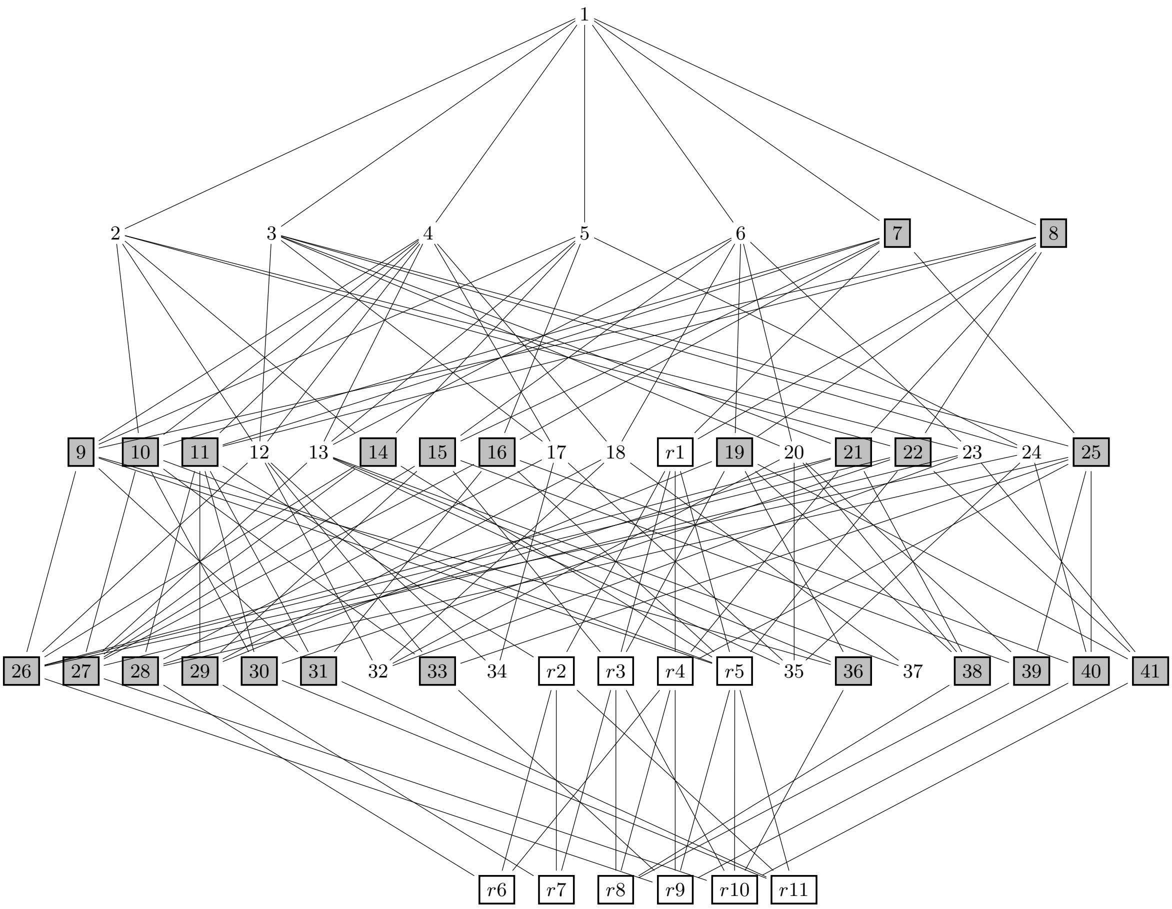


fig. A.122: Degeneration diagram of $b_4 \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_3(1) \oplus \tau^{-5}b_4$
2	Reg_2	$S_3(1) \oplus \tau^{-5}b_1 \oplus \tau^{-4}b_3$
	Reg_3	$S_3(1) \oplus \tau^{-8}b_1 \oplus \tau^{-2}c$
	Reg_4	$S_3(1) \oplus \tau^{-5}a_1 \oplus \tau^{-1}a_1$
	Reg_5	$S_3(1) \oplus \tau^{-8}b_2 \oplus b_2$
	Reg_6	$S_3(1) \oplus \tau^{-5}b_2 \oplus \tau^{-3}b_2$
1	Reg_7	$S_3(1) \oplus \tau^{-17}b_1 \oplus b_3$
	Reg_8	$S_3(1) \oplus \tau^{-14}b_1 \oplus c$
	Reg_9	$S_3(1) \oplus \tau^{-8}b_1 \oplus a_1 \oplus \tau^{-5}b_1$
	Reg_{10}	$S_3(1) \oplus \tau^{-8}b_1 \oplus b_2 \oplus \tau^{-7}b_1$
	Reg_{11}	$S_3(1) \oplus \tau^{-5}b_1 \oplus \tau^{-3}b_2 \oplus \tau^{-4}b_1$
	Reg_{12}	$S_3(2) \oplus \tau^{-3}b_2 \oplus b_2$

Table 62: Deformations with nonzero regular part of $b_4 \oplus S_3(3)$ in Period 3

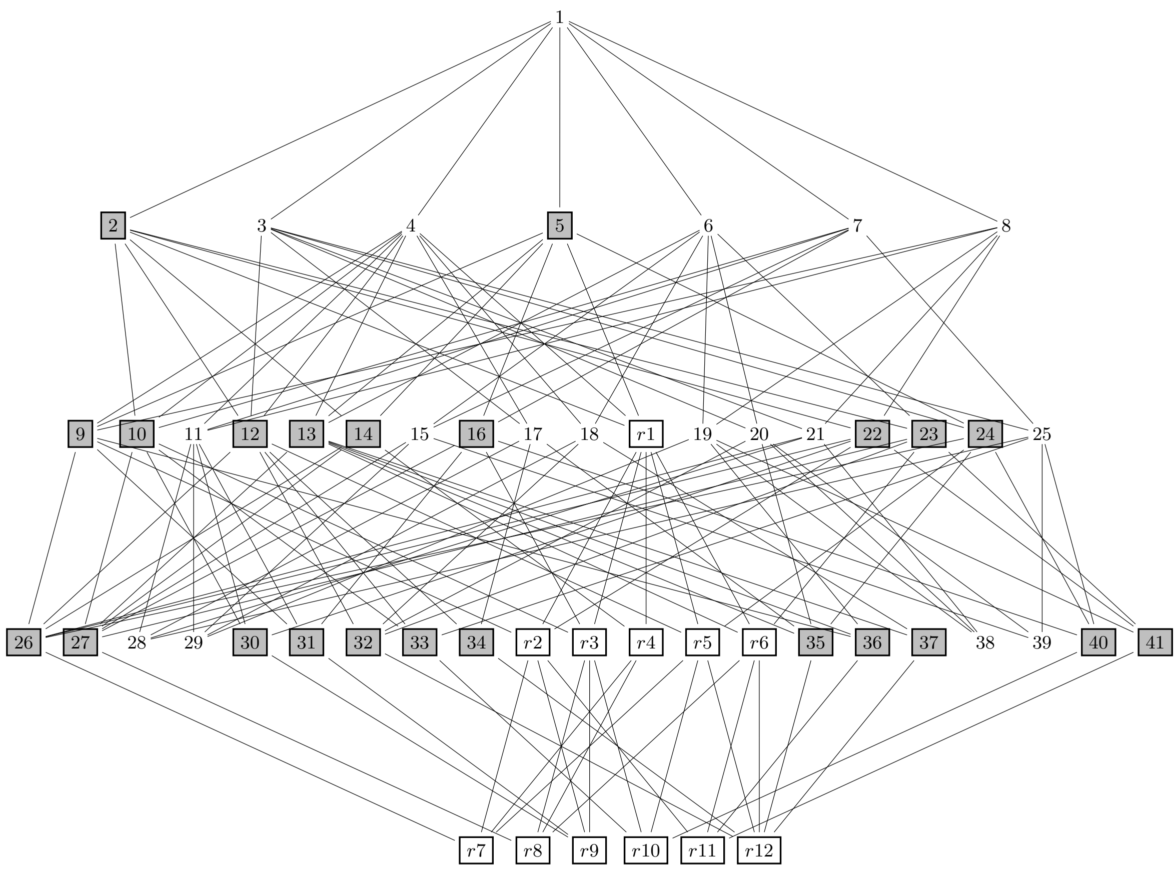


fig. A.123: Degeneration diagram of $b_4 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1&2&3&3&2&2&1&1\end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&2&3&2&2&1&1&0\end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(1)$	$\oplus$	$\tau^{-4}b_3$	$\oplus$	b_1
1	Reg_2			$S_1(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	b_3
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	a_1	$\oplus$	b_1
	Reg_4	$S_1(1)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-3}b_2$	$\oplus$	b_1

Table 63: Deformations with nonzero regular part of $b_4 \oplus S_1(2)$ in Period 2

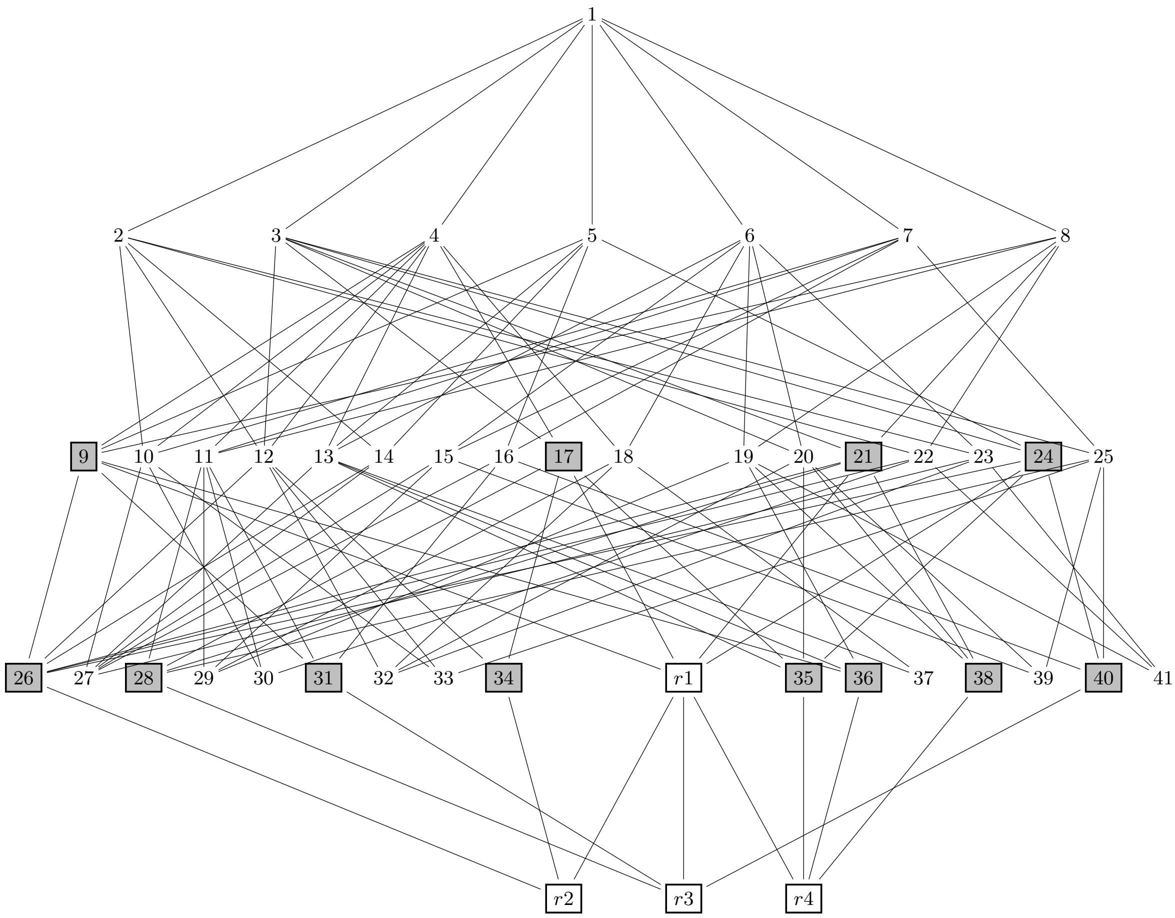


fig. A.124: Degeneration diagram of $b_4 \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_2(1)$	$\oplus$	$\tau^{-3}b_1$	$\oplus$	$\tau^{-2}c$
1	Reg_2			$S_2(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	c
	Reg_3	$S_2(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	a_1	$\oplus$	$\tau^{-3}b_1$
	Reg_4	$S_2(1)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_2	$\oplus$	$\tau^{-3}b_1$

Table 64: Deformations with nonzero regular part of $b_4 \oplus S_2(2)$ in Period 2

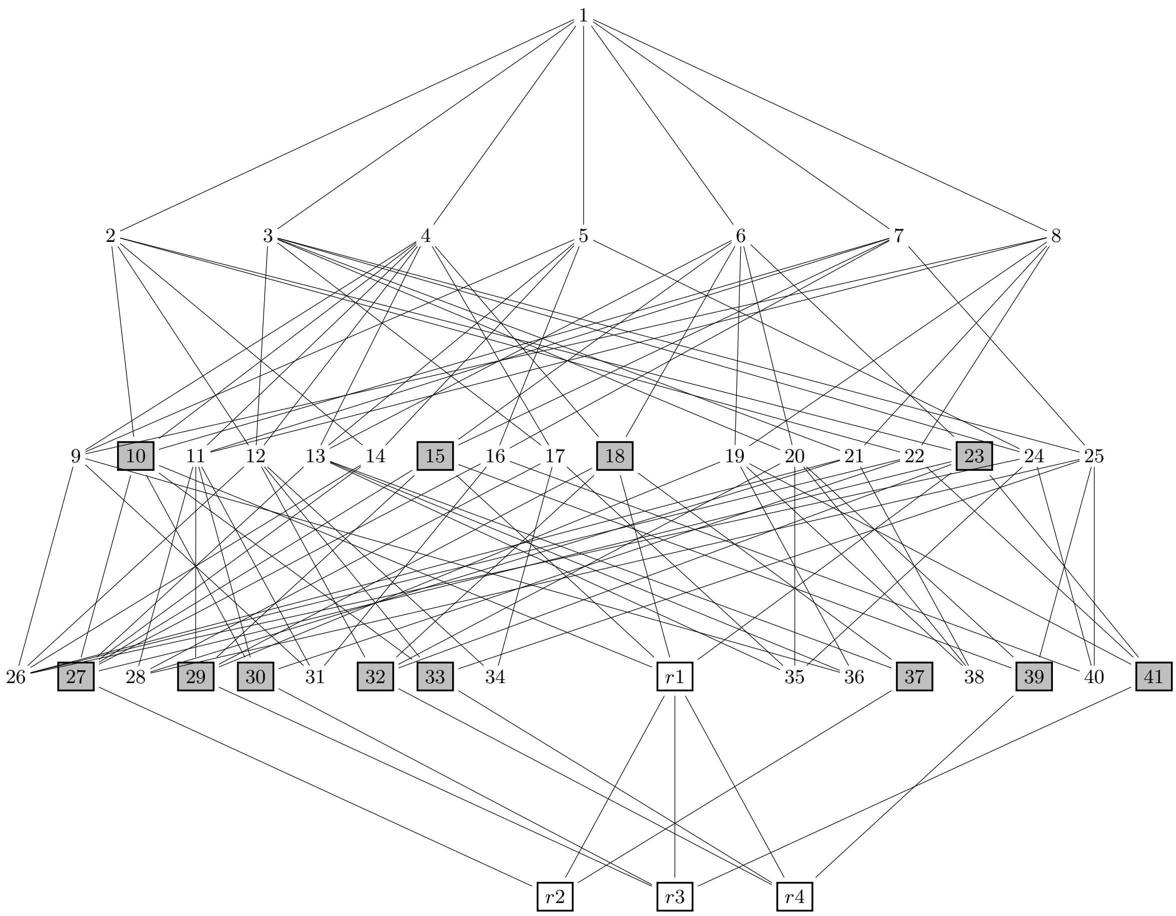


fig. A.125: Degeneration diagram of $b_4 \oplus S_2(2)$ in Period 2

A.11.6 $\mathbf{U = b_3}$

$$\mathbf{Codim}(U \oplus V, \tau^{-10}b_3) = 4$$

Codimension	Dot	Preprojective deformations
4	1	$\tau^{-10}b_3$
3	2	$\tau^{-14}b_1 \oplus \tau^{-5}a_1$
	3	$\tau^{-10}a_1 \oplus \tau^{-4}b_1$
	4	$\tau^{-18}b_1 \oplus \tau^{-5}b_2$
	5	$\tau^{-10}b_1 \oplus \tau^{-9}b_2$
	6	$\tau^{-10}b_2 \oplus \tau^{-8}b_1$
	7	$\tau^{-14}b_2 \oplus b_1$
2	8	$\tau^{-24}b_1 \oplus a_1$
	9	$\tau^{-28}b_1 \oplus b_2$
	10	$\tau^{-20}b_1 \oplus \tau^{-4}b_2$
	11	$\tau^{-23}b_1 \oplus b_1 \oplus \tau^{-4}b_1$
	12	$\tau^{-19}b_1 \oplus b_1 \oplus \tau^{-8}b_1$
	13	$\tau^{-14}b_1 \oplus b_1 \oplus \tau^{-13}b_1$
	14	$\tau^{-18}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	15	$\tau^{-15}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-8}b_1$
	16	$\tau^{-10}b_1 \oplus \tau^{-8}b_1 \oplus \tau^{-9}b_1$

Table 65: Preprojective deformations of $b_3 \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

Period 5:

$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0\end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 1 & 1 & 1 & 0 & 0\end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0\end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\end{smallmatrix}\right]$ $S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0\end{smallmatrix}\right]$

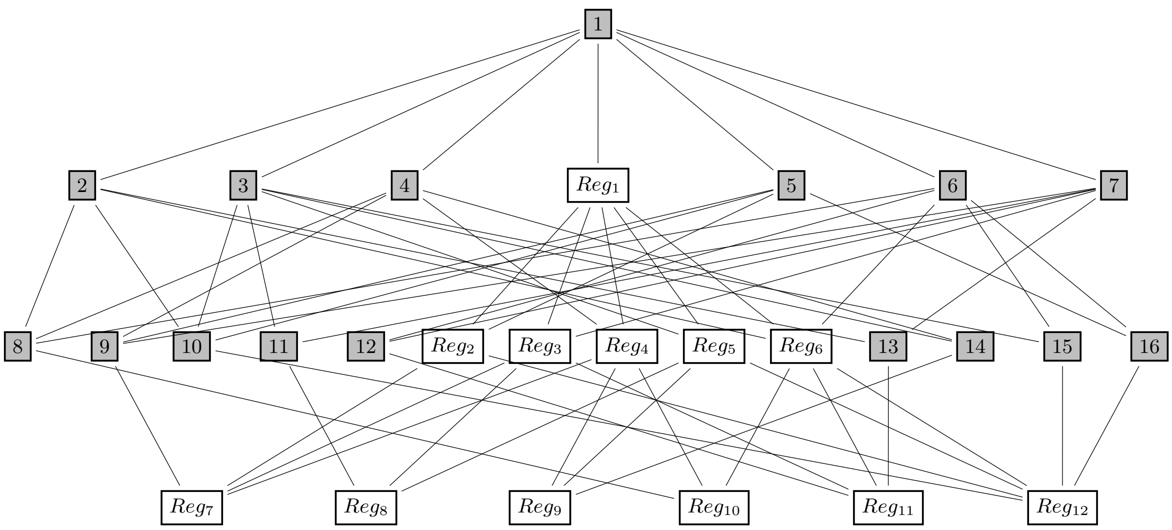


fig. A.126: Degeneration diagram of $b_3 \oplus S_1(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_1(2) \oplus \tau^{-6}b_3$
2	Reg_2	$S_1(2) \oplus \tau^{-10}b_1 \oplus \tau^{-1}a_1$
	Reg_3	$S_1(2) \oplus \tau^{-6}a_1 \oplus b_1$
	Reg_4	$S_1(2) \oplus \tau^{-6}b_1 \oplus \tau^{-5}b_2$
	Reg_5	$S_1(2) \oplus \tau^{-6}b_2 \oplus \tau^{-4}b_1$
	Reg_6	$S_1(3) \oplus \tau^{-2}c$
1	Reg_7	$S_1(2) \oplus \tau^{-16}b_1 \oplus b_2$
	Reg_8	$S_1(2) \oplus \tau^{-11}b_1 \oplus b_1 \oplus \tau^{-4}b_1$
	Reg_9	$S_1(2) \oplus \tau^{-6}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	Reg_{10}	$S_1(3) \oplus \tau^{-6}b_1 \oplus a_1$
	Reg_{11}	$S_1(3) \oplus \tau^{-3}a_1 \oplus b_1$
	Reg_{12}	$S_1(4) \oplus c$

Table 66: Deformations with nonzero regular part of $b_3 \oplus S_1(5)$ in Period 5

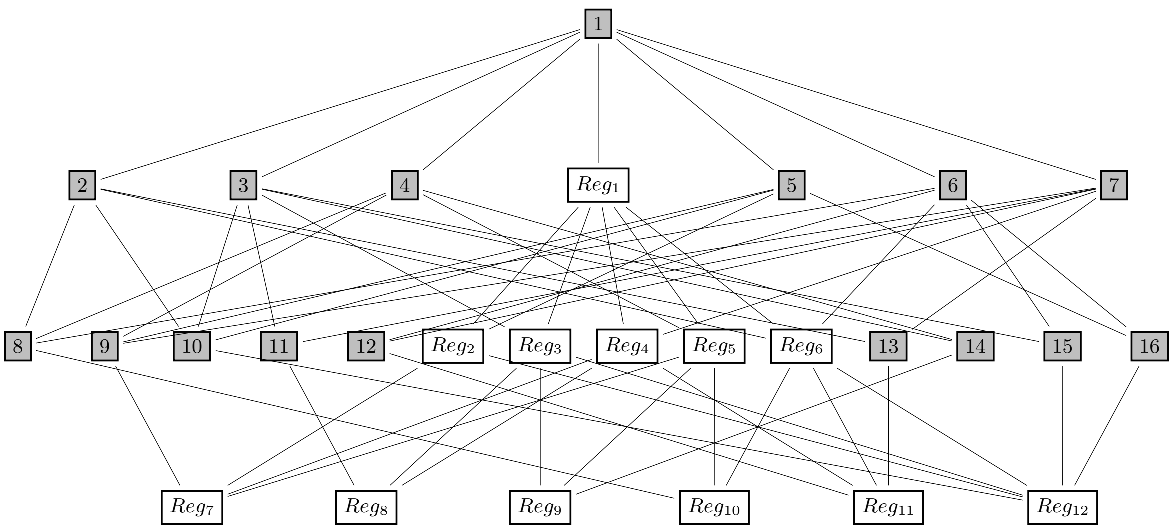


fig. A.127: Degeneration diagram of $b_3 \oplus S_2(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part						
3	Reg_1					$S_2(1)$	$\oplus$	$\tau^{-6}c$
2	Reg_2			$S_2(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	$\tau^{-4}a_1$
	Reg_3			$S_2(1)$	$\oplus$	$\tau^{-7}a_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_4			$S_2(1)$	$\oplus$	$\tau^{-9}a_1$	$\oplus$	b_1
	Reg_5			$S_2(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	$\tau^{-5}b_2$
	Reg_6					$S_2(2)$	$\oplus$	$\tau^{-4}c$
1	Reg_7			$S_2(1)$	$\oplus$	$\tau^{-22}b_1$	$\oplus$	b_2
	Reg_8	$S_2(1)$	$\oplus$	$\tau^{-17}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	Reg_9	$S_2(1)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	Reg_{10}			$S_2(2)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_1
	Reg_{11}			$S_2(2)$	$\oplus$	$\tau^{-8}b_2$	$\oplus$	b_1
	Reg_{12}					$S_2(3)$	$\oplus$	$\tau^{-4}b_3$

Table 67: Deformations with nonzero regular part of $b_3 \oplus S_2(5)$ in Period 5

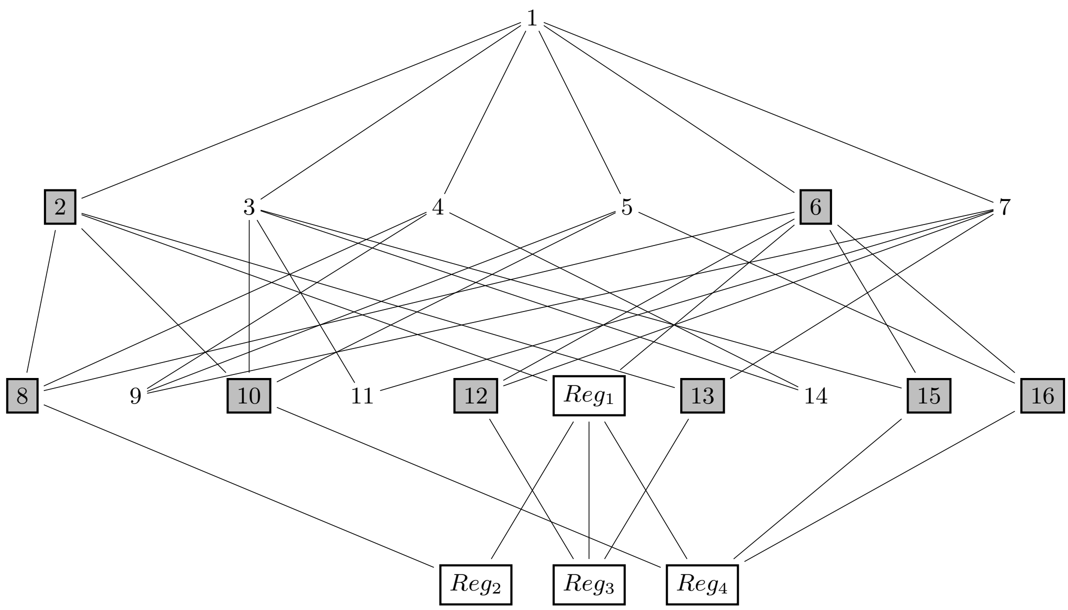


fig. A.128: Degeneration diagram of $b_3 \oplus S_3(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_3(1) \oplus \tau^{-8}b_1 \oplus \tau^{-5}a_1$
1	Reg_2	$S_3(1) \oplus \tau^{-18}b_1 \oplus a_1$
	Reg_3	$S_3(1) \oplus \tau^{-13}b_1 \oplus b_1 \oplus \tau^{-8}b_1$
	Reg_4	$S_3(2) \oplus \tau^{-8}b_1 \oplus \tau^{-4}b_2$

Table 68: Deformations with nonzero regular part of $b_4 \oplus S_3(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_4(1) \oplus \tau^{-9}b_2 \oplus \tau^{-4}b_1$
1	Reg_2	$S_4(1) \oplus \tau^{-14}b_1 \oplus \tau^{-4}b_2$
	Reg_3	$S_4(1) \oplus \tau^{-9}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-8}b_1$
	Reg_4	$S_4(4) \oplus \tau^{-4}b_1 \oplus b_2$

Table 69: Deformations with nonzero regular part of $b_4 \oplus S_4(5)$ in Period 5

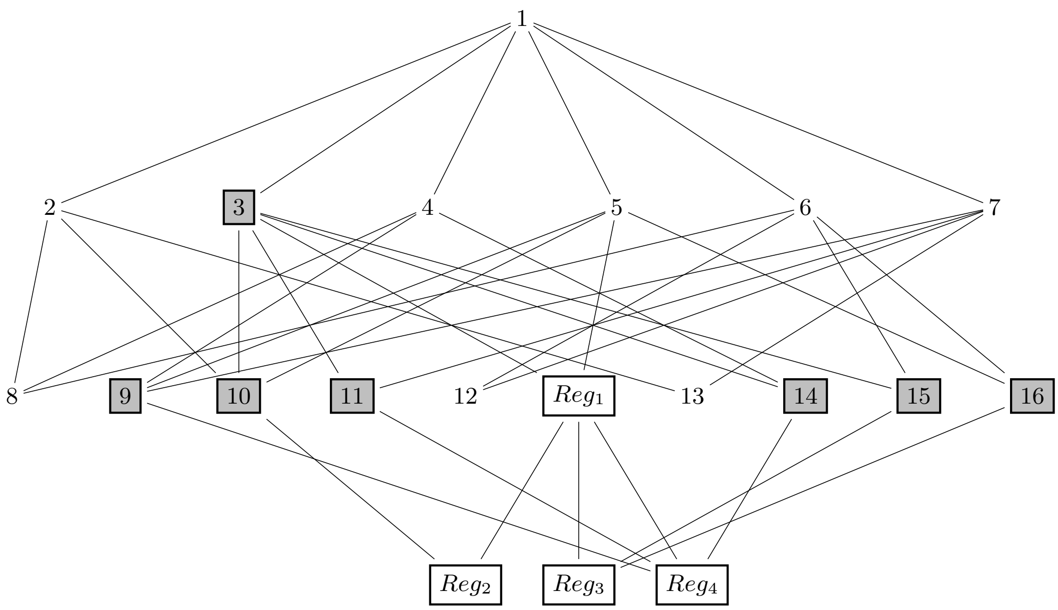


fig. A.129: Degeneration diagram of $b_3 \oplus S_4(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_5(3)$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	b_1
1	Reg_2			$S_5(3)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	b_2
	Reg_3	$S_5(3)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	Reg_4			$S_5(4)$	$\oplus$	a_1	$\oplus$	b_1

Table 70: Deformations with nonzero regular part of $b_4 \oplus S_5(5)$ in Period 5

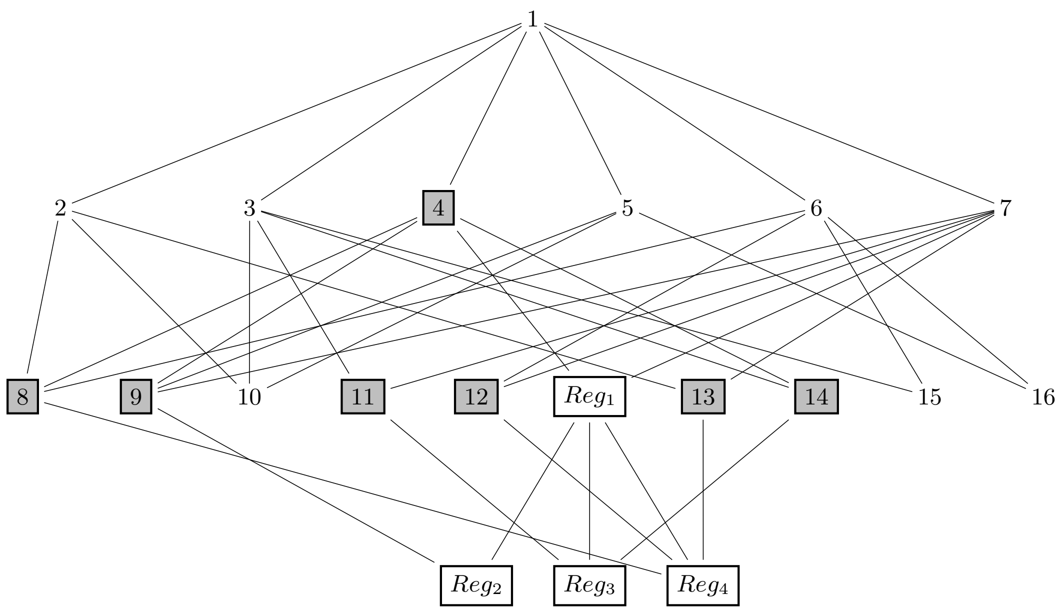


fig. A.130: Degeneration diagram of $b_3 \oplus S_5(5)$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 2 & 2 & 1 & 0 & 0 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(1)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-4}b_1$
1	Reg_2			$S_1(1)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	$\tau^{-4}b_2$
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	Reg_4			$S_1(2)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	a_1

Table 71: Deformations with nonzero regular part of $b_4 \oplus S_1(3)$ in Period 3

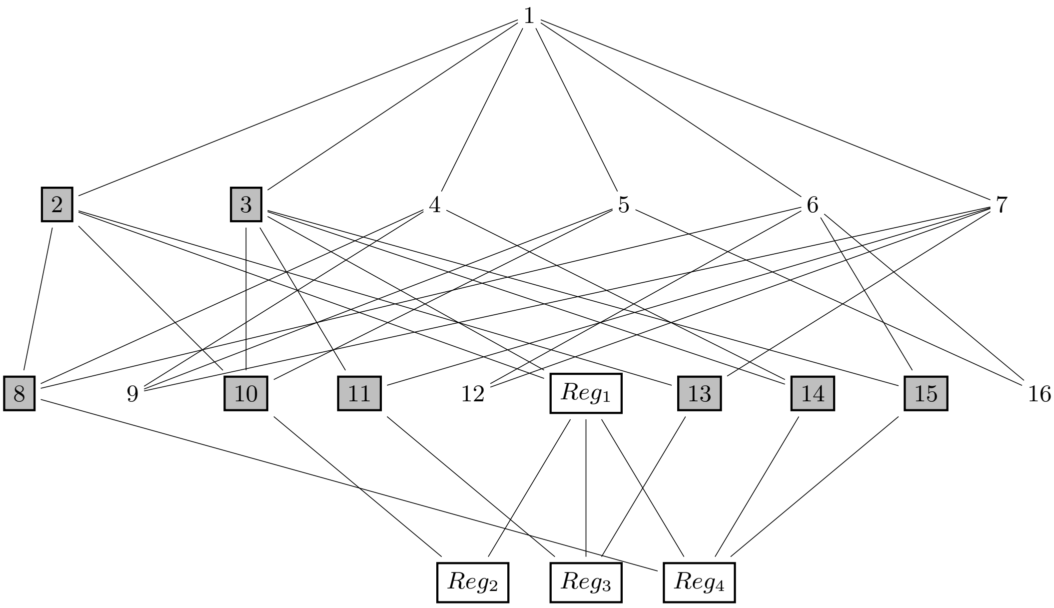


fig. A.131: Degeneration diagram of $b_3 \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	$\tau^{-5}b_2$
1	Reg_2			$S_2(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	a_1
	Reg_3	$S_2(1)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$
	Reg_4			$S_2(2)$	$\oplus$	$\tau^{-8}b_1$	$\oplus$	b_2

Table 72: Deformations with nonzero regular part of $b_4 \oplus S_2(3)$ in Period 3

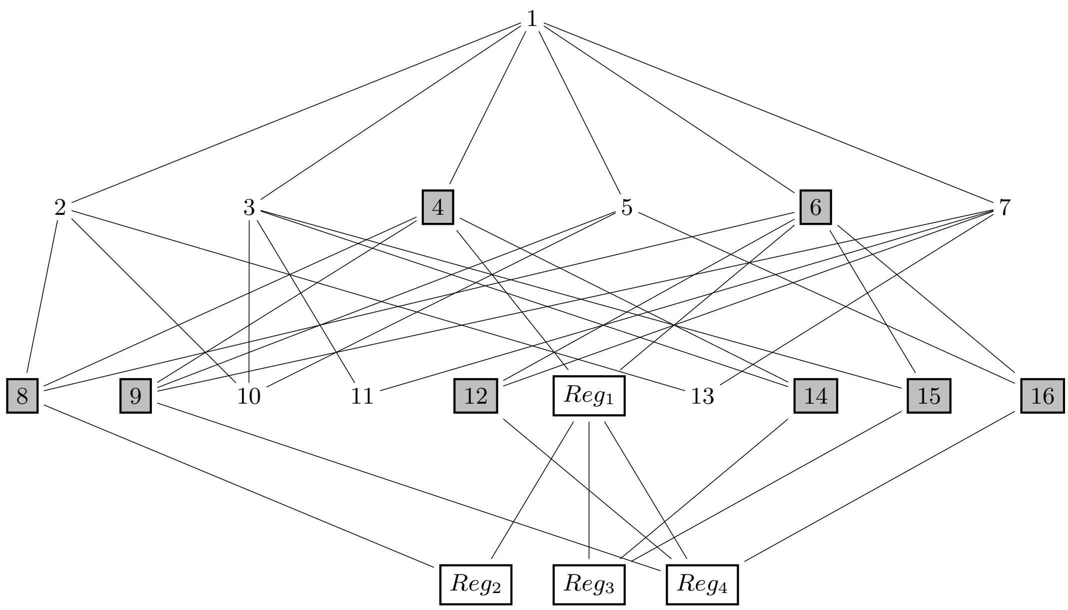


fig. A.132: Degeneration diagram of $b_3 \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_3(1)$	$\oplus$	$\tau^{-9}b_2$	$\oplus$	b_1
1	Reg_2			$S_3(1)$	$\oplus$	$\tau^{-18}b_1$	$\oplus$	b_2
	Reg_3	$S_3(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-8}b_1$
	Reg_4			$S_3(2)$	$\oplus$	$\tau^{-4}b_2$	$\oplus$	b_1

Table 73: Deformations with nonzero regular part of $b_4 \oplus S_3(3)$ in Period 3

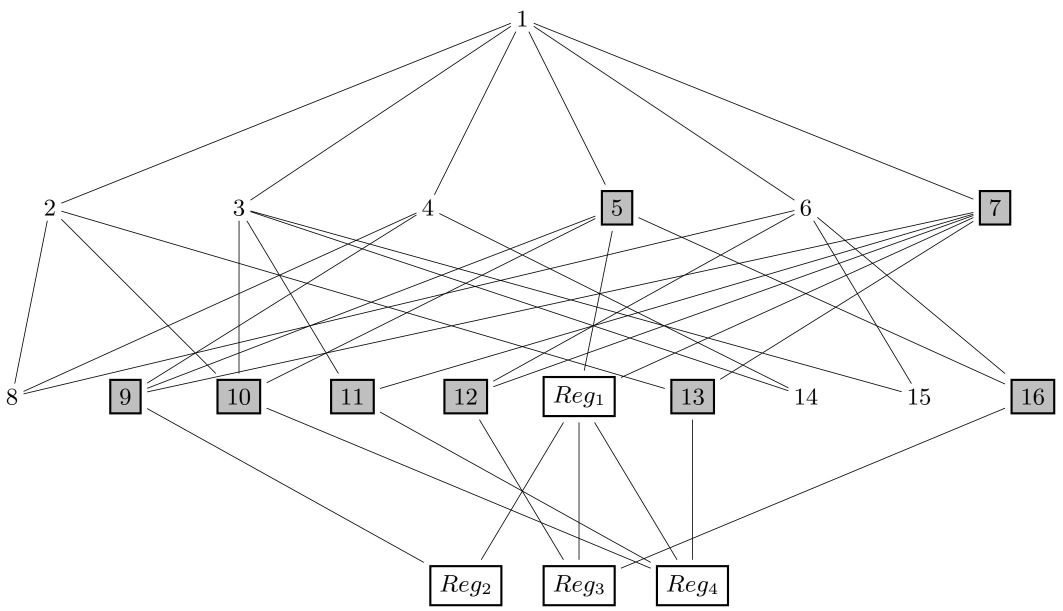


fig. A.133: Degeneration diagram of $b_3 \oplus S_3(3)$ in Period 3

Period 2:	$S_1(1) = [\,{}_{1\ 2\ \overset{2}{3}\ 3\ 2\ 2\ 1\ 1}\,]$	$S_2(1) = [\,{}_{1\ 2\ \overset{1}{3}\ 2\ 2\ 1\ 1\ 0}\,]$

Codimension	Dot	Deformations with nonzero regular part				
2	Reg_1			$S_1(1)$	$\oplus$	$\tau^{-5}b_3$
1	Reg_2	$S_1(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	a_1
	Reg_3	$S_1(1)$	$\oplus$	$\tau^{-13}b_1$	$\oplus$	b_2
	Reg_4	$S_1(1)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	$\tau^{-4}b_2$

Table 74: Deformations with nonzero regular part of $b_3 \oplus S_1(2)$ in Period 2

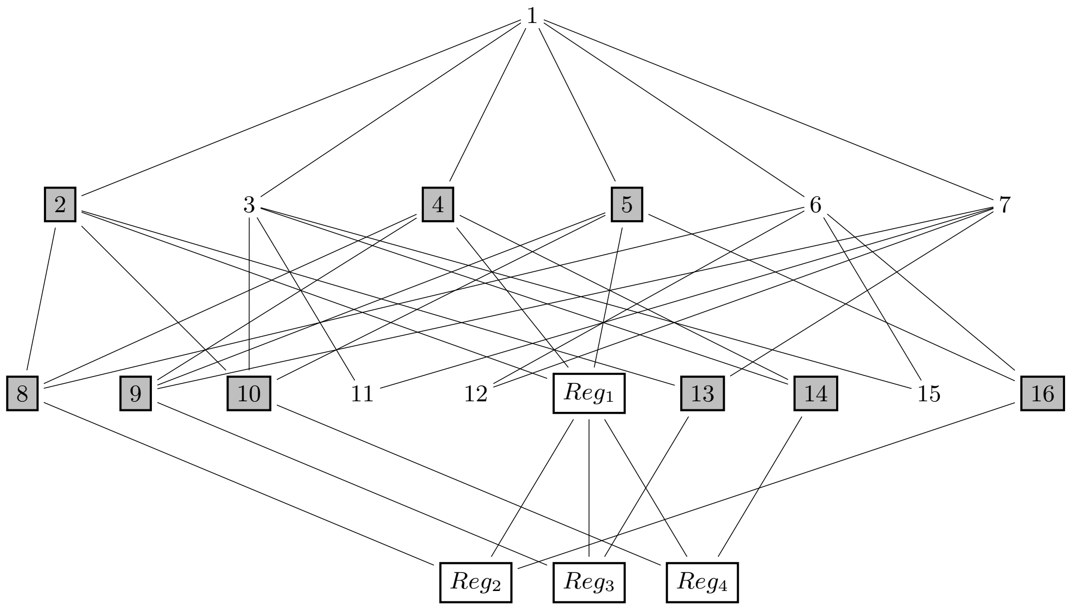


fig. A.134: Degeneration diagram of $b_3 \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part
1	Reg_1	$S_2(1) \quad \oplus \quad \tau^{-8}b_1 \quad \oplus \quad b_1 \quad \oplus \quad \tau^{-4}b_1$

Table 75: Deformations with nonzero regular part of $b_3 \oplus S_2(2)$ in Period 2

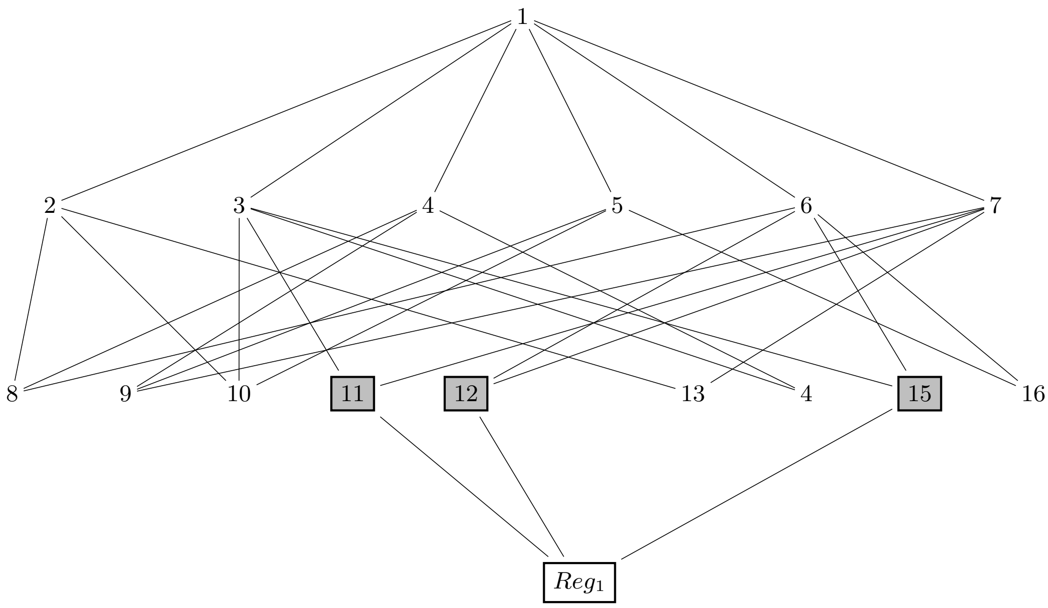


fig. A.135: Degeneration diagram of $b_3 \oplus S_2(2)$ in Period 2

A.11.7 $\mathbf{U = b_2}$

$$\mathbf{Codim}(U \oplus V, \tau^{-15}b_2) = 3$$

Period 5:

$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{smallmatrix}\right]$ $\quad S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$ $\quad S_4(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$ $\quad S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$

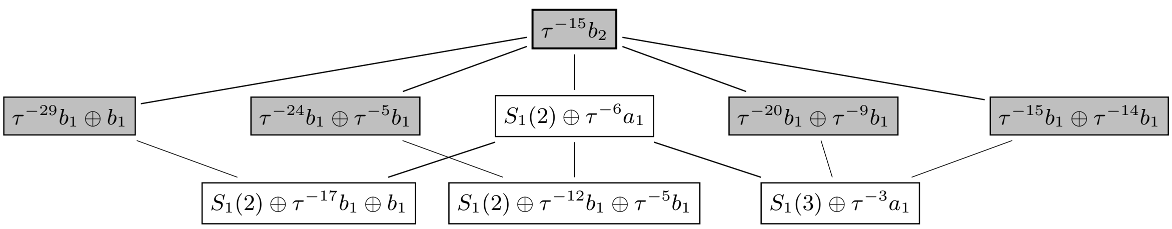


fig. A.136: Degeneration diagram of $b_2 \oplus S_1(5)$ in Period 5

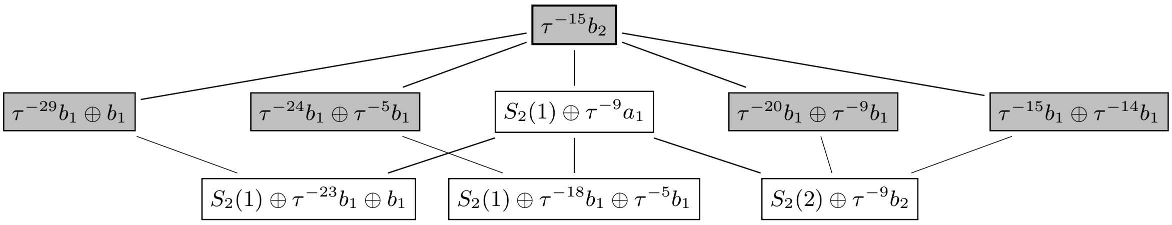


fig. A.137: Degeneration diagram of $b_2 \oplus S_2(5)$ in Period 5

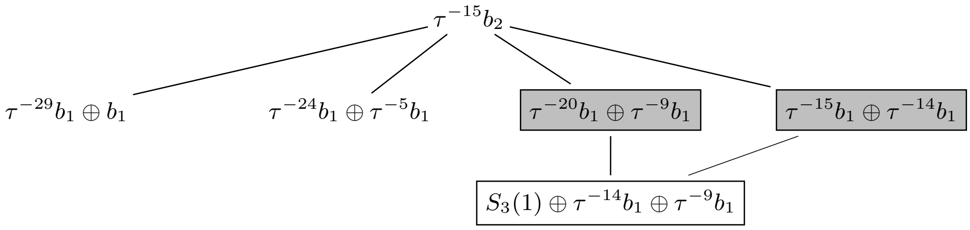


fig. A.138: Degeneration diagram of $b_2 \oplus S_3(5)$ in Period 5

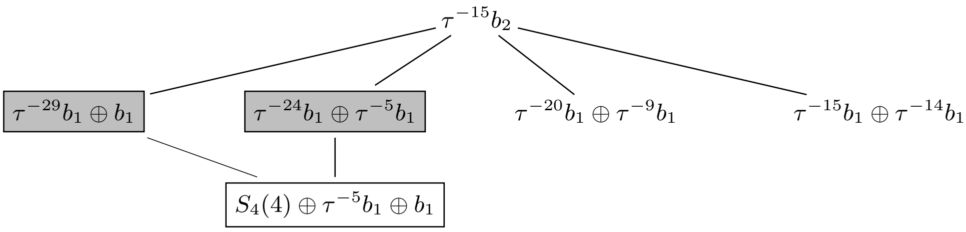


fig. A.139: Degeneration diagram of $b_2 \oplus S_4(5)$ in Period 5

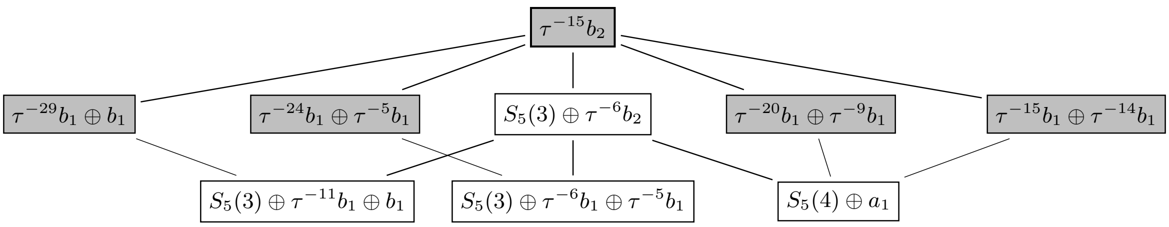


fig. A.140: Degeneration diagram of $b_2 \oplus S_5(5)$ in Period 5

Period 3:	$S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 2 & 2 & 1 & 1 & 0 \end{smallmatrix}\right]$	$S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$	$S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$

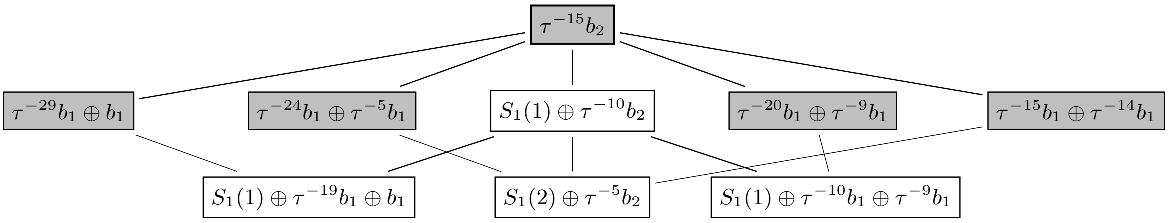


fig. A.141: Degeneration diagram of $b_2 \oplus S_1(3)$ in Period 3

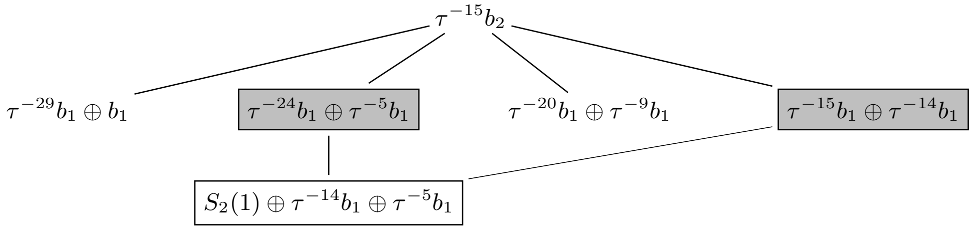


fig. A.142: Degeneration diagram of $b_2 \oplus S_2(3)$ in Period 3

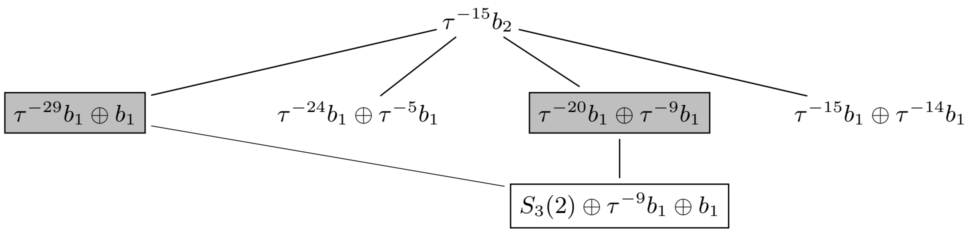


fig. A.143: Degeneration diagram of $b_2 \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = [\,1\;2\;\substack{2\\3}\;3\;2\;2\;1\;0\,]$ $S_2(1) = [\,1\;2\;\substack{1\\3}\;2\;2\;1\;1\;1\,]$

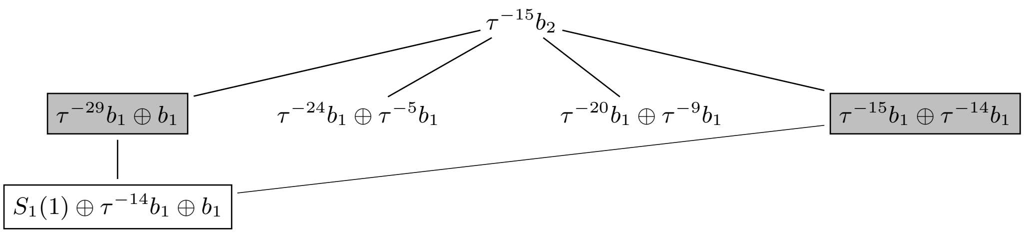


fig. A.144: Degeneration diagram of $b_2 \oplus S_1(2)$ in Period 2

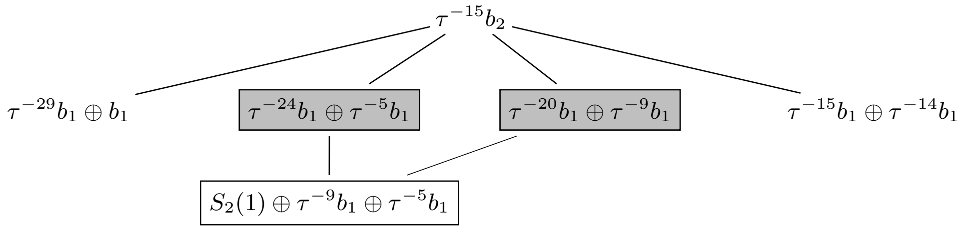


fig. A.145: Degeneration diagram of $b_2 \oplus S_2(2)$ in Period 2

A.11.8 $\mathbf{U = b_1}$

$$\mathbf{Codim}(U \oplus V, \tau^{-30}b_1) = 2$$

Period 5:

$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0&0&1&1&1&0&0&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&1&1&1&1&0&0 \end{smallmatrix}\right]$

$S_3(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0&1&1&1&1&1&1&0 \end{smallmatrix}\right]$ $S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&2&1&1&1&1&1 \end{smallmatrix}\right]$ $S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0&1&1&1&0&0&0&0 \end{smallmatrix}\right]$

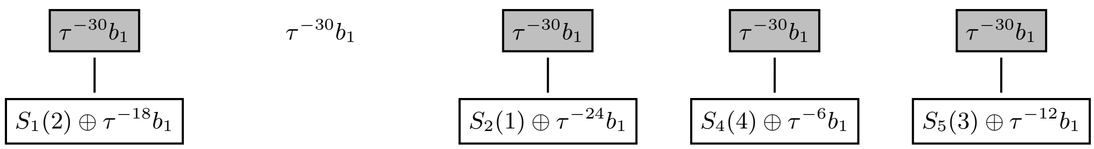


fig. A.146: Degeneration diagrams of $b_1 \oplus S_i(5)$, $i \in \{1, 2, 3, 4, 5\}$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0&1&2&1&1&1&0&0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&1&2&2&1&1&1&0 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&2&2&2&2&1&1&1 \end{smallmatrix}\right]$

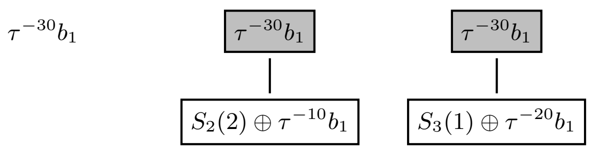


fig. A.147: Degeneration diagrams of $b_1 \oplus S_i(3)$, $i \in \{1, 2, 3\}$ in Period 3

Period 2: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1&2&3&3&2&2&1&1 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 2 & & & & & \\ 1&2&3&2&2&1&1&0 \end{smallmatrix}\right]$

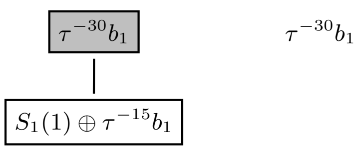


fig. A.148: Degeneration diagrams of $b_1 \oplus S_i(2)$, $i \in \{1, 2\}$ in Period 2

Codimension	Dot	Preprojective deformations				
4	1					$\tau^{-10}c$
3	2			$\tau^{-14}b_1$	$\oplus$	$\tau^{-5}a_1$
	3			$\tau^{-10}b_1$	$\oplus$	$\tau^{-7}a_1$
	4			$\tau^{-10}a_1$	$\oplus$	$\tau^{-4}b_1$
	5			$\tau^{-12}a_1$	$\oplus$	b_1
	6			$\tau^{-12}b_1$	$\oplus$	$\tau^{-5}b_2$
	7			$\tau^{-10}b_2$	$\oplus$	$\tau^{-2}b_1$
2	8			$\tau^{-24}b_1$	$\oplus$	a_1
	9			$\tau^{-20}b_1$	$\oplus$	$\tau^{-2}a_1$
	10			$\tau^{-22}b_1$	$\oplus$	b_2
	11	$\tau^{-19}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	12	$\tau^{-17}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	13	$\tau^{-14}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-7}b_1$
	14	$\tau^{-15}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	15	$\tau^{-10}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-9}b_1$
	16	$\tau^{-12}b_1$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	$\tau^{-5}b_1$

Table 76: Preprojective deformations of $c \oplus V$, with V indecomposable and $\underline{\dim}\, V = \underline{\delta}$

A.11.9 $\mathbf{U = c}$

$$\mathbf{Codim}(U \oplus V, \tau^{-10}c) = 4$$

Period 5:

$$S_1(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{smallmatrix}\right] \quad S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{smallmatrix}\right]$$

$$S_3(1) = \left[\begin{smallmatrix} & & 0 & & & & & \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right] \quad S_4(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{smallmatrix}\right] \quad S_5(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 \end{smallmatrix}\right]$$

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_1(2)$	$\oplus$	$\tau^{-5}b_2$	$\oplus$	b_1
1	Reg_2			$S_1(2)$	$\oplus$	$\tau^{-10}b_1$	$\oplus$	b_2
	Reg_3	$S_1(2)$	$\oplus$	$\tau^{-5}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-4}b_1$
	Reg_4			$S_1(4)$	$\oplus$	a_1	$\oplus$	b_1

Table 77: Deformations with nonzero regular part of $c \oplus S_1(5)$ in Period 5

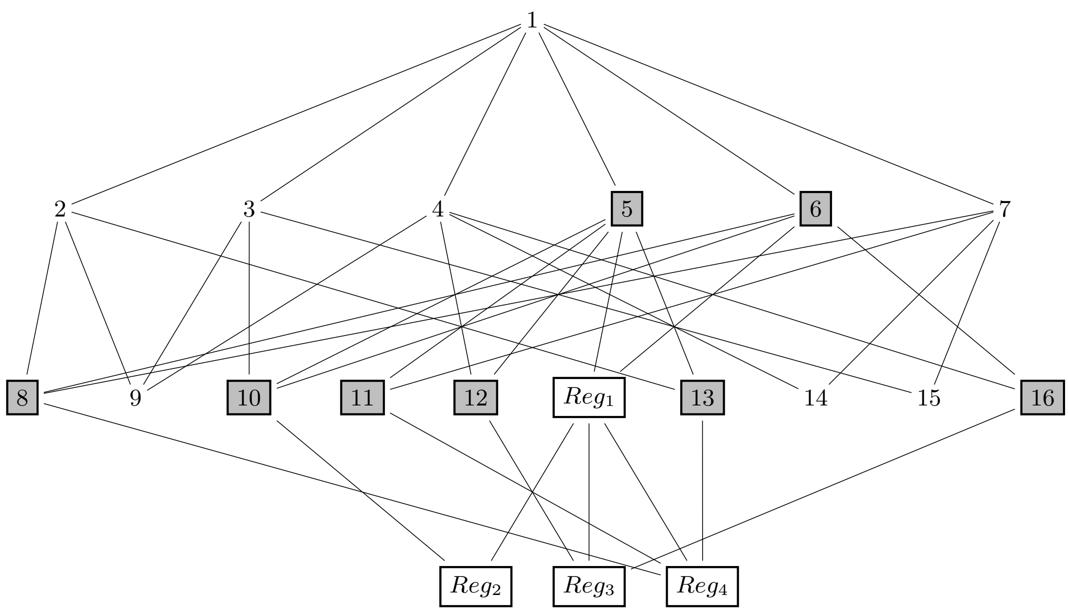


fig. A.149: Degeneration diagram of $c \oplus S_1(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_2(1) \oplus \tau^{-6}b_3$
2	Reg_2	$S_2(1) \oplus \tau^{-10}b_1 \oplus \tau^{-4}a_1$
	Reg_3	$S_2(1) \oplus \tau^{-9}a_1 \oplus b_1$
	Reg_4	$S_2(1) \oplus \tau^{-6}b_1 \oplus \tau^{-5}b_2$
	Reg_5	$S_2(1) \oplus \tau^{-6}b_2 \oplus \tau^{-4}b_1$
	Reg_6	$S_2(3) \oplus \tau^{-2}b_3$
1	Reg_7	$S_2(1) \oplus \tau^{-16}b_1 \oplus b_2$
	Reg_8	$S_2(1) \oplus \tau^{-11}b_1 \oplus b_1 \oplus \tau^{-4}b_1$
	Reg_9	$S_2(1) \oplus \tau^{-6}b_1 \oplus \tau^{-4}b_1 \oplus \tau^{-5}b_1$
	Reg_{10}	$S_2(3) \oplus \tau^{-6}b_1 \oplus a_1$
	Reg_{11}	$S_2(3) \oplus \tau^{-2}b_2 \oplus b_1$
	Reg_{12}	$S_2(4) \oplus \tau^{-2}c$

Table 78: Deformations with nonzero regular part of $c \oplus S_2(5)$ in Period 5

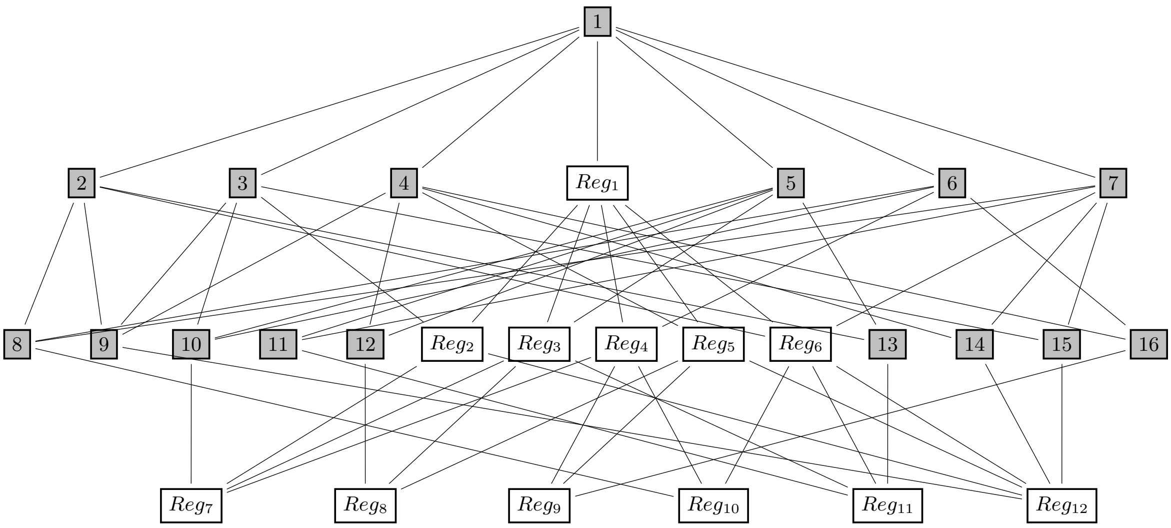


fig. A.150: Degeneration diagram of $c \oplus S_2(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_3(2)$	$\oplus$	$\tau^{-5}a_1$	$\oplus$	$\tau^{-2}b_1$
1	Reg_2			$S_3(2)$	$\oplus$	$\tau^{-12}b_1$	$\oplus$	a_1
	Reg_3	$S_3(2)$	$\oplus$	$\tau^{-7}b_1$	$\oplus$	b_1	$\oplus$	$\tau^{-2}b_1$
	Reg_4			$S_3(3)$	$\oplus$	$\tau^{-2}a_1$	$\oplus$	$\tau^{-2}b_1$

Table 79: Deformations with nonzero regular part of $c \oplus S_3(5)$ in Period 5

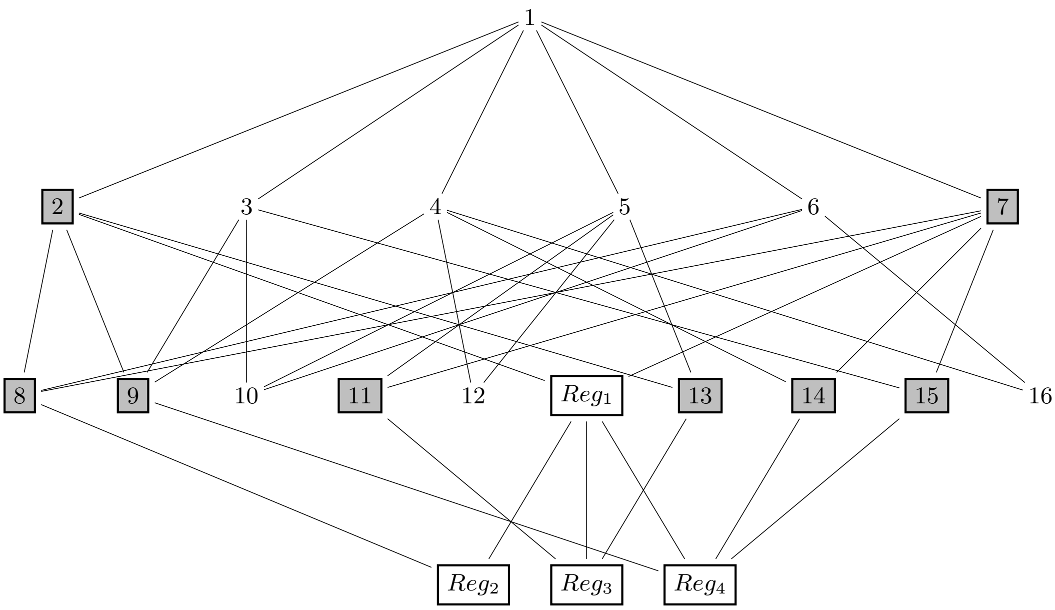


fig. A.151: Degeneration diagram of $c \oplus S_3(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part
3	Reg_1	$S_4(1) \oplus \tau^{-8}c$
2	Reg_2	$S_4(1) \oplus \tau^{-12}b_1 \oplus \tau^{-3}a_1$
	Reg_3	$S_4(1) \oplus \tau^{-8}b_1 \oplus \tau^{-5}a_1$
	Reg_4	$S_4(1) \oplus \tau^{-8}a_1 \oplus \tau^{-2}b_1$
	Reg_5	$S_4(1) \oplus \tau^{-8}b_2 \oplus b_1$
	Reg_6	$S_4(2) \oplus \tau^{-4}b_3$
1	Reg_7	$S_4(1) \oplus \tau^{-18}b_1 \oplus a_1$
	Reg_8	$S_4(1) \oplus \tau^{-13}b_1 \oplus b_1 \oplus \tau^{-2}b_1$
	Reg_9	$S_4(1) \oplus \tau^{-8}b_1 \oplus b_1 \oplus \tau^{-7}b_1$
	Reg_{10}	$S_4(2) \oplus \tau^{-8}b_1 \oplus \tau^{-2}a_1$
	Reg_{11}	$S_4(2) \oplus \tau^{-4}b_2 \oplus \tau^{-2}b_1$
	Reg_{12}	$S_4(4) \oplus b_3$

Table 80: Deformations with nonzero regular part of $c \oplus S_4(5)$ in Period 5

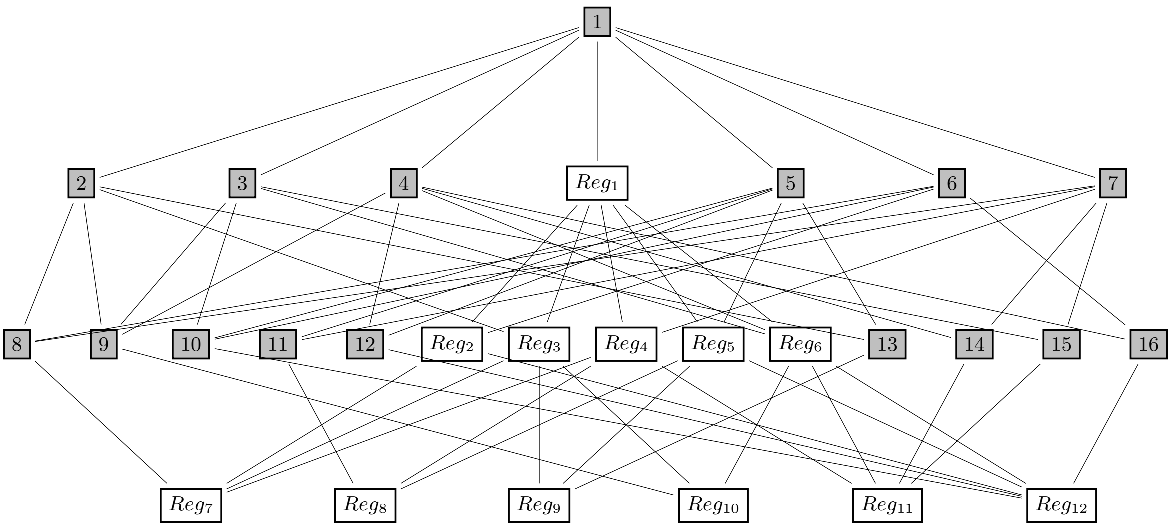


fig. A.152: Degeneration diagram of $c \oplus S_4(5)$ in Period 5

Codimension	Dot	Deformations with nonzero regular part						
2	Reg_1			$S_5(1)$	$\oplus$	$\tau^{-7}a_1$	$\oplus$	$\tau^{-4}b_1$
1	Reg_2			$S_5(1)$	$\oplus$	$\tau^{-14}b_1$	$\oplus$	$\tau^{-2}a_1$
	Reg_3	$S_5(1)$	$\oplus$	$\tau^{-9}b_1$	$\oplus$	$\tau^{-2}b_1$	$\oplus$	$\tau^{-4}b_1$
	Reg_4			$S_5(3)$	$\oplus$	$\tau^{-4}b_1$	$\oplus$	b_2

Table 81: Deformations with nonzero regular part of $c \oplus S_5(5)$ in Period 5

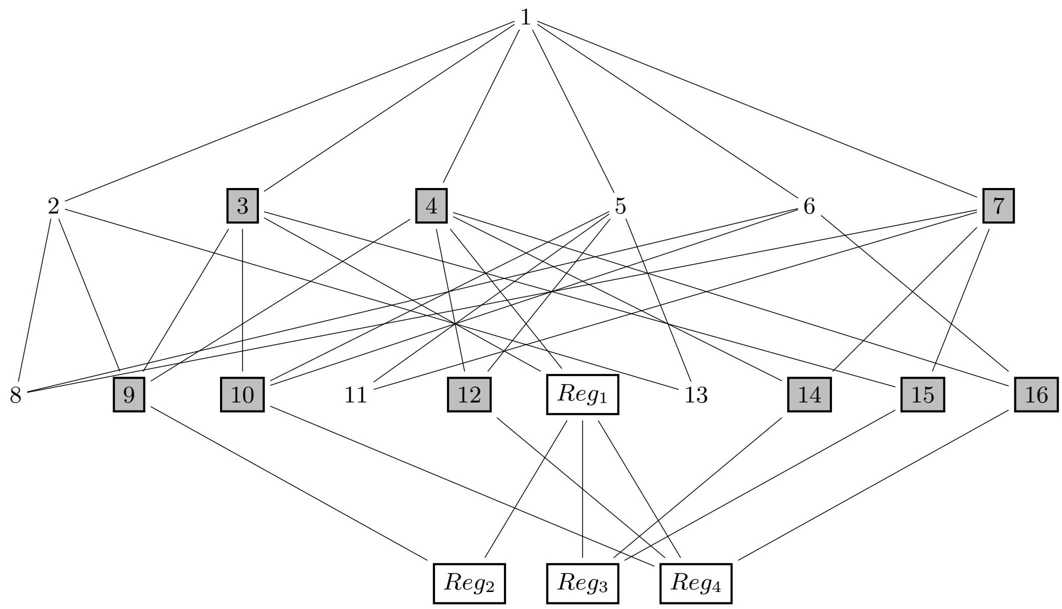


fig. A.153: Degeneration diagram of $c \oplus S_5(5)$ in Period 5

Period 3: $S_1(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 & 0 \end{smallmatrix}\right]$ $S_2(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 0 & 1 & 2 & 2 & 2 & 1 & 1 & 1 \end{smallmatrix}\right]$ $S_3(1) = \left[\begin{smallmatrix} & & 1 & & & & & \\ 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 \end{smallmatrix}\right]$

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_1(1) \oplus \tau^{-5}a_1 \oplus \tau^{-4}b_1$
1	Reg_2	$S_1(1) \oplus \tau^{-10}b_1 \oplus \tau^{-2}a_1$
	Reg_3	$S_1(1) \oplus \tau^{-7}b_1 \oplus b_1 \oplus \tau^{-4}b_1$
	Reg_4	$S_1(2) \oplus \tau^{-4}b_1 \oplus a_1$

Table 82: Deformations with nonzero regular part of $c \oplus S_1(3)$ in Period 3

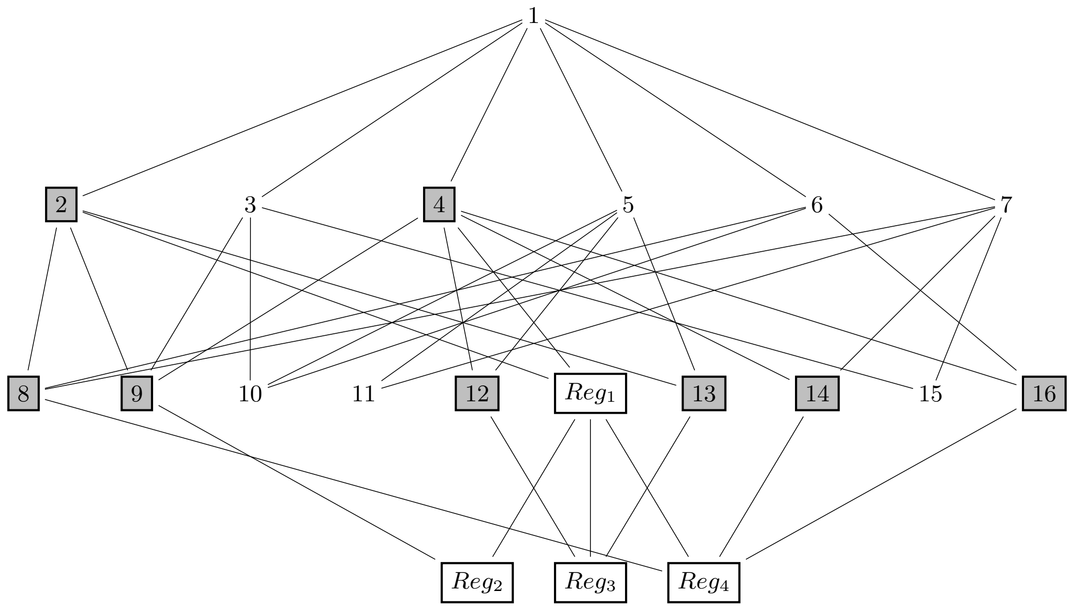


fig. A.154: Degeneration diagram of $c \oplus S_1(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_2(1) \oplus \tau^{-5}b_2 \oplus \tau^{-2}b_1$
1	Reg_2	$S_2(1) \oplus \tau^{-14}b_1 \oplus a_1$
	Reg_3	$S_2(1) \oplus \tau^{-5}b_1 \oplus \tau^{-2}b_1 \oplus \tau^{-4}b_1$
	Reg_4	$S_2(2) \oplus \tau^{-2}b_1 \oplus b_2$

Table 83: Deformations with nonzero regular part of $c \oplus S_2(3)$ in Period 3

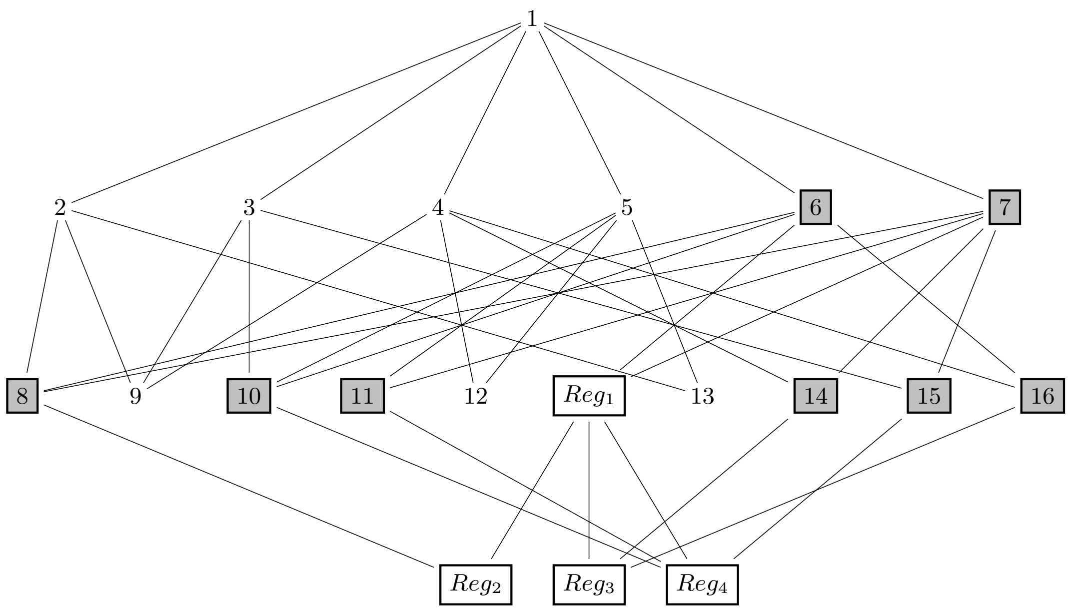


fig. A.155: Degeneration diagram of $c \oplus S_2(3)$ in Period 3

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_3(1) \oplus \tau^{-7}a_1 \oplus b_1$
1	Reg_2	$S_3(1) \oplus \tau^{-12}b_1 \oplus b_2$
	Reg_3	$S_3(1) \oplus \tau^{-9}b_1 \oplus b_1 \oplus \tau^{-2}b_1$
	Reg_4	$S_3(2) \oplus \tau^{-2}a_1 \oplus b_1$

Table 84: Deformations with nonzero regular part of $c \oplus S_3(3)$ in Period 3

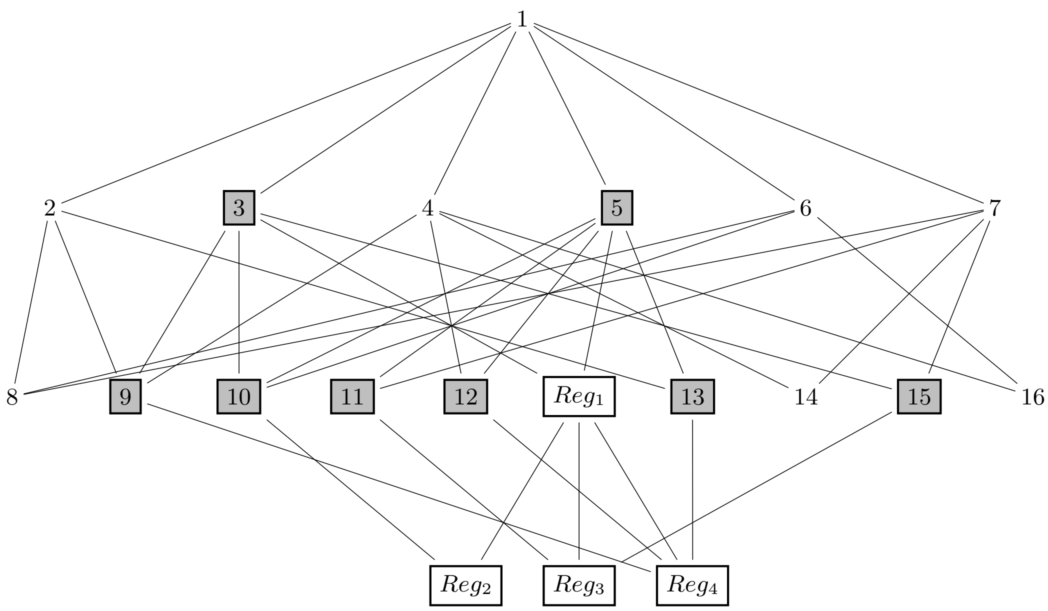


fig. A.156: Degeneration diagram of $c \oplus S_3(3)$ in Period 3

Period 2: $S_1(1) = [\,1\ 2\ \overset{2}{3}\ 3\ 2\ 2\ 1\ 1\,]$ $S_2(1) = [\,1\ 2\ \overset{1}{3}\ 2\ 2\ 1\ 1\ 0\,]$

Codimension	Dot	Deformations with nonzero regular part
2	Reg_1	$S_1(1) \oplus \tau^{-5}c$
1	Reg_2	$S_1(1) \oplus \tau^{-9}b_1 \oplus a_1$
	Reg_3	$S_1(1) \oplus \tau^{-5}b_1 \oplus \tau^{-2}a_1$
	Reg_4	$S_1(1) \oplus \tau^{-7}b_1 \oplus b_2$

Table 85: Deformations with nonzero regular part of $c \oplus S_1(2)$ in Period 2

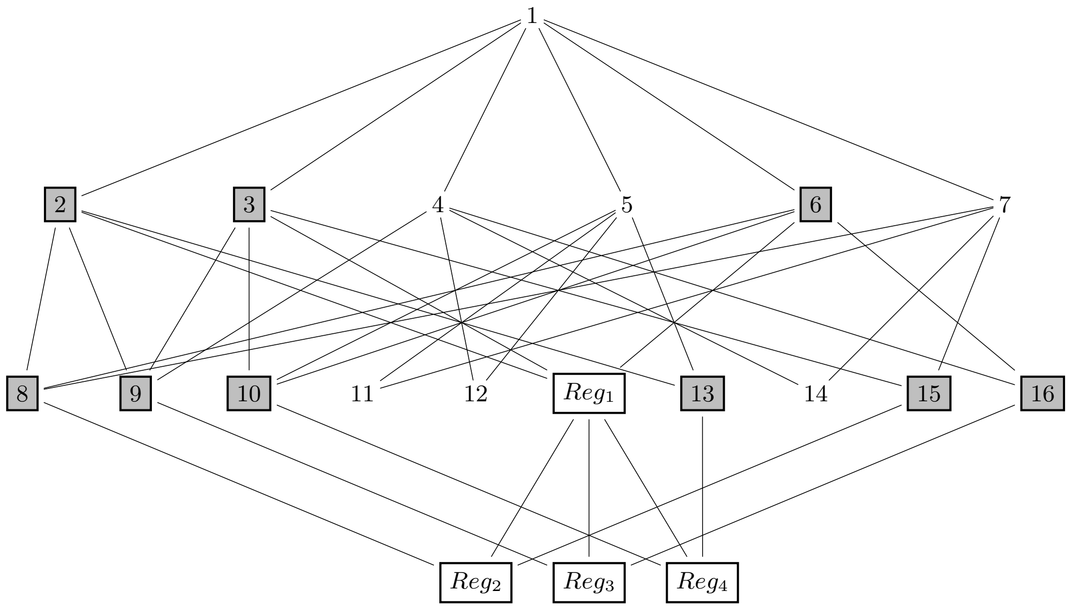


fig. A.157: Degeneration diagram of $c \oplus S_1(2)$ in Period 2

Codimension	Dot	Deformations with nonzero regular part
1	Reg_1	$S_2(1) \oplus \tau^{-4}b_1 \oplus b_1 \oplus \tau^{-2}b_1$

Table 86: Deformations with nonzero regular part of $c \oplus S_2(2)$ in Period 2

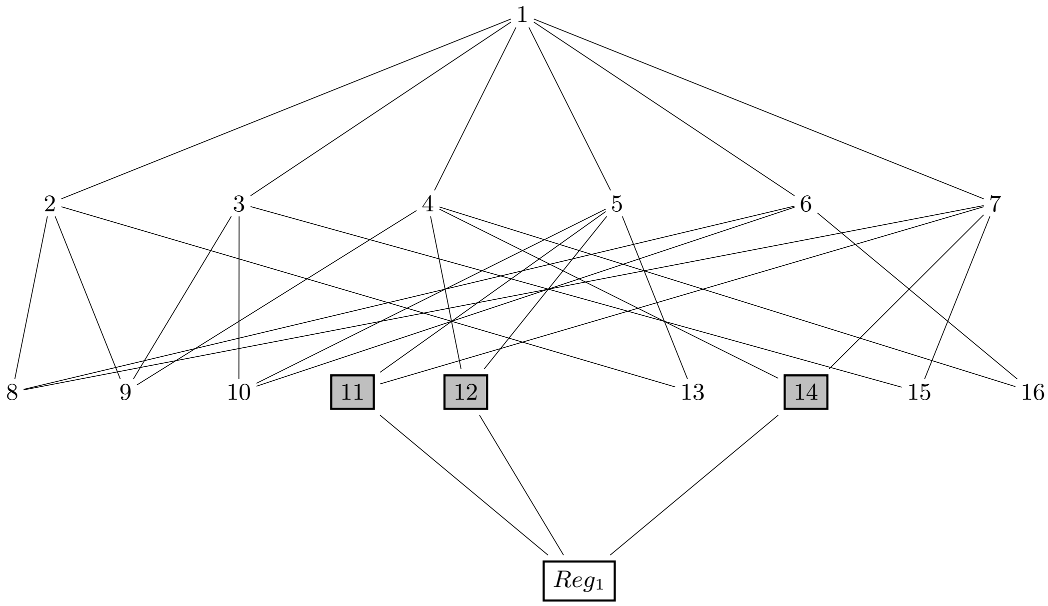


fig. A.158: Degeneration diagram of $c \oplus S_2(2)$ in Period 2

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