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Analysis of non-forward quark-quark correlator

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Thesis

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Abstract

In this work we define forward and off-forward quark-quark correlation functions both in a spin and in a light-cone helicity basis. The properties of quark-quark correlators, which play a crucial role in the investigation of the internal structure of hadrons, are here examined. We derive constraints on forward and off-forward quark-quark correlation functions, implementing the known properties of the fundamental fields of Quantumchromodynamics (QCD), quarks and gluons, under parity and time reversal transformations and applying hermiticity.

We develop a new method to construct ansätze for these correlators both in a spin and in a light-cone helicity basis. These ansätze are based on general principles and are obtained as tensor products of the set of independent Dirac matrices with the independent hadronic spinorial products. These are further saturated by tensors dependent on the available vectors, which occur in the definition of the correlators. The constraints obtained are applied to reduce the number of independent terms forming the ansätze. Furthermore we express ordinary and skewed parton distributions (SPDs) in terms of the independent amplitudes of which the ansätze for forward and off-forward quark-quark correlators consist. Finally we present the complete leading order analysis of ordinary and skewed parton distributions and we conclude that the number of independent skewed parton helicity changing distributions is four.

Contents

1 Introduction

One of the most important aims of high energy physics is to understand the structure of hadrons, in particular of nucleons. Despite the many improvements in the knowledge of the internal hadronic structure achieved in the last thirty years, it is essentially unknown how this structure emerges dynamically. Though Quantum Chromo-dynamics (QCD) offers a re-normalizable quantum field theory of colour interaction between quarks and gluons, we do not know how to handle QCD rigorously in the non-perturbative regime where *confinement* of quarks and gluons inside hadrons takes place.
Since 1969, when the first Deep Inelastic Scattering (DIS) experiments were carried out at SLAC, the standard approach pursued to investigate hadrons has consisted in collecting many experimental data on hard scattering processes. The findings from these experiments were compared to the predictions of various models.
In quantum field theory the structure of complex particles like hadrons is described by hadronic *matrix elements* of all possible quark and gluon operators. Matrix elements of quark and gluon operators are therefore fundamental and universal objects in understanding the internal hadronic structure.
Forward matrix elements of quark and gluon operators, i.e. quark and gluon operators evaluated between hadronic states of equal momenta, are investigated in inclusive processes, for instance in deep inelastic scattering, as they enter in the definition of *parton (quark and gluon) distributions*. Forward hard processes have thus clarified only those aspects of the hadronic dynamics where initial and final hadronic states are the same.
Recently non-forward high-energy processes, namely Compton scattering experiments in the deeply virtual kinematical limit and hard diffractive vector-meson productions, have shown to give access to a new type of nucleon quantities: *the skewed parton distributions* (SPDs). These functions are defined as Dirac projections of off-forward quark and gluon correlators and generalise the ordinary parton distributions. The matrix elements involved in these processes are *non-diagonal* in initial and final state, i.e. quark and gluon operators are evaluated between hadronic states of unequal momenta.
Ordinary and skewed parton distributions are crucial objects in parametrising our "ignorance" about the long-distance physics, since they are related to the experimental observables in hard processes, but at the same time they can be expressed in QCD as matrix elements of non local quark and gluon operators within the hadron. Forward hard processes like DIS have brought much enlightenment in the problem of understanding hadrons and many researchers have great hope in the possibilities of the new non-forward high-energy reactions like Deeply Virtual Compton Scattering (DVCS). Because of the partial similarities of DVCS to DIS the same general methods can be applied, because of the differences new aspects are to be explored.
A main task of this work is to carefully investigate the properties of the forward and off-forward hadronic matrix elements and to carry out a so-called *twist-analysis* of SPDs. Following and generalising the method developed for the conventional forward distribution functions by Mulders' group in Amsterdam [MT96] we investigate the Dirac content of the distribution functions in terms of different Dirac projections of the quark correla-

tion functions. To this aim we first formulate the most general *ansatz* for the quark-quark correlation function, then we analyse its property and finally we trace it with the different Dirac structures.

An important motivation is that a twist-analysis allows to determine the number of independent Skewed Parton Distributions (SPDs) and their pre-factors. According to Hoodbhoy and Ji [HJ98], the number of independent chiral-odd SPDs should be two, whereas Diehl recently argued that the independent leading order parton helicity flip SPDs are four. [Die01]

The outline of the work is the following.

In the Chapter,“Hadronic structure in hard processes”, in order to introduce the reader to definitions which we will need from the very beginning we briefly review the concepts of hadronic matrix elements and we define both forward and off-forward distribution functions, the phenomenological objects which store up the information about non-perturbative long-distance dynamics. All these concepts will be discussed further in subsequent Chapters. Later on in the Chapter “Hadronic structure in hard processes” we introduce the recent developments in Virtual Compton Scattering focusing on the properties of this exclusive hard reaction in the Bjorken regime where Virtual Compton Scattering is analogous to the well-known DIS. At last we mention the so called “spin crisis” and Ji’s proposal to use the second moment of off-forward distribution functions to disantagle the different contributions to the spin of the nucleon.[Ji99]

In the Chapter called “Definition of correlation functions” we define the forward and off-forward quark-quark correlation functions, which are Fourier transforms of forward and off-forward matrix elements of quark operators and are the fundamental objects whose properties are investigated in this thesis.

In the Chapter called “Constraints on quark-quark correlators” we derive constraints on the forward and off-forward quark-quark correlation functions, assuming that the elementary fields of QCD occurring in the definition of quark-quark correlations satisfy general requirement of hermiticity and behave in a definite way under parity and time reversal transformations. These constraints are implemented when building the ansätze for the correlators.

In the Chapter “Choice of spinors and evaluation of spinorial products” we describe spinors which are light-cone helicity eigenstates and we derive the corresponding covariant spin vectors. Later on it will be necessary to deduce many results in the basis of light-cone helicity. At last we show how to evaluate spinorial products.

In the Chapter, named “Forward Quark-Quark Correlators”, we deduce the expressions of the forward quark-quark correlators both in the basis of spin and in the basis of light-cone helicity. Further we develop two methods to construct ansätze for the correlators.

Ansätze for the off-forward quark-quark correlators are obtained in the Chapter “Off-forward Quark-Quark Correlators”.

Forward and off-forward distribution functions are briefly discussed in the Chapter “Definition of distribution functions”.

In “Twist-analysis of quark-quark correlators”, we treat the twist-analysis of leading order forward and off-forward distribution functions.

Finally in the Appendix we briefly review DVCS kinematics and we show how to implement the constraints deriving from hermiticity, parity and time reversal on the ansätze for the forward and off-forward correlators. Lastly we report the Dirac matrices in Weyl representation, used to discuss the chiral properties of the forward quark-quark correlators.
This Ph.D. thesis was written partly during my stay at Universitá di Pavia and partly during my stay at the Bergische Universität in Wuppertal.

2 Hadronic structure in hard processes

2.1 Hadronic matrix elements of quark and gluon operators

From the point of view of quantum field theory all we can do in order to give a description of complex particles like hadrons is to evaluate the hadronic matrix elements of all possible quark and gluon operators. Matrix elements of quark and gluon operators are therefore fundamental objects in understanding the internal hadronic structure and have the nice property that, once they have been measured in a hard process, the result can be plugged into the calculation of observables for other hard processes. This property is called *universality*.

Factorization theorems state that in many hard scattering processes, when the hard scale Q^2 goes to infinity, it is possible to divide the process into two parts, one completely described by perturbative theories and the other related to the non-perturbative nature of hadrons. For finite Q^2 this factorization still holds approximately. It is the non-perturbative soft hadronic part which can be described by hadronic matrix elements of quark and gluon field operators. These are the objects whose properties are investigated in this thesis.

A complete description of the hadron structure in terms of fundamental quanta of QCD requires to consider matrix elements of quark and gluon operators, both diagonal and non diagonal in the hadronic states. In particular we will consider quark operators that contribute at leading order in appropriate hard processes. They are, in fact, relatively simple to handle and are accessible in experimental measurements.

In inclusive DIS (see Figures 2.1 and 2.2) one measures *structure functions* which are observables derived from *parton (quark and gluon) distributions*, which represent not directly observable quantities.

Parton distribution functions (PDFs) are given by projections with Dirac matrix structures of hadron state expectation values, i.e., *diagonal* matrix elements, of *bi-local* combinations of field operators at a light-like distance z, e.g. quarks described by the fields ψ,

$$\Phi^{[\Gamma]}(x) = \frac{1}{2}\int dk^- \, d^2\vec{k}_T \, \mathrm{Tr}(\Gamma \, \Phi_{ij}(k,P,S)|_{k^+=xP^+}, \tag{2.1}$$

where Γ is one of the 4×4 Dirac matrices given in the Fierz decomposition and

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4}\int d^4z \, e^{i\,k\cdot z} \, \langle P,S|\, \overline{\psi}_j(-\frac{z}{2})\, \psi_i(\frac{z}{2})\,|P,S\rangle. \tag{2.2}$$

is the quark-quark correlation function. In the following we will consider only quark-quark correlation functions.

When tracing the quark-quark correlator with γ^+ and $\gamma^+\gamma_5$ matrices and integrating over three components of the quark momentum, for instance, one obtains the unpolarized and polarized parton distributions $f_1(x)$ and $g_1(x)$, respectively.

Another type of observable, related to nucleon structure, are *elastic form factors*, like the Dirac and Pauli form factors $F_1(Q^2)$ and $F_2(Q^2)$ of the nucleon. (see Figures 2.3 and 2.4)

Form factors are defined from matrix elements of *local* combinations of quark field operators

$$\langle P', S'|\,\overline{\psi}_j(0)\,\gamma_\mu\psi_i(0)\,|P,S\rangle = \bar{u}(P',S')\,[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(Q^2)]\,u(P,S). \quad (2.3)$$

In contrast to the above mentioned matrix elements of the inclusive processes, the matrix elements involved in the definition of form factors are *non-diagonal* in initial and final state; initial and final states in the process are characterized by different momenta, P and P', and in general different spin states, characterized by spin vectors S and S'.
These exclusive observables like SPDs provide information about moments of *off-forward* matrix elements of quark and gluon operators.
The skewed parton distributions (SPDs) for the case of quarks are defined as Dirac projections of *non-diagonal* hadronic matrix elements of *bi-local* quark field operators

$$\widetilde{\Phi}^{[\Gamma]}(x,\xi,t) = \frac{1}{2}\int d\bar{k}^-\; d^2\vec{\bar{k}}_\perp\; \mathrm{T}r(\Gamma\;\widetilde{\Phi}_{ij}(k,k',P,P',S,S')|_{\bar{k}^+=x\bar{P}^+}, \quad (2.4)$$

where $\widetilde{\Phi}_{ij}(k,k',P,P',S,S')$ is the off-forward quark-quark correlator

$$\widetilde{\Phi}_{ij}(k,k',P,P',S,S') = \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,\bar{k}\cdot z}\;\langle P',S'|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,S\rangle. \quad (2.5)$$

The trace of $\widetilde{\Phi}_{ij}(k,k',P,P',S,S')$ with γ^+ give a linear combination of the SPDs $H(x,\xi=0,\Delta=0)$ and $\tilde{H}(x,\xi=0,\Delta=0)$. Tracing $\widetilde{\Phi}_{ij}(k,k',P,P',S,S')$ with $\gamma^+\gamma_5$ we have a linear combination of the SPDs $E(x,\xi=0,\Delta=0)$ and $\tilde{E}(x,\xi=0,\Delta=0)$.
We remind that in (2.5) k and k' are the momenta of the initial and final quark, while $\bar{k}$ is defined as the average quark momentum $\bar{k} = \frac{k+k'}{2}$. Since off-forward matrix elements of quark-quark correlators are evaluated between initial and final states carrying different momenta P and P', one introduces the average momentum $\bar{P} = \frac{1}{2}(P+P')$ and the variable $\Delta = P' - P$, whose square Δ^2 coincides with the Mandelstam variable t.
Skewed parton distributions, as the forward ones, are phenomenological functions that characterize certain properties of the nucleon exhibited in a class of high-energy scattering; they reflect the low-energy internal structure of hadrons; the long distance information about nucleons' structure is stored up in non-forward matrix elements of quark and gluon light-cone operators.
The new distributions generalize and interpolate between the ordinary parton distributions and elastic form factors and therefore contain a great deal of information on the nucleon structure.
The following reduction formulae link SPDs to forward distribution functions and to elastic form factors

1. SPDs in the *forward limit* are identical to the conventional PDFs

$$H^q(x,\xi=0,\Delta=0) = f_1^q(x) \quad (2.6)$$
$$\tilde{H}^q(x,\xi=0,\Delta=0) = g_1^q(x) \quad (2.7)$$

2. the *lowest moments* of SPD are the contributions from quarks of flavor q to the nucleon elastic FF

$$\sum_q e_q \int_{-1}^{1} dx\ H^q(x,\xi,\Delta) \quad = \quad F_1(\Delta^2) \tag{2.8}$$

$$\sum_q e_q \int_{-1}^{1} dx\ E^q(x,\xi,\Delta) \quad = \quad F_2(\Delta^2)\,. \tag{2.9}$$

The exclusive process of Deeply Virtual Compton Scattering (DVCS), where one can measure SPDs, thus provides a link between inclusive processes like DIS, where one measures PDFs, and exclusive processes, where one measures form factors.

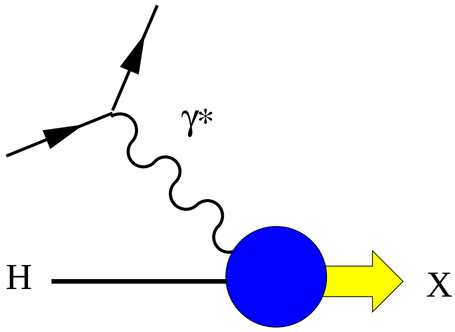

Figure 2.1: *Deep inelastic Scattering. Lepton-hadron scattering via a virtual photon*

There are also important differences between the forward distributions and off-forward ones.
First of all usual parton distributions, besides a logarithmic scale dependence, depend only on the momentum fraction x, while SPDs depend on three kinematical variables, x, ξ and t, which characterize the non-forward nature of the new distributions. In particular, the skewness parameter ξ is defined through the ratio $-2\,\xi = \Delta^+/\bar{P}^+$ between the light-cone plus components of the vectors Δ and $\bar{P}$. (for the definition of light-cone components of a vector refer to the Appendix).
Secondly, how one hadron is built up from the fundamental fields of QCD, quarks and gluons, can be seen through the corresponding hadronic wave functions. If we represent the partonic distributions in terms of hadronic wave functions, usual parton distributions represent classical probabilities to find a parton with a specified momentum fraction x within a hadron. They are obtained considering the wave functions for all configurations

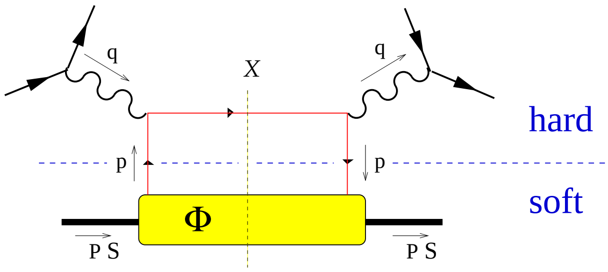

Figure 2.2: *The square amplitude for the leading contribution to DIS can be factorized into hard lepton-parton scattering and a soft part. Inclusive scattering allows summing over all unobservable final states X of the reaction.*

containing this parton with given momentum fraction x, by squaring each wave function and summing over all possible configurations of the spectator partons. In contrast, the SPDs cannot be regarded as particle densities, but rather their physical interpretation is given in terms of a probability amplitude. SPDs are, in fact, the interference of different hadronic wave functions. One parton with momentum fraction x is extracted from the hadron and re-inserted with different momentum fraction x', while the spectator configurations remain the same. What SPDs give access to is the interference between hadronic states which differ only in the amount of hadron momentum fraction the parton carries, that actually takes active part in the hard process. Additionally there is also the possibility that the partonic content between the initial and final state changes , an option which will not be further pursued here. In the forward limit, when initial and final hadronic states are equal and $x = x'$, SPDs become the usual partonic distributions, providing then boundary conditions for SPDs.

Compared to PDFs, SPDs give access to a great deal of information concerning partonic and hadronic degrees of freedom. The different properties of PDFs and SPDs reflect the differences in hard forward and off-forward processes.

The optical theorem (see Fig. 2.5) relates the hadronic part of DIS cross section, $W^{\mu\nu}$, to the imaginary part of forward virtual Compton scattering amplitude $T^{\mu\nu}$

$$2\pi W^{\mu\nu} = \mathrm{I}m T^{\mu\nu}, \tag{2.10}$$

where the forward Compton scattering amplitude is

$$T^{\mu\nu} = i \int d^4z\, e^{i\bar{q}\cdot z} < P'|\, [J^\mu(-\frac{z}{2}) J^\nu(\frac{z}{2})]\, |P > , \tag{2.11}$$

and the hadronic tensor $W^{\mu\nu}$ is defined as

$$W^{\mu\nu} = \frac{i}{4\pi} \int d^4z\, e^{i\bar{q}\cdot z} < P, S \mid T\, [\, J^\mu(-\frac{z}{2}) J^\nu(\frac{z}{2})\,] \mid P, S > , \tag{2.12}$$

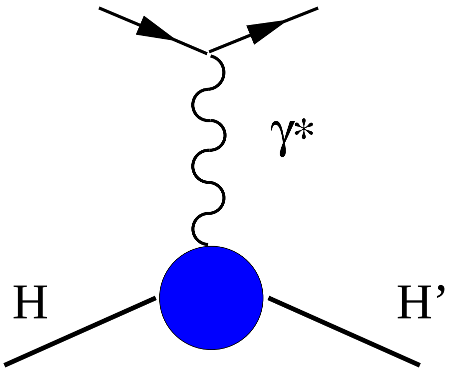

Figure 2.3: *Elastic nucleon form factors*

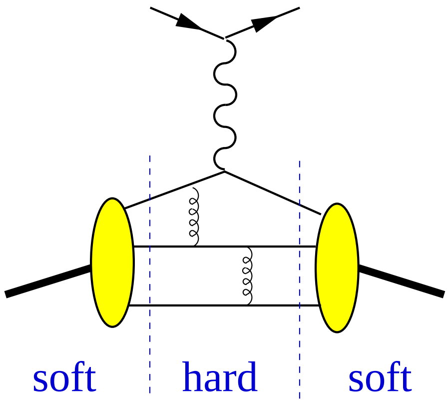

Figure 2.4: *Elastic nucleon form factors. Factorization of the process.*

with $\bar{q} = \frac{1}{2}(q + q')$.
In the case of forward Compton scattering the helicity of the hadron is not flipped and the statement (2.10) leads us to conclude that the flipping of the hadron helicity is likewise forbidden in the case of inclusive DIS.
On the contrary DVCS, that represents in general an off-forward reaction, allows for flipping of the hadron helicity and SPDs provide an exhaustive description of all possible cases, where helicities of the initial and final hadron may be either equal or unequal.
SPDs contain rich information about hadronic and partonic spin degrees of freedom, as

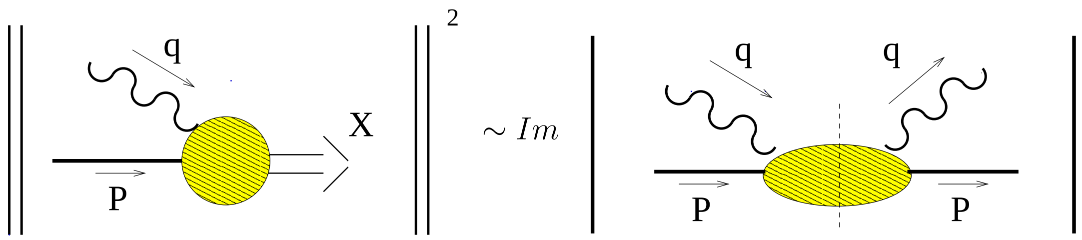

Figure 2.5: *Diagrammatical representation of the optical theorem*

these functions involve after all orbital momentum of the partons and this is the reason why SPDs seem to play such a relevant role to understand how the spin carried by partons contributes to built the total spin of the nucleon.

2.2 Deeply Virtual Compton Scattering

As already mentioned, the process where SPDs most naturally emerge is Compton scattering in deeply virtual limit, analogous to the Bjorken limit of DIS.
Compton process, which is the elastic scattering of a photon off a charged object, is a well-known reaction since it provided one of the first evidences that the electromagnetic wave is quantized and has the nature of particles.
Compton process was used afterwards to investigate the structure of hadrons, although this reaction on a composite particle is in general a complicated process.
When a point-like constituent inside the hadron absorbs the photon, the system becomes excited and propagates in time. Indeed to determine quantum-mechanical propagation of a composite particle is a difficult task.
Nevertheless in special kinematical regions Compton scattering can be relatively easily described.
As Low showed, at sufficiently low energy the intermediate propagation is dominated by the nucleon itself (other resonances do not contribute) and the spin dependent part of Compton amplitude depends on the anomalous magnetic moment of the nucleon and, at higher order terms in the low-energy expansion, on the electric and magnetic polarizabilities [Low54].
Another region, where the process is relatively easy to handle, is where the t-channel momentum transfer is large i.e. where the nucleon has a large recoil, and indeed in this case only the valence Fock states contribute dominantly and one describes the propagation of virtual quarks and gluons instead of the propagation of a composite system. This is the well-known Brodsky-Lepage mechanism, where the soft physics, parameterized by distribution amplitudes, factorizes from the hard physics.

The description of DVCS reaction, in deep inelastic kinematics and at the leading order in perturbative QCD, is also simplified since the process is dominated by the propagation of only one quark as shown by the hand-bag diagram. (see Fig.2.6).
In the Bjorken limit, where energy and momentum of the virtual photon go to infinity at the same rate (s and Q^2 are large but Q^2/s is finite and the scattering angle $\theta_{\gamma^*\gamma}$ is close to zero), the reaction consists in the scattering of a highly virtual photon on a nucleon, close to the forward direction. Inside the nucleon a quark absorbs the virtual photon, becomes highly virtual and propagates perturbatively, then radiates a real photon.
Recently it has been shown that factorization holds for the DVCS amplitude in QCD, up to power suppressed terms, to all orders in perturbation theory [CF99], [Ji98b]. It has also been proven that factorization remains valid independent of the virtuality of the emitted photon, so that it can be also applied to the production of a real photon like in DVCS.
On one side we consider the hard scattering on partons inside the nucleon - *hard physics* - that can be calculated perturbatively. Higher-order (loop) corrections can also be included in the perturbative series, at least in principal, to an arbitrary order in the strong coupling. On the other side we describe the low energy internal structure of nucleon itself - *soft physics* -, parameterizing it in terms of distribution functions. The amplitude of the process depends then on few skewed parton distributions.
Discussing VCS it is also worthwhile remarking the principal difficulties the experimentalists have to cope with in order to make measurements of observables appearing in DVCS.
In the $(e,\, e',\, \gamma)$ reaction on a proton

$$e\,+\,p\longrightarrow e'\,+\,p'\,+\,\gamma, \tag{2.13}$$

the final photon can be emitted either by the proton, giving access to the VCS process, or by the electrons, in the so called Bethe-Heitler process.
The BH amplitude can be calculated exactly in QED, if one knows the elastic form factors of the proton. Light particles such as electrons radiate much more than the heavy proton. For this reason BH process usually dominates or interferes strongly with the VCS process. One way to overcome this difficulty is to find kinematical regions where the BH is suppressed. Otherwise one could take advantage of the interference between BH and VCS, as BH amplitude can be exactly calculated in QED.
Moreover the VCS cross-section is suppressed by a factor $\alpha\,\sim\,1/137$ with respect to the elastic FF case, as one needs to detect the out-coming photon. Another experimental difficulty is related to the fact that a pion may be emitted in the process and this in turn decays into two photons, that may jeopardize the VCS result.

2.3 Light-cone dominance of VCS in the Bjorken limit

One introduces SPDs to parameterize the soft physics part in the Compton scattering amplitude.
The leading twist contribution to the DVCS amplitude in the forward direction is given by the handbag diagram shown in 2.6.[Ji97b]

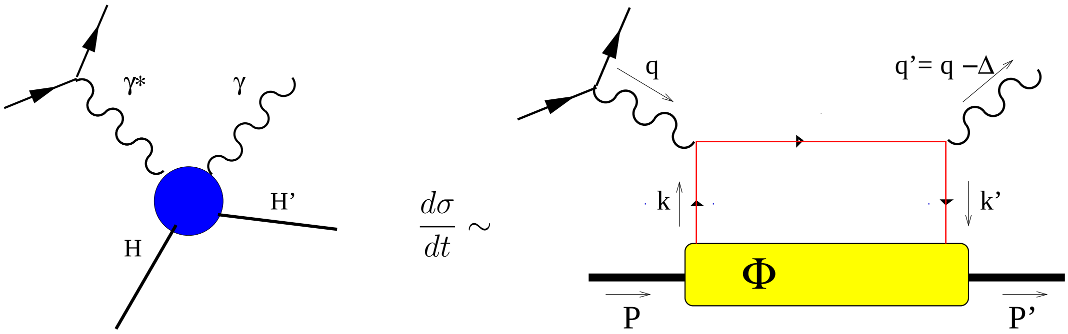

Figure 2.6: *Deeply virtual Compton scattering amplitude. The external process resembles the one of the elastic nucleon form factors, its parton level description is a generalization of the one for DIS.*

In the hand-bag diagram a nucleon of momentum P^{μ} absorbs a virtual photon of momentum q^{μ}, producing an outgoing real photon of momentum $q'^{\mu} = q^{\mu} - \Delta^{\mu}$ and a recoil nucleon of momentum $P'^{\mu} = q^{\mu} + \Delta^{\mu}$.
To calculate the corresponding amplitude, it is convenient to use a frame where the virtual photon momentum q^{μ} and the average nucleon momentum $\bar{P}^{\mu}$ are collinear and along the z-axis.
In the Bjorken limit in the rest frame, the q^{-} component of the virtual photon momentum is of the order Q^2, whereas the component q^{+} is of the order 1. Therefore the operator product in DVCS tensor

$$T^{\mu\nu} = i \int d^4 z e^{i\bar{q}\cdot z} < P'|[\, J^{\mu}(-\frac{z}{2}) J^{\nu}(\frac{z}{2})\,]|P > \tag{2.14}$$

is dominated, as in DIS, by free quark currents separated by a light-like distance.
This is due on the one hand to the dominance of the 4-fold integral in (2.11) by the region $y^{+} \sim 1/|q^{-}|$ and on the other hand to causality which forces $y^2 > 0$ [Ell77].
In the Bjorken regime Eq.(2.14) reduces to a one dimensional integral along a light-like line. DVCS in the Bjorken regime will therefore select the leading twist part of the matrix element of the bi-local quark operator represented by the lower blob in Fig. (2.6).

$$T^{\mu\nu} = i \int \frac{d^4 z}{(2\pi)^4} \left\{ Tr[\frac{i\gamma^{\mu}\gamma^{\nu}}{\not{k} - \frac{1}{2}\not{\Delta} + \not{q} + i\epsilon} + \frac{i\gamma^{\nu}\gamma^{\mu}}{\not{k} + \frac{1}{2}\not{\Delta} + \not{q} + i\epsilon}] M(k) \right\} \tag{2.15}$$

where

$$M(k) = \int d^4 z e^{i\bar{k}z} < P'|\bar{\psi}(-\frac{z}{2})\psi(\frac{z}{2})|P > \tag{2.16}$$

where ψ is the quark field.

$$
\begin{aligned}
&T^{\mu\nu}(\bar{P}, q, \Delta) = \\
\tfrac{1}{2} \quad &(g^{\mu\nu} - v^{\nu}v'^{\mu} - v^{\mu}v'^{\nu}) \int_{-1}^{+1} dx \left(\frac{1}{x - \xi/2 + i\epsilon} + \frac{1}{x + \xi/2 - i\epsilon}\right) \times \\
&[H^q(x,\xi,t)\bar{u}(P')\gamma^+ u(P) + E^q(x,\xi,t)\bar{u}(P') i\sigma^{+\alpha}\frac{\Delta_\alpha}{2m} u(P)] + \\
&\frac{i}{2}\epsilon^{\mu\nu\alpha\beta} v_\alpha v'_\beta \int_{-1}^{+1} dx \left(\frac{1}{x - \xi/2 + i\epsilon} + \frac{1}{x + \xi/2 - i\epsilon}\right) \times \\
&[\tilde{H}^q(x,\xi,t)\bar{u}(P')\gamma^+\gamma_5 u(P) + \tilde{E}^q(x,\xi,t)\bar{u}(P')\gamma_5 \frac{\Delta^+}{2m} u(P)] \qquad (2.17)
\end{aligned}
$$

where one introduces the two light-like vectors $v^\mu = (1, 0, 0, 1)$ and $v'^\mu = (1, 0, 0, -1)$.
The leading twist matrix element of (2.17) is parameterized in terms of four SPD $H^q(x, \xi, t)$, $E^q(x, \xi, t)$, $\tilde{H}^q(x, \xi, t)$ and $\tilde{E}^q(x, \xi, t)$, defined for each flavor ($q = u, d, s, \cdots$) that depend upon the variables x, ξ and $t = \Delta^2$.
The SPDs $H^q(x, \xi, t)$ and $\tilde{H}^q(x, \xi, t)$ conserve the helicity of the hadron, while $E^q(x, \xi, t)$ and $\tilde{E}^q(x, \xi, t)$ allow for helicity flips.
The light-cone momentum fraction, defined as $x = \bar{k}^+/\bar{P}^+$ takes values in the interval $[-1, 1]$. A negative momentum fraction corresponds to an anti-quark. ξ is the longitudinal momentum transfer and is bounded by

$$
0 < \xi < \frac{\sqrt{-\Delta^2}/2}{\bar{m}} < 1 \, , \qquad (2.18)
$$

because the momentum fractions of the nucleons cannot be negative. In (2.18) $\bar{m}$ is defined by the following relation $\bar{m}^2 = m^2 - \frac{\Delta^2}{4}$.
The active quark with momentum $k - \Delta/2$ has longitudinal plus-component momentum fraction $x + \xi$, whereas the one with momentum $k + \Delta/2$ has longitudinal momentum fraction $x - \xi$. Negative momentum fractions correspond to anti-quarks and for this reason we can identify two different regions according to whether $|x| > \xi$ or $|x| < \xi$. If $x > \xi$ both propagators in Fig.2.6 are quarks and when $x < -\xi$ they both represent anti-quarks. In these regions SPD are generalizations of the usual parton distributions. In the region $-\xi < x < \xi$ one quark propagator represents a quark and the other one an anti-quark. In this region the SPD behave like a meson distribution amplitude. [GV98]

2.4 Form factors of the QCD energy-momentum tensor and nucleon spin structure

We have already mentioned how important is to measure observables related to matrix elements of hadronic quark and gluon operators, since this seems to be the only way to

understand the internal structure of hadrons. But SPDs turn out to be very useful also in order to clarify the long-standing problem of the nucleon spin. Moments of the DVCS skewed parton distributions are in fact related to form factors of the energy-momentum tensor, from which one can extract the fractions of nuclear spin carried by quarks and gluons. Since there is no fundamental probe (except the graviton) which couples directly to the energy-momentum tensor, it appeared hopeless to measure this form factor.
The problem of understanding the spin of the nucleon arose when in 1987 the European Muon Collaboration (EMC) measured with an unprecedented precision the proton's spin dependent structure function $G_1(Q^2)$ in polarized deep-inelastic scattering.
Combining their data with the hyperon beta decay rates, augmented with the assumption of the flavor SU(3) symmetry, EMC extracted the fraction of the spin nucleon carried in the spin of quarks [Ji98b]

$$\Delta\Sigma(Q^2 = 10GeV^2) = 0.12 \pm 0.17. \tag{2.19}$$

This result, in flat contradiction with the quark model prediction

$$\Delta\Sigma = 1 \ , \tag{2.20}$$

gave rise to the so called *spin crisis*.

To better understand the problem one considers the angular momentum operator in QCD, that, following Ji's proposal [Ji98b], can be written as the sum of the quark and gluon contributions, respectively J_q and J_g

$$\vec{J}_{QCD} = \vec{J}_q + \vec{J}_g \tag{2.21}$$

where

$$\begin{aligned} \vec{J}_q &= \int d^3r \ \vec{r} \times \vec{T}_q \\ &= \int d^3r \ [\frac{1}{2}\psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{r} \times (-i\vec{D})\psi] \end{aligned} \tag{2.22}$$

and

$$\vec{J}_g = \int d^3r \ \vec{r} \times (\vec{E} \times \vec{B}) \tag{2.23}$$

The quark and gluon parts of the angular momentum are generated from the quark and gluon momentum densities $\vec{T}_q$ and $\vec{E} \times \vec{B}$, respectively ($\vec{E}$ and $\vec{B}$ are the fields). $\vec{\Sigma} = \gamma^5\gamma^0\vec{\gamma}$ represents the Dirac spin matrix and the corresponding term is clearly the quark spin contribution. $\vec{D} = \vec{\partial} + ig\vec{A}$ is the covariant derivative and the associated term can be interpreted as the gauge invariant quark orbital angular momentum contribution.
Consider now a nucleon moving in the z direction and polarized in an helicity eigenstate $\lambda = 1/2$. The expectation value of J_z in the nucleon state is

$$\frac{1}{2} = J_g(\mu) + L_q(\mu) + \frac{1}{2}\Delta\Sigma(\mu) \tag{2.24}$$

where the three contributions, depending on the scale μ, denote the matrix elements of three parts of the angular momentum operator in (2.22) and (2.23).
On the other hand by examining the definition of $J_{q,g}$

$$J_{q,g}(\mu) =< P\frac{1}{2}| \int d^3r(\vec{r}\times\vec{t}_{q,g})|P\frac{1}{2} > \tag{2.25}$$

one realizes that the fractions of nucleon spin carried by quarks and gluons can be extracted from the form factors of the quark and gluon parts of the QCD energy-momentum tensor $t^{\mu\nu}_{q,g}$.
Using Lorentz covariance and invariance under the discrete symmetries, one can expand the matrix elements of $t^{\mu\nu}$ in terms of four form factors

$$\begin{aligned} < P'|t^{\mu\nu}_{q,g}|P > &= \bar{u}(P')[A_{q,g}(t)\gamma^{(\mu)}\bar{P}^{(\nu)} + B_{q,g}(t)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2m \\ &+ C_{q,g}(t)(\Delta^\mu\Delta^\nu - g^{\mu\nu}t)/m + \bar{C}_{q,g}(t)g^{\mu\nu}M]u(P) \end{aligned}$$

Taking the forward limit in the $\mu = 0$ component and integrating over 3-space one finds that $A_{q,g}(0)$ give the momentum fractions of the nucleon carried by quarks and gluons. Substituting the expression of $t^{\mu\nu}_{q,g}$ into the nucleon matrix element of (2.25), one finds

$$J_{q,g} = \frac{1}{2}(A_{q,g}(0) + B_{q,g}(0)) \tag{2.26}$$

The matrix elements of the energy-momentum tensor provide the fractions of the nucleon spin carried by quarks and gluons.
It is quite difficult to measure the form factors of the energy-momentum tensor, being the graviton the only particle that could couple to it. Nevertheless, since the quark and gluon energy-momentum tensors are just the twist two operators occurring in DVCS, one has the following sum rules [Ji98b]

$$\begin{aligned} \int_{-1}^{1} dx\; xH(x,\xi,t) &= A(t) + \xi^2 C(t) \\ \int_{-1}^{1} dx\; xE(x,\xi,t) &= B(t) - \xi^2 C(t) \end{aligned}$$

and combining the two preceding relations

$$\int_{-1}^{1} dx\; x(H(x,\xi,t) + E(x,\xi,t)) = A(t) + B(t). \tag{2.27}$$

Provided one is able to extrapolate the sum rule to $t = 0$, the separate total quark and total gluon contribution to the nucleon spin is obtained.

3 Definition of correlation functions

Quark-quark (gluon-gluon) correlation functions are the main objects of interest, which encode complete information on the hadronic structure in terms of quark (gluon) degrees of freedom.
From dimensional reasoning, the leading order contribution to a given hard process must involve the minimum number of independent parton fields, which for QCD quantized on the light-cone is two. Therefore to leading order, one needs to consider only the matrix elements of bilinear operators at two different points on the light-cone.
In the following we will often refer to the operators which contribute at leading order in hard processes as *twist-two* operators.
This *effective definition* of twist, due to Jaffe [Jaf96b], denotes the leading order in $1/Q^2$ (modulo logarithms) at which a particular effect is seen in a particular experiment. In general, if one object behaves like $(1/Q^2)^p$, then it is said to have twist $t = 2 + 2p$. We remark that this working definition of twist is not exactly the one used in the context of operator product expansion, where twist refers to the difference between canonical dimension and spin of local operators.
Quark-quark correlators are defined as Fourier transforms of the simplest connected diagonal and off-diagonal matrix elements of non-local operators, constructed from two quark fields. Quark-quark correlators are thus 4×4 matrices in the quark chirality space and 2×2 matrices in the Dirac hadronic space, since we take only nucleons into consideration. Anti-nucleons can be described separately.
Within usual factorization schemes quark-quark correlators parameterize the contribution from soft physics, that is a priori unknown. They describe how the fundamental quanta of QCD can arrange themselves to build up the hadron.

3.1 Forward quark-quark correlation function

We consider the most general form of the forward quark-quark correlation function in light-cone gauge $A^+ = 0$, i.e. the bi-local product of two quark fields

$$\Phi_{ij}(k, P, S) = \frac{1}{(2\pi)^4} \int d^4z \; e^{i\,k\cdot z} \; \langle P, S|\, \overline{\psi}_j(-\frac{z}{2})\, \psi_i(\frac{z}{2})\, |P, S\rangle, \tag{3.28}$$

where a summation over color indices is implicit. The incoming and outgoing hadron have equal four-momenta P and spin vector S, while the momentum of the quark is denoted by k.
The product of quark fields ψ that appear in (3.28), is not gauge-invariant unless one replaces it by

$$\overline{\psi}_j(\frac{z}{2})\, G(-\frac{z}{2} \longrightarrow \frac{z}{2})\, \psi_i(\frac{z}{2}) \tag{3.29}$$

where the *link operator* is defined as

$$G(-\frac{z}{2} \longrightarrow \frac{z}{2}) = P\exp(\, -ig \int_C dy^\mu A_\mu(y)\,) \tag{3.30}$$

and P is the path ordering.
In (3.30) the product in the path integral is evaluated along a curve C that links the two points $-\frac{z}{2}$ and $\frac{z}{2}$ and A_μ is the color gauge field.
Because of light-cone dominance of the hard processes we are interested in, we will always refer to the definition of the quark-quark correlator $\Phi_{ij}(k,P,S)$ on the light-cone.
On the light-cone it is possible to choose a convenient path in hyperplane $y^+ = 0$,$y_T = 0$ and a gauge where in each point $A^+ = 0$, such that the link operator reduces to identity.
We consider the trace of the light-cone correlation function with the Dirac matrix γ^+, $f_1(x)$, defined as

$$\begin{aligned} f_1(x) &= \int d^2\,\vec{k}_\perp\, d\,k^+\ \mathrm{T}r(\Phi\gamma^+) \\ &= \int \frac{dz^-}{2\pi}\, e^{ik^+z^-}\, \langle P,S|\, \psi^\dagger(-\frac{z}{2})\gamma^0\,\gamma^+\,\psi(\frac{z}{2})\,|P,S\rangle|_{z^+=z_T=0}\ . \end{aligned} \tag{3.31}$$

Introducing good components of the quark fields in the light-cone quantization, $\psi_+ \equiv P_+\psi = \frac{1}{2}\gamma^-\gamma^+\psi$, (3.31) becomes

$$f_1(x) = \int \frac{dz^-}{2\pi\sqrt{2}}\, e^{ik^+z^-}\, \langle P,S|\, {\psi_+}^\dagger(-\frac{z}{2})\,\psi_+(\frac{z}{2})\,|P,S\rangle|_{z^+=z_T=0} \tag{3.32}$$

In (3.32) one can insert a complete set of intermediate states X and integrate over $\vec{k}_\perp$ and k^+

$$f_1(x) = \int \frac{dz^-}{2\pi\sqrt{2}} \sum_X < X\ |\ \psi_+(-\frac{z^-}{2})\ |\ P,S >< P,S\ |\ {\psi_+}^\dagger(\frac{z^-}{2})\ |\ X > \tag{3.33}$$

obtaining finally with a translation of light-cone coordinates

$$f_1(x) = \frac{1}{\sqrt{2}} \sum_X \delta(\,P^+ + q^+ - P_X^+\,)|< X\ |\ \psi_+(0)\ |\ P,S >|^2 \tag{3.34}$$

that shows that DIS correlation functions have a natural interpretation as light-cone probability densities. The quantity in (3.34) represents the probability that a quark is annihilated from $|\ P >$ giving a state $|\ X >$ with momentum $P_X^+ - (1-x)\,P^+$.
The quark-quark correlation function for DIS is diagrammatically represented in Fig. 3.7.

Measurements of polarization observables, although quite complicated from the experimental point of view, are a unique source of information on the nuclear structure and, in this sense, the forward quark-quark correlation functions defined contain complete information about polarization of the beams of incoming and outgoing particles.

3.2 Off-forward quark-quark correlation function

A complete parameterization of hadrons' internal structure at leading order needs also off-diagonal matrix elements of quark-quark operators, which occur in the description of

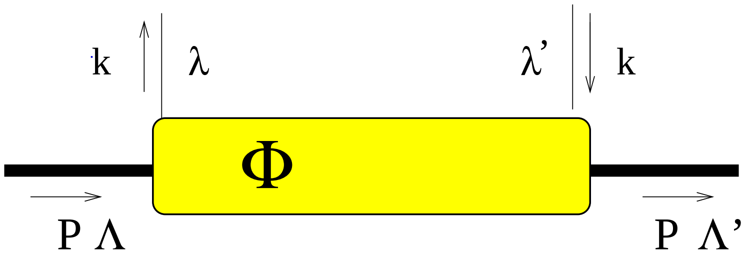

Figure 3.7: *Diagrammatic representation of the correlation function* Φ. Λ *and* Λ' *denote initial and final helicity of the hadron and* λ *and* λ' *the helicities of the parton extracted and replaced.*

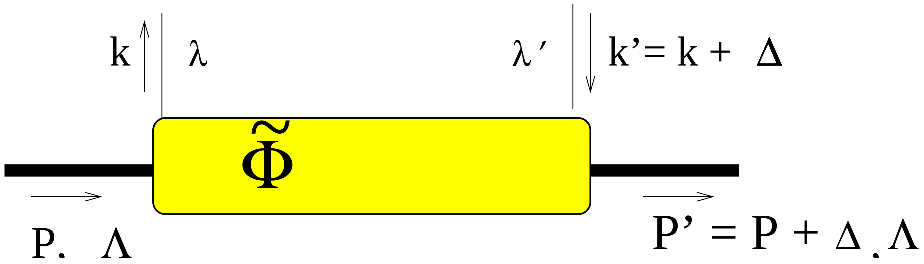

Figure 3.8: *Diagrammatic representation of the correlation function* $\tilde{\Phi}$

hard processes like DVCS or hard meson production where the initial and final hadron have different momenta and spin vectors.
In complete analogy with the usual treatment [MT96] we define off-forward quark-quark correlation functions as Fourier transform of bi-local product of quark fields at two different points.
Non-diagonal quark-quark correlation functions are then given by

$$\widetilde{\Phi}_{ij}(k,k',P,P',S,S') = \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,\bar{k}\cdot z}\;\langle P',S'|\,\overline{\psi}_j(-\frac{z}{2})\,G(-\frac{z}{2},\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,S\rangle, \tag{3.35}$$

where the incoming hadron has momentum P and spin S, the outgoing one has momentum P' and spin S', while $\bar{k}$ is the average parton momentum $\bar{k} = \frac{1}{2}(k+k')$; in the definition above note the presence of the link operators $G(-\frac{z}{2},\frac{z}{2})$, which in light-cone gauge ($A^+ = 0$) reduces to unity.
The matrix elements of light-cone operators are evaluated between different final and initial states, characterized by momenta which differ by a quantity $\Delta^\mu = P'^\mu - P^\mu$ absorbed by the emitted real photon. The momenta for the parton extracted and for the one re-inserted into the hadron, respectively denoted by k^μ and k'^μ, differ by the same quantity $\Delta^\mu = k'^\mu - k^\mu$.
While diagonal correlators are expectation values of quark-quark operators, off-diagonal correlators are transition matrix elements of the same operators and provide more general

information about hadrons compared to usual quark-quark correlators, to which they reduce whenever one considers the forward limit (all four components of the four-vector Δ^μ identically equal to zero and $S' = S$).
The quark-quark correlator is diagrammatically represented in 3.8.

4 Constraints on quark-quark correlators

We discuss some constraints on the correlation functions Φ and $\widetilde{\Phi}$ which appear if one performs hermitian conjugation, parity and time reversal transformations.
The explicit proof of these constraints descends from properties of the Dirac quark fields and of the hadronic states under these transformations. We know how the states behave under such transformations, and we know how the fields transform. This gives consistency conditions (we will use the following notation for momenta and spin vectors: $\tilde{a} = (a^0, -\vec{a})$).

4.1 Hermiticity constraint for quark-quark correlators

4.1.1 Hermiticity constraint for forward quark-quark correlators

Given the correlator

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4} \int d^4z \, e^{i\,k\cdot z} \, \langle P,S|\, \bar{\psi}_j(-z/2)\, \psi_i(z/2)\, |P,S\rangle \tag{4.36}$$

let us consider its adjoint $(\Phi^\dagger)_{ij}$, defined as

$$\begin{aligned}
(\Phi^\dagger)_{ij} = \Phi^*_{ji} \;&=\; \frac{1}{(2\pi)^4} \int d^4z \, e^{-i\,k\cdot z} \, \langle P,S|\, \psi^\dagger_k(-z/2)\, (\gamma_0)_{ki}\, \psi_j(z/2)\, |P,S\rangle^* \\
&=\; \frac{1}{(2\pi)^4} \int d^4z \, e^{-i\,k\cdot z} \, \langle P,S|\, \psi^\dagger_j(z/2)\, (\gamma_0)_{ik}\, \psi_k(-z/2)\, |P,S\rangle \\
&=\; \frac{1}{(2\pi)^4} \int d^4z \, e^{i\,k\cdot z} \, \langle P,S|\, \psi^\dagger_j(-z/2)\, (\gamma_0)_{ik}\, \psi_k(z/2)\, |P,S\rangle \\
&=\; \frac{1}{(2\pi)^4} \int d^4z \, e^{i\,k\cdot z} \, \langle P,S|\, \bar{\psi}_l(-z/2)\, (\gamma_0)_{lj}\, (\gamma_0)_{ik}\, \psi_k(z/2)\, |P,S\rangle \\
&=\; (\gamma_0)_{ik}\, \Phi_{kl}\, (\gamma_0)_{lj} \,.
\end{aligned}$$

We end up with the following constraint for the quark-quark correlator

$$\Phi^\dagger(k,P,S) = \gamma_0\, \Phi(k,P,S)\, \gamma_0 \tag{4.37}$$

4.1.2 Hermiticity constraint for off-forward quark-quark correlators

Given the correlator

$$\widetilde{\Phi}_{ij}(k,k',P,P',S,S') = \frac{1}{(2\pi)^4} \int d^4z \, e^{i\,\bar{k}\cdot z} \, \langle P',S'|\, \bar{\psi}_j(-z/2)\, \psi_i(z/2)\, |P,S\rangle \tag{4.38}$$

its adjoint $(\widetilde{\Phi}^\dagger)_{ij}$ is given as

$$(\widetilde{\Phi}^\dagger)_{ij} = \Big(\widetilde{\Phi}^\dagger(k,k',P,P',S,S')\Big)_{ij} = \widetilde{\Phi}^*_{ji} \tag{4.39}$$

$$
\begin{aligned}
&\left(\widetilde{\Phi}^{\dagger}(k,k',P,P',S,S')\right)_{ij} = \widetilde{\Phi}^{*}_{ji} \\
&= \frac{1}{(2\pi)^4}\int d^4z\; e^{-i\,\bar{k}\cdot z}\; \langle P',S'|\,\psi^{\dagger}_{k}(-z/2)\,(\gamma_0)_{ki}\,\psi_j(z/2)\,|P,S\rangle^{*} \\
&= \frac{1}{(2\pi)^4}\int d^4z\; e^{-i\,\bar{k}\cdot z}\; \langle P,S|\,\psi^{\dagger}_{j}(z/2)\,(\gamma_0)_{ik}\,\psi_k(-z/2)\,|P',S'\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,\bar{k}\cdot z}\; \langle P,S|\,\psi^{\dagger}_{j}(-z/2)\,(\gamma_0)_{ik}\,\psi_k(z/2)\,|P',S'\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,\bar{k}\cdot z}\; \langle P,S|\,\bar{\psi}_{l}(-z/2)\,(\gamma_0)_{lj}\,(\gamma_0)_{ik}\,\psi_k(z/2)\,|P',S'\rangle \\
&= (\gamma_0)_{ik}\,\widetilde{\Phi}_{kl}(k',k,P',P,S',S)\,(\gamma_0)_{lj}\,.
\end{aligned}
$$

We finally obtain the following symmetry relation

$$
\widetilde{\Phi}^{\dagger}(k,k',P,P',S,S') = \gamma_0\,\widetilde{\Phi}(k',k,P',P,S',S)\,\gamma_0 \tag{4.40}
$$

Note that the order of the arguments in (4.40) is the opposite of the one in (4.37).

4.2 Parity constraint

Parity operator $\mathcal{P}$ transforms four-vectors according to the following rule

$$
x^{\mu} = (t,\vec{r}) \longrightarrow \tilde{x}^{\mu} \equiv x_{\mu} = (t,-\vec{r}). \tag{4.41}
$$

On the other hand three dimensional spin vectors $\vec{S}$, which are axial vectors, do not change under parity transformation; this implies that helicity, defined as $\lambda = \vec{S}\cdot\hat{\vec{P}}$, with $\hat{\vec{P}} = \frac{\vec{P}}{|\vec{P}|}$, changes sign under parity transformation since $\vec{P}$, the spatial component of a four vector, transforms according to (4.41).
As far as the transformation property for a fermion field $\psi(x)$ is concerned, one looks for a unitary operator P satisfying

$$
\psi(x) \longrightarrow \mathcal{P}\;\psi(x)\;\mathcal{P}^{-1} = \eta_{\mathcal{P}}\;A\;\psi^{\mathcal{P}}(\tilde{x}) \tag{4.42}
$$

where $\eta_{\mathcal{P}}$ is the intrinsic parity of the field and A is a 4×4 matrix in Dirac space. Assuming that both $\psi(x)$ and $\psi^{\mathcal{P}}(\tilde{x})$ satisfy the Dirac equation, after some steps, one obtains that the transformed field is

$$
\psi^{\mathcal{P}}(x) = \gamma_0\psi(\tilde{x}) \tag{4.43}
$$

4.2.1 Parity constraint for the forward quark-quark correlator

Applying unitarity of the parity operator $\mathcal{P}$ and implementing transformation properties of quark fields and hadronic states

$$
\begin{aligned}
\Phi_{ij}(k,P,S) &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\,\langle P,S|\,\mathcal{P}^\dagger\mathcal{P}\,\bar\psi_j(-z/2)\,\mathcal{P}^\dagger\mathcal{P}\,\psi_i(z/2)\,\mathcal{P}^\dagger\mathcal{P}\,|P,S\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\langle\tilde P,-\tilde S|\,\bar\psi_l(-\tilde z/2)(\gamma_0)_{lj}\,(\gamma_0)_{ik}\psi_k(\tilde z/2)\,|\tilde P,-\tilde S\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\tilde k\cdot z}\,\langle\tilde P,-\tilde S|\,\bar\psi_l(-z/2)(\gamma_0)_{lj}\,(\gamma_0)_{ik}\psi_k(z/2)\,|\tilde P,-\tilde S\rangle \\
&= (\gamma_0)_{ik}\,\Phi_{kl}(\tilde k,\tilde P,-\tilde S)\,(\gamma_0)_{lj}
\end{aligned}
$$

where $\mathcal{P}\,\psi(z)\mathcal{P}^\dagger = \gamma_0\psi(\tilde z)$ and from second to third line $k\cdot z = \tilde k\cdot\tilde z$ and $d^4z = d^4\tilde z$ has been used, one gets the following constraint for quark-quark correlator

$$
\Phi(k,P,S) = \gamma_0\,\Phi(\tilde k,\tilde P,-\tilde S)\,\gamma_0 \tag{4.44}
$$

4.2.2 Parity constraint for the off-forward quark-quark correlator

Let us make use of unitarity of the parity operator to off-forward quark-quark correlator

$$
\begin{aligned}
&\widetilde\Phi_{ij}(k,k',P,P',S,S') \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\bar k\cdot z}\,\langle P',S'|\,\mathcal{P}^\dagger\mathcal{P}\,\bar\psi_j(-z/2)\,\mathcal{P}^\dagger\mathcal{P}\,\psi_i(z/2)\,\mathcal{P}^\dagger\mathcal{P}\,|P,S\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\bar k\cdot z}\langle\tilde P',-\tilde S'|\,\bar\psi_l(-\tilde z/2)(\gamma_0)_{lj}\,(\gamma_0)_{ik}\psi_k(\tilde z/2)\,|\tilde P,-\tilde S\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\tilde{\bar k}\cdot z}\,\langle\tilde P',-\tilde S'|\,\bar\psi_l(-z/2)(\gamma_0)_{lj}\,(\gamma_0)_{ik}\psi_k(z/2)\,|\tilde P,-\tilde S\rangle \\
&= (\gamma_0)_{ik}\,\widetilde\Phi(\tilde k,\tilde k',\tilde P,\tilde P',-\tilde S,-\tilde S')\,(\gamma_0)_{lj}
\end{aligned}
$$

where $\mathcal{P}\,\psi(z)\mathcal{P}^\dagger = \gamma_0\psi(\tilde z)$ and from second to third line $\bar k\cdot z = \tilde{\bar k}\cdot\tilde z$ and $d^4z = d^4\tilde z$ has been used,

$$
\widetilde\Phi(k,k',P,P',S,S') = \gamma_0\,\widetilde\Phi(\tilde k,\tilde k',\tilde P,\tilde P',-\tilde S,-\tilde S')\,\gamma_0 \tag{4.45}
$$

4.3 Time reversal constraint

Time reversal operator $\mathcal{T}$ transforms four-vectors according to the following rule

$$
x^\mu = (t,\vec r) \longrightarrow -\tilde x^\mu \equiv -x_\mu = (-t,\vec r). \tag{4.46}
$$

Three dimensional spin vectors therefore change sign under this transformation, while helicity does not. The transformed fermion field is

$$
\psi^{\mathcal{T}}(x) = i\,\gamma_5\,C\,\psi(-\tilde x) \tag{4.47}
$$

where $C = i\gamma^2\gamma^0$.

4.3.1 Time reversal constraint for the forward quark-quark correlator

Anti-unitarity [1] of time reversal operator $\mathcal{T}$ means that

$$\begin{aligned}\Phi_{ij}(k,P,S) &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\,\langle P,S|\,\bar\psi_j(-z/2)\,\psi_i(z/2)\,|P,S\rangle \\ &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\,\langle P,S|\,\mathcal{T}^\dagger\mathcal{T}\,\bar\psi_j(-z/2)\,\mathcal{T}^\dagger\mathcal{T}\,\psi_i(z/2)\,\mathcal{T}^\dagger\mathcal{T}\,|P,S\rangle^*\end{aligned}$$

Considering the complex conjugate of the correlator $\Phi_{ij}(k,P,S)$ and implementing transformation properties of quark fields and hadronic states

$$\begin{aligned}\Phi^*_{ij}(k,P,S) &= \frac{1}{(2\pi)^4}\int d^4z\, e^{-i\,k\cdot z}\,\langle P,S|\,\mathcal{T}^\dagger\mathcal{T}\,\bar\psi_j(-z/2)\,\mathcal{T}^\dagger\mathcal{T}\,\psi_i(z/2)\,\mathcal{T}^\dagger\mathcal{T}\,|P,S\rangle \\ &= \frac{1}{(2\pi)^4}\int d^4z\, e^{-i\,k\cdot z}\,\langle \tilde P,\tilde S|\,\overline{(-i\gamma_5C\psi)}_j(\tilde z/2)\,(-i\gamma_5C\psi)_i(-\tilde z/2)\,|\tilde P,\tilde S\rangle \\ &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\tilde k\cdot z}\langle \tilde P,\tilde S|\,\overline{(-i\gamma_5C\psi)}_j(-z/2)\,(-i\gamma_5C\psi)_i(z/2)\,|\tilde P,\tilde S\rangle \\ &= (-i\gamma_5C)_{ik}\,\Phi_{kl}(\tilde k,\tilde P,\tilde S)\,(-i\gamma_5C)_{lj},\end{aligned}$$

where $\mathcal{T}\,\psi(z)\mathcal{T}^\dagger = -i\gamma_5C\psi(-\tilde z)$ and from the second to the third line $k\cdot z = \tilde k\cdot\tilde z$ and $d^4z = d^4\tilde z$ has been used together with a renaming $-\tilde z \to z$, one obtains

$$\Phi^*(k,P,S) = (-i\gamma_5C)\,\Phi(\tilde k,\tilde P,\tilde S)\,(-i\gamma_5C) \tag{4.48}$$

4.3.2 Time reversal constraint for the off-forward quark-quark correlator

$$\begin{aligned}\widetilde\Phi_{ij}(k,k',P,P',S,S') &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\bar k\cdot z}\,\langle P',S'|\,\bar\psi_j(-z/2)\,\psi_i(z/2)\,|P,S\rangle \\ &= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\bar k\cdot z}\,\langle P',S'|\,\mathcal{T}^\dagger\mathcal{T}\,\bar\psi_j(-z/2)\,\mathcal{T}^\dagger\mathcal{T}\,\psi_i(z/2)\,\mathcal{T}^\dagger\mathcal{T}\,|P,S\rangle^*\end{aligned}$$

or

[1] A is anti-linear if $A(\lambda|\phi\rangle + \mu|\psi\rangle) = \lambda^*\,A|\phi\rangle + \mu^*A\,|\psi\rangle$.
An anti-linear operator is *anti-unitary* if $A^\dagger = A^{-1}$.
One thus has $\langle A\phi|A\psi\rangle = \langle A\psi|A\phi\rangle^* = \langle\psi|A^\dagger A\phi\rangle^* = \langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$.

$$
\begin{aligned}
&\widetilde{\Phi}^{*}_{ij}(k,k',P,P',S,S') \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{-i\,\bar{k}\cdot z}\,\langle P',S'|\,\mathcal{T}^\dagger\mathcal{T}\,\bar{\psi}_j(-z/2)\,\mathcal{T}^\dagger\mathcal{T}\,\psi_i(z/2)\,\mathcal{T}^\dagger\mathcal{T}\,|P,S\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{-i\,\bar{k}\cdot z}\,\langle \tilde{P}',\tilde{S}'|\,\overline{(-i\gamma_5 C\psi)}_j(\tilde{z}/2)\,(-i\gamma_5 C\psi)_i(-\tilde{z}/2)\,|\tilde{P},\tilde{S}\rangle \\
&= \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,\tilde{\bar{k}}\cdot z}\langle \tilde{P}',\tilde{S}'|\,\overline{(-i\gamma_5 C\psi)}_j(-z/2)\,(-i\gamma_5 C\psi)_i(z/2)\,|\tilde{P},\tilde{S}\rangle \\
&= (-i\gamma_5 C)_{ik}\,\widetilde{\Phi}_{kl}(\tilde{k},\tilde{k}',\tilde{P},\tilde{P}',\tilde{S},\tilde{S}')\,(-i\gamma_5 C)_{lj},
\end{aligned}
$$

where $\mathcal{T}\,\psi(z)\mathcal{T}^\dagger = -i\gamma_5 C\psi(-\tilde{z})$ and $\mathcal{T}\,|P,S\rangle = \langle\tilde{P},\tilde{S}|$, and from the second to the third line $k\cdot z = \tilde{k}\cdot\tilde{z}$ and $d^4z = d^4\tilde{z}$ has been used together with a renaming $-\tilde{z}\to z$. Note that $\overline{(-i\gamma_5 C\psi)}_j(z) = \bar{\psi}_l(z)(-i\gamma_5 C)_{lj}$.
Finally we obtain

$$
\widetilde{\Phi}^{*}_{ij}(k,k',P,P',S,S') = (-i\gamma_5 C)\,\widetilde{\Phi}\,(\tilde{k},\tilde{k}',\tilde{P},\tilde{P}',\tilde{S},\tilde{S}')\,(-i\gamma_5 C) \tag{4.49}
$$

We will make use of this constraints later on when building the ansätze for forward and off-forward quark-quark correlators.

5 Choice of spinors and evaluation of spinorial products

We review different possible choices for spin states and their corresponding spin vectors in hard processes and introduce the concept of *light-cone helicity*. Spin vectors for the eigenstates of light-cone helicity (LC helicity) will be also defined.
We calculate moreover various spinorial products necessary to work out the form factor decomposition of the vector, axial and tensor currents of the proton and to write down ansätze for the forward and off-forward quark-quark correlation functions. We will perform many calculations in the basis of light-cone helicity since, as we will show, one can easily build ansätze for the off-forward quark-quark correlators if the correlators are evaluated between eigenstates of LC hadron helicity. On the contrary ansätze for the forward quark-quark correlation functions will be written both in the basis of eigenstates of LC helicity and for generic spin states, characterized by a covariant spin vector.

5.1 Definition of spin and helicity states

In a relativistic covariant theory spin emerges automatically, as can be seen for instance from the Dirac Equation for spin $1/2$; it is not an additional degree of freedom as in non-relativistic quantum mechanics. Nevertheless the relativistic description of spin is non-trivial and only in the rest frame of the particle one can identify a set of spin operators and use the spin formalism developed for non-relativistic quantum mechanics[CJ80].
To this aim one introduces the Pauli-Lubanski operators W^σ which are constructed in terms of the generators of translations P^ρ and the generators of the homogeneous Lorentz transformations $J^{\mu\nu}$

$$W^\sigma = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}J^{\mu\nu}P^\rho \, , \tag{5.50}$$

with

$$J_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu} + P_\mu x_\nu - P_\nu x_\mu \tag{5.51}$$

and the angular momentum operators are

$$J^i = -\frac{1}{2}\,\epsilon^{i\mu\nu}\,J_{\mu\nu}\,. \tag{5.52}$$

The Pauli-Lubanski operators satisfy the following commutation relations

$$[W_\mu,\, W_\nu] = i\,\epsilon_{\mu\nu\rho\sigma}\,W^\rho P^\sigma \tag{5.53}$$

which do not generate the algebra one would expect for spin operators but thanks to the Pauli-Lubanski operators in the particle rest frame one can define $\hat{s}^i$

$$\hat{s}^i = \frac{1}{m}W^i \tag{5.54}$$

which indeed satisfy the commutation relations $[s_j,\, s_k] = i\epsilon_{jkl}\, s^l$.

Thus one approaches the problem of defining spin vectors and spin states for particles of arbitrary momentum by acting upon the state of a particle at rest with suitable Lorentz transformations.
Starting from the particle rest frame, we now look for various solutions of the free Dirac equation.
For instance we could seek solutions of the Dirac equation which are eigenstates of the operators of the spin or of the helicity $\Lambda = \frac{\vec{S}\cdot\vec{P}}{|\vec{P}|}$.
The difference between eigenstates of spin and eigenstates of helicity lies in the different choice of reference frame the observer of the particle adopts.
Let us assume that a particle is at rest in a given reference frame $\overset{\circ}{R}$ in which the particle momentum is called $\overset{\circ}{P}$. This frame is moving with momentum P with respect to the observer's frame R. There are many possible frames $\overset{\circ}{R}$ which differ as they are rotated from each other.
One can for instance decide to observe the particle from a reference frame R with respect to which the particle state of motion is obtained by a pure Lorentz transformation ("boost"). In this case the observer must have before rotated his reference frame in order to have his z axis aligned with the direction of motion of the particle. After the boost he will be compelled to do an inverse rotation, applied to the boosted system, obtaining thus a set of eigenstates of the spin operators.
Alternatively the observer can apply a boost with speed v to his reference frame R and then apply a Jacob and Wick rotation such that the particle momentum appears as having polar angles (θ, ϕ). This is the choice usually adopted if one considers helicity eigenstates.
Let us now explicity construct these states.
For any space-like four-vector n ($n^2 = -1$), orthogonal to the particle momentum P, we have from (5.50) that

$$W \cdot n = -\frac{1}{4}\epsilon_{\mu\nu\rho\sigma} n^\mu P^\nu \sigma^{\rho\sigma} = -\frac{1}{2}\gamma_5 \not{n} \not{P}. \tag{5.55}$$

In the rest frame of the particle where $\overset{\circ}{P}{}^{\mu}$ is given as

$$\overset{\circ}{P}{}^{\mu} = (m, 0, 0, 0) \tag{5.56}$$

one can evaluate $W \cdot n$ obtaining

$$W^0 = 0 \qquad \frac{\vec{W}}{m} = \frac{1}{2}\gamma^5\gamma^0\vec{\gamma} \, . \tag{5.57}$$

If n is chosen along the z axis, then a set of four independent eigenvectors of the spin projector with eigenvalues $+1/2$ for u_1 and v_1 (spin up) and $-1/2$ for u_2 and v_2 (spin down) is given by

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad u_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{5.58}$$

where v_1 and v_2 correspond to antiparticles solutions.
One can then boost these solutions up to a velocity $v = |\vec{P}|/P^0$ by a pure Lorentz transformation obtaining eigenstates of $-W \cdot n'/m$ where n' is now the transform of n with respect to the boost

$$u_S(P,+) = \sqrt{\frac{1}{\sqrt{2}}(P^+ + P^-) + M} \begin{pmatrix} \frac{1}{\sqrt{2}}(P^+ + P^-) + M \\ 0 \\ \frac{1}{\sqrt{2}}(P^+ - P^-) \\ P^1 + i\,P^2 \end{pmatrix} \tag{5.59}$$

and

$$u_S(P,-) = \sqrt{\frac{1}{\sqrt{2}}(P^+ + P^-) + M} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(P^+ + P^-) + M \\ P^1 - i\,P^2 \\ -\frac{1}{\sqrt{2}}(P^+ - P^-) \end{pmatrix} \tag{5.60}$$

and similar for $u_S(P,+)$ and $u_S(P,-)$.
There is a special choice of the light-like vector n such that its spatial part $\vec{n}$ is proportional to $\vec{P}$ in the reference frame. Such a choice leads to define states which are eigenstates of helicity

$$u_H(P,+) \;=\; \sqrt{\frac{1}{\sqrt{2}}(P^+ + P^-) + M} \begin{pmatrix} \frac{1}{\sqrt{2}}(P^+ + P^-) + M \\ 0 \\ \frac{1}{\sqrt{2}}(P^+ + P^-) - M \\ 0 \end{pmatrix} \tag{5.61}$$

and

$$u_H(P,-) \;=\; \sqrt{\frac{1}{\sqrt{2}}(P^+ + P^-) + M} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(P^+ + P^-) + M \\ 0 \\ -\frac{1}{\sqrt{2}}(P^+ + P^-) - M \end{pmatrix} \tag{5.62}$$

In the framework of light-cone quantization one usually considers states which are eigenstates of light-cone helicity, as for example the spinors of Kogut and Soper [KS70] or Brodsky and Lepage [LB80].
For the construction of ordinary helicity eigenstates the second step after the longitudinal boost is a rotation in space which leaves the energy unchanged. For the construction of LC helicity states instead a so called "transverse boost" is employed, defined such that the LC "plus" component of the momentum is unchanged. Accordingly, ordinary helicity eigenstates are invariant under spatial rotations, whereas LC helicity eigenstates are invariant under transverse boosts.
We remark in passing that when speaking of helicity in the following we will always refer to the light-cone helicity. In the target rest frame the light-cone helicity is the z-component of the spin vector, while it coincides with the standard definition of helicity in the infinite momentum frame ($P^+ \to \infty$) or if the particle has zero mass $m = 0$.
A particularly useful spinor basis is obtained from the eigenstates of the projection operators $\mathcal{P}_+ = \frac{1}{2}\gamma^-\gamma^+$ and $\mathcal{P}_- = \frac{1}{2}\gamma^+\gamma^-$. In the standard representation of Dirac matrices the eigenstates of $\mathcal{P}_+$ are given by

$$u_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad u_- = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \tag{5.63}$$

Then one performs a Lorentz boost and subsequently a transverse boost, that leaves the plus components of vectors unchanged, which together carry the rest momentum $\overset{\circ}{P}$ into an arbitrary momentum P

$$\begin{aligned} u_{LC}(P,+) &= \frac{1}{\sqrt{P^+}}\,(\,P^+ + \gamma^0 m + \gamma^0\vec{\gamma}_\perp\,)\,u_+ \\ u_{LC}(P,-) &= \frac{1}{\sqrt{P^+}}\,(\,P^+ + \gamma^0 m + \gamma^0\vec{\gamma}_\perp\,)\,u_- \end{aligned}$$

obtaining

$$\begin{aligned} u_{LC}(P,+) &= \frac{1}{(2\sqrt{2}\,P^+)^{1/2}}\begin{pmatrix} \sqrt{2}\,P^+ + M \\ P^1 + i\,P^2 \\ \sqrt{2}\,P^+ - M \\ P^1 + i\,P^2 \end{pmatrix} \\ u_{LC}(P,-) &= \frac{1}{(2\sqrt{2}\,P^+)^{1/2}}\begin{pmatrix} -P^1 + i\,P^2 \\ \sqrt{2}\,P^+ + M \\ P^1 - i\,P^2 \\ -\sqrt{2}\,P^+ + M\,. \end{pmatrix} \end{aligned} \tag{5.64}$$

Since $\bar{u}(p,\pm) = u^\dagger(p,\pm)\gamma_0$ it follows that

$$\bar{u}(P,+)_{LC} = \frac{1}{(2\sqrt{2}\,P^+)^{1/2}} \Big(\sqrt{2}\,P^+ + m\,,\ P_1 - i\,P_2\,,\ -\sqrt{2}\,P^+ + m\,,\ -P_1 + i\,P_2\Big) \tag{5.65}$$

$$\bar{u}(P,-)_{LC} = \frac{1}{(2\sqrt{2}\,P^+)^{1/2}} \Big(-P_1 - i\,P_2\,,\ \sqrt{2}\,P^+ + m\,,\ -P_1 - i\,P_2\,,\ \sqrt{2}\,P^+ - m\Big) \tag{5.66}$$

which are normalized according to

$$\begin{aligned} \bar{u}(P,+)\,u(P,+) &= \bar{u}(P,-)\,u(P,-) = 2\,m \\ \bar{u}(P,-)\,u(P,+) &= \bar{u}(P,+)\,u(P,-) = 0\,. \end{aligned} \tag{5.67}$$

The spinors for the antiparticles are defined as

$$\begin{aligned} v_{LC}(P,+) &= \frac{1}{\sqrt{P^+}}\,(\ P^+ - \gamma^0 m + \gamma^0\vec{\gamma}_\perp\ )\,u_- \\ v_{LC}(P,-) &= \frac{1}{\sqrt{P^+}}\,(\ P^+ - \gamma^0 m + \gamma^0\vec{\gamma}_\perp\ )\,u_+ \end{aligned}$$

With respect to the eigenstates of ordinary helicity, Brodsky and Lepage's spinor representations are obtained without acting upon the rest frame states with a Wigner rotation. For zero mass the light-cone helicity spinors are identical to the ordinary helicity spinors, to which they are mapped by the following transformation

$$\begin{pmatrix} u_H(P,+) \\ u_H(P,-) \end{pmatrix} = U \begin{pmatrix} u_{LC}(P,+) \\ u_{LC}(P,-) \end{pmatrix} \tag{5.68}$$

where the unitary U matrix, representing a rotation, in standard representation is given as

$$U = \frac{1}{\sqrt{2\,(P^0+P^3)|\,\vec{P}\,|\,(|\,\vec{P}\,|+P^3)}} \begin{pmatrix} (|\,\vec{P}\,|+P^3)\sqrt{P^0+|\,\vec{P}\,|} & (P^1+iP^2)\sqrt{P^0-|\,\vec{P}\,|} \\ -(P^1-iP^2)\sqrt{P^0-|\,\vec{P}\,|} & (|\,\vec{P}\,|+P^3)\sqrt{P^0+|\,\vec{P}\,|} \end{pmatrix}$$

In the ultra relativistic limit the difference between usual and light-cone helicity vanishes. Taking the limit $P \longrightarrow \infty$ we find that the light-cone helicity eigenstates are helicity eigenstates when viewed from the infinite-momentum frame.

We introduce now the spin vectors corresponding to the helicity eigenstates as introduced by Brodsky and Lepage.

In the rest frame the particle four-momentum $\overset{\circ}{P}{}^{\mu}$ in light-cone coordinates is

$$\overset{\circ}{P}{}^{\mu} = \left[\ \frac{m}{\sqrt{2}}\,,\ \frac{m}{\sqrt{2}}\,,\ \vec{0}_\perp\ \right] \tag{5.69}$$

and, assuming that the particle is polarized along the z axis, the spin vector is given by

$$S^\mu = (\ 0,\ 0,\ 0,\ \Lambda\ ) \tag{5.70}$$

or in light-cone coordinates

$$S^\mu = \left[\ \frac{\Lambda}{\sqrt{2}}\ ,\ -\frac{\Lambda}{\sqrt{2}}\ ,\ \vec{0}_\perp\ \right] \tag{5.71}$$

where the light-cone helicity Λ can assume values $+1$ or -1.
Let us now boost the particle along the z direction with a speed equal to v. This produces a change in the particle momentum

$$P^\mu = \left[\ \frac{me^\phi}{\sqrt{2}}\ ,\ \frac{me^{-\phi}}{\sqrt{2}}\ ,\ \vec{0}_\perp\ \right] \tag{5.72}$$

where $\phi = \frac{1}{2} ln \frac{1+v}{1-v}$, so that $v = tanh\,\phi$.
As a consequence of the boost the spin vector becomes

$$S^\mu = \left[\ \frac{\Lambda\, e^\phi}{\sqrt{2}}\ ,\ -\frac{\Lambda\, e^{-\phi}}{\sqrt{2}}\ ,\ \vec{0}_\perp\ \right]\ . \tag{5.73}$$

One can therefore set a relation between the spin vector and the four-momentum vector in any frame obtained by boosting the particle from the rest frame

$$S^\mu = \frac{\Lambda}{m}\ (\ P^\mu - \frac{m^2}{P^+} v'^\mu\ ) \tag{5.74}$$

where $v'^\mu = \left[\ 0\ ,\ 1\ ,\ \vec{0}_\perp\ \right]$.
In order to obtain vectors for particles in any other frame related by a transverse boost, one performs Lorentz transformations which leave v'^μ invariant. For instance, to define the spin vector in the average frame (see the Appendix), we still have to do a transverse boost. Nevertheless one can easily show that such a transverse boost does not change the relation between the spin and the momentum vectors.
Spin vectors for either generic pure states or for impure states, so called mixtures of states, contain also a transverse component of the spin given $\left[\ 0\ ,\ 0\ ,\ \vec{S}_\perp\ \right]$.
The complete spin vector will then be written as

$$S^\mu = \frac{\Lambda}{m}(P^\mu - \frac{m^2}{P^+} {v'}^\mu) + S^\mu_\perp\ . \tag{5.75}$$

The covariant spin vector in (5.75) satisfies

$$P^\mu S_\mu = 0 \tag{5.76}$$

and if the spin vector S^μ describes a pure state, it is also true that S^μ has length one

$$S^2 = -(\ \Lambda^2 + {S_\perp}^2\ ) = -1\ , \tag{5.77}$$

as it can be shown in the particle rest frame, where $S^\mu = (0, \vec{S}_\perp, \Lambda)$, that $S^2 = \Lambda^2 + \vec{S}^2_\perp < 1$ for a mixed state and $\Lambda^2 + \vec{S}^2_\perp = 1$ for a pure state.

5.2 Spinorial products

We calculate the spinorial products

$$\bar{u}(P',S')\,\Gamma^{\alpha\beta}\,u(P,S) \qquad \text{where} \qquad \Gamma = \mathbb{1},\,\gamma^{\alpha},\,\gamma_5,\,\gamma^{\alpha}\gamma_5,\sigma^{\alpha\beta}\,, \tag{5.78}$$

which we will need later on to build ansätze for forward and off-forward quark-quark correlation functions. $\bar{u}(P',S')$ and $u(P,S)$ are assumed to be eigenstates of light-cone helicity whose corresponding covariant vectors are defined in (5.74).
Studying the form factor decomposition of the tensor current of the proton [Die01], we will need additionally the Dirac bilinears formed by the following Γ structures

$$\begin{aligned}
\Gamma^{+j} &= \sigma^{+j} \\
\Gamma^{+j} &= \epsilon^{+j\rho\sigma}\,\Delta_{\rho}\bar{P}_{\sigma} \\
\Gamma^{+j} &= \epsilon^{+j\rho\sigma}\,\Delta_{\rho}\gamma_{\sigma} \\
\Gamma^{+j} &= \epsilon^{+j\rho\sigma}\,\bar{P}_{\rho}\gamma_{\sigma}\,,
\end{aligned} \tag{5.79}$$

which will be also computed below.
We report two methods to calculate spinorial products.

5.2.1 Method via traces

With the method, which we name "via traces", the spinorial products in (5.78) are derived as

$$\begin{aligned}
\bar{u}(P',S')\,\Gamma\,u(P,S) &= \frac{(\bar{u}(P',S')\Gamma\,u(P,S))(\bar{u}(P',S')\,u(P,S))^*}{|\bar{u}(P',S')\,u(P,S)|} \times \frac{|\bar{u}(P',S')\,u(P,S)|}{(\bar{u}(P',S')\,u(P,S))^*} \\
&= \frac{\mathrm{Tr}\left[(\not{P}+m)\frac{1}{2}(1+\gamma_5\,\not{S})(\not{P}'+m)\frac{1}{2}(1+\gamma_5\,\not{S}')\,\Gamma\right]}{\sqrt{\mathrm{Tr}\left[(\not{P}+m)\frac{1}{2}(1+\gamma_5\,\not{S})(\not{P}'+m)\frac{1}{2}(1+\gamma_5\,\not{S}')\right]}} \\
&\qquad \times \frac{|\bar{u}(P',S')\,u(P,S)|}{(\bar{u}(P',S')\,u(P,S))^*}
\end{aligned} \tag{5.80}$$

where the first term in (5.80) is calculated from the traces

$$\frac{\mathrm{Tr}\left[(\not{P}+m)\frac{1}{2}(1+\gamma_5\,\not{S})(\not{P}'+m)\frac{1}{2}(1+\gamma_5\,\not{S}')\,\Gamma\right]}{\sqrt{\mathrm{Tr}\left[(\not{P}+m)\frac{1}{2}(1+\gamma_5\,\not{S})(\not{P}'+m)\frac{1}{2}(1+\gamma_5\,\not{S}')\right]}} \tag{5.81}$$

obtained multiplying $\bar{u}(P',S')u(P,S)$ by $(\bar{u}(P',S')\;\;u(P,S))^*/\sqrt{|\bar{u}(P',S')\,u(P,S)|^2}$ and the second term in (5.80) is a phase given by

$$\bar{u}(P',S')\,u(P,S) = phase \times |\bar{u}(P',S')\,u(P,S)| \tag{5.82}$$

which depends upon the initial and final spinor, i.e. we need to know the result of $\bar{u}(P',S')\,u(P,S)$ for any combination of helicities in the initial and final spinor.

The calculation of the phase

$$phase = \frac{1}{(phase)^*} \tag{5.83}$$

is done by substituting the explicit spinors, quoted in (5.64), in the spinorial product $\bar{u}(P', S')\, u(P, S)$. Here we quote only the results

$$\begin{aligned}
\bar{u}(P', S')\, \Gamma\, u(P, S) &= |\bar{u}(P', S')\, \Gamma\, u(P, S)| \times 1 \qquad\qquad \text{for } \Lambda' = \Lambda = \pm 1 \\
&= |\bar{u}(P', S')\, \Gamma\, u(P, S)| \times \begin{cases} -\eta & \text{for } \Lambda' = -\Lambda = -1 \\ \eta^* & \text{for } \Lambda' = -\Lambda = +1 \end{cases}
\end{aligned}$$

having defined the phase factor η as

$$\eta = \frac{\Delta_1 + i\,\Delta_2}{|\vec{\Delta}_\perp|}\,, \tag{5.84}$$

where

$$|\vec{\Delta}_\perp| = \sqrt{\frac{-4\xi^2 m^2}{1-\xi^2} + \frac{4\xi^2 m^2 + \vec{\Delta}_\perp^2}{1-\xi^2}}\,\sqrt{1-\xi^2} = \sqrt{t_0 - t}\,\sqrt{1-\xi^2}\,. \tag{5.85}$$

5.2.2 Spinor products evaluated with the trace for the forward case

With the parameterization (B.255), defined in the Appendix and $P \cdot P' = m^2 - t/2$ and for LC helicity eigenstates with spin vector as in (5.74), the results for helicity non-flip, i.e., $\Lambda' = \Lambda$ are

$$\begin{aligned}
\text{scalar}: &\qquad |\,\bar{u}(P', S')\, u(P, S)\,| = \frac{2\,m}{\sqrt{1-\xi^2}} \\
\text{vector}: &\qquad |\,\bar{u}(P', S')\, \gamma^+\, u(P, S)\,| = 2\,\bar{P}^+\,\sqrt{1-\xi^2} \\
\text{pseudo}: &\qquad |\,\bar{u}(P', S')\, \gamma_5\, u(P, S)\,| = \Lambda\,\frac{2\,m\,\xi}{\sqrt{1-\xi^2}} \\
\text{axial}: &\qquad |\,\bar{u}(P', S')\, \gamma^+ \gamma_5\, u(P, S)\,| = \Lambda\, 2\,\bar{P}^+\,\sqrt{1-\xi^2}
\end{aligned} \tag{5.86}$$

and for helicity flip, i.e., $\Lambda' = -\Lambda$,

$$\begin{aligned}
\text{scalar}: &\qquad |\,\bar{u}(P', S')\, u(P, S)\,| = \left(\frac{-4\,\xi^2\,m^2}{1-\xi^2} - t\right)^{1/2} = \sqrt{t_0 - t} \\
\text{pseudo}: &\qquad |\,\bar{u}(P', S')\, \gamma_5\, u(P, S)\,| = \Lambda\,\left(\frac{-4\,\xi^2\,m^2}{1-\xi^2} - t\right)^{1/2} = \Lambda\sqrt{t_0 - t}\,.
\end{aligned} \tag{5.87}$$

The traces involving γ^+ (vector) and $\gamma^+\gamma_5$ (axial) vanish.

5.2.3 Spinor products evaluated with the trace method for the off-forward case

With the parameterization (B.255) and $P \cdot P' = m^2 - t/2$ the results for helicity non-flip, i.e., $\Lambda' = \Lambda$ are

$$
\begin{aligned}
\text{scalar :} \qquad & |\,\bar{u}(P',\Lambda')\; u(P,\Lambda)\,| = \frac{2\,m}{\sqrt{1-\xi^2}} \\
\text{vector :} \qquad & |\,\bar{u}(P',\Lambda')\;\gamma^\mu\; u(P,\Lambda)\,| = \frac{2\bar{P}^\mu}{\sqrt{1-\xi^2}} + \frac{\xi\Delta^\mu}{\sqrt{1-\xi^2}} + \frac{t\; v'^\mu}{2\bar{P}^+\sqrt{1-\xi^2}} \\
& - \Lambda \frac{i\,\epsilon^{\mu\nu\rho\sigma}\bar{P}_\nu \Delta_\rho v'_\sigma}{\bar{P}^+\sqrt{1-\xi^2}} \\
\text{pseudo :} \qquad & |\,\bar{u}(P',\Lambda')\;\gamma_5\; u(P,\Lambda)\,| = \Lambda\,\frac{2\,m\,\xi}{\sqrt{1-\xi^2}} \\
\text{axial :} \qquad & \bar{u}(P',\Lambda')\;\gamma^\mu\gamma_5\; u(P,\Lambda) = \frac{\Lambda\,2\,\bar{P}^\mu}{\sqrt{1-\xi^2}} + \frac{\Lambda\xi\Delta^\mu}{\sqrt{1-\xi^2}} + \frac{\Lambda(t-4m^2)\; v'^\mu}{2\,\bar{P}^+\sqrt{1-\xi^2}} \\
& - \frac{i\,\epsilon^{\mu\nu\rho\sigma}\bar{P}_\nu \Delta_\rho v'_\sigma}{\bar{P}^+\sqrt{1-\xi^2}} \\
\text{tensor :} \qquad & |\,\bar{u}(P',\Lambda')\;\sigma^{\mu\nu}\; u(P,\Lambda)\,| = \frac{i\,m\;(\Delta^\mu v'^\nu - \Delta^\nu v'^\mu)}{\bar{P}^+\,\sqrt{1-\xi^2}} \\
& + \Lambda\,\frac{2m\,\epsilon^{\mu\nu\rho\sigma}\bar{P}_\rho v'_\sigma}{\bar{P}^+\sqrt{1-\xi^2}}
\end{aligned}
\tag{5.88}
$$

and for helicity flip, i.e., $\Lambda' = -\Lambda$,

$$
\begin{aligned}
\text{scalar :} \qquad & |\,\bar{u}(P',\Lambda')\; u(P,\Lambda)\,| = \left(\frac{-4\,\xi^2\,m^2}{1-\xi^2} - t\right)^{1/2} = \sqrt{t_0 - t} \\
\text{vector :} \qquad & |\,\bar{u}(P',\Lambda')\;\gamma^\mu\; u(P,\Lambda)\,| = \frac{4m\xi^2\,\bar{P}^\mu}{(1-\xi^2)\sqrt{t_0-t}} - \frac{2m\xi\Delta^\mu}{(1-\xi^2)\sqrt{t_0-t}}
\end{aligned}
$$

$$- \frac{m\,t\,v'^{\mu}}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}} + \Lambda \frac{i\,2m\xi\epsilon^{\mu\nu\rho\sigma}\bar{P}_\nu\Delta_\rho v'_\sigma}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}}$$

pseudo : $$|\,\bar{u}(P',\Lambda')\;\gamma_5\;u(P,\Lambda)\,| = \Lambda\sqrt{t_0-t}$$

axial : $$|\,\bar{u}(P',\Lambda')\;\gamma^{\mu}\gamma_5\;u(P,\Lambda)\,| = -\frac{\Lambda\,4m\xi\,\bar{P}^{\mu}}{(1-\xi^2)\sqrt{t_0-t}} - \frac{\Lambda\,2m\Delta^{\mu}}{(1-\xi^2)\sqrt{t_0-t}}$$

$$- \frac{\Lambda\,m\xi(t-4m^2)\;v'^{\mu}}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}} - \frac{i\,2m\epsilon^{\mu\nu\rho\sigma}\bar{P}_\nu\Delta_\rho v'_\sigma}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}}$$

tensor : $$|\,\bar{u}(P',\Lambda')\;\sigma^{\mu\nu}\;u(P,\Lambda)\,| = -\frac{i\,2\,\left(\bar{P}^{\mu}\Delta^{\nu}-\bar{P}^{\nu}\Delta^{\mu}\right)}{\sqrt{t_0-t}}$$

$$- \frac{i\,2\,m^2\;(\Delta^{\mu}v'^{\nu}-\Delta^{\nu}v'^{\mu})}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}} - \frac{\Lambda\,4m^2\xi\;\epsilon^{\mu\nu\rho\sigma}\bar{P}_\rho v'_\sigma}{\bar{P}^{+}(1-\xi^2)\sqrt{t_0-t}}$$

$$+ \frac{2\Lambda\;\epsilon^{\mu\nu\rho\sigma}\bar{P}_\rho\Delta_\sigma}{\sqrt{t_0-t}} \tag{5.89}$$

Here we quote also the results for the bilinears mentioned in (5.79) which we will need later on to expand the form factor decomposition of the tensor current of the proton. Notice that latin indices $i,j=1,2$, while greek ones run from 0 to 3.
For the helicity non-flip case, i. e., $\Lambda'=\Lambda$, we have

$$\begin{aligned}
|\,\bar{u}(P',S')\;\epsilon^{+j\rho\sigma}\bar{P}_\rho\gamma_\sigma\;u(P,S)\,| &= \frac{-i\,\bar{P}^{+}\,\Lambda\Delta^{j}+\xi\,\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma}{\sqrt{1-\xi^2}} \\
|\,\bar{u}(P',S')\;\sigma^{+j}\,\gamma^{5}\;u(P,S)\,| &= \frac{\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma\,\xi\,\sqrt{1-\xi^2}}{(1+\xi)\,m} \\
|\,\bar{u}(P',S')\,\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma\;u(P,S)\,| &= -2m\,\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma\,\sqrt{-1+\xi^2} \\
|\,\bar{u}(P',S')\;\epsilon^{+j\rho\sigma}\Delta_\rho\gamma_\sigma\;u(P,S)\,| &= \frac{2\,(\bar{P}^{+}\Lambda\Delta^{j}i\,\xi-\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma)}{\sqrt{-1+\xi^2}}
\end{aligned} \tag{5.90}$$

The results for the helicity flip case, i.e., $\Lambda'=-\Lambda$, are

$$\begin{aligned}
|\,\bar{u}(P',S')\;\frac{\epsilon^{+j\rho\sigma}\bar{P}_\rho\gamma_\sigma}{m}\;u(P,S)\,| &= -2\,\frac{m\xi(i\,\bar{P}^{+}\,\Delta^{j}\Lambda-\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma)}{\sqrt{-1+\xi^2}\sqrt{t_0-t}} \\
|\,\bar{u}(P',S')\;\sigma^{+j}\,\gamma^{5}\;u(P,S)\,| &= -2\,\frac{i\bar{P}^{+}\Lambda\Delta^{j}+\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma}{\sqrt{t_0-t}}
\end{aligned}$$

$$
\begin{aligned}
|\,\bar{u}(P',S')\,\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma\,u(P,S)\,| &= -\frac{\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma\,(4m^2\xi^2-\xi^2\,t+t)}{\sqrt{1-\xi^2}\sqrt{t_0-t}} \\
|\,\bar{u}(P',S')\,\epsilon^{+j\rho\sigma}\Delta_\rho\gamma_\sigma\,u(P,S)\,| &= 4m\xi^2\,\frac{\bar{P}^+\Lambda\Delta^j i-\epsilon^{+j\rho\sigma}\bar{P}_\rho\Delta_\sigma}{\sqrt{-1+\xi^2}\,\sqrt{t_0-t}}
\end{aligned} \tag{5.91}
$$

Notice that we assume that $\epsilon^{+1-2}=1$ since $\epsilon^{0123}=1$.

5.2.4 Evaluation from explicit representation of LC-helicity spinors

The method “via traces” discussed above allows to determine spinorial products if the factor

$$
\sqrt{\mathrm{Tr}\left[(\not{P}+m)\frac{1}{2}(1+\gamma_5\,\not{S})(\not{P}'+m)\frac{1}{2}(1+\gamma_5\,\not{S}')\right]}=\sqrt{|\bar{u}(P',S')\,u(P,S)|^2} \tag{5.92}
$$

by which we divide the trace in (5.80) in order to normalize it is different from zero. This factor is always different from zero except in the forward case when the initial and final states carry opposite helicities. Indeed the factor in (5.92) corresponds to the probability to find a spinor, carrying for instance helicity $-$, which spontaneously transforms to a spinor having helicity $+$ and this probability is clearly zero. In this case we need an alternative method to calculate the spinorial products which does not rely on calculating traces.
Actually we can determine the spinorial products also by substituting directly the explicit expressions of the spinors defined by Brodsky and Lepage [LB80], quoted in (5.64), directly in the definition of the various spinorial products (5.78). By evaluating the spinorial products through this direct method we have also the correct phase information.

6 Forward quark-quark correlators

We discuss forward quark-quark correlators in both spin and light-cone helicity basis.
We then provide two methods to model ansätze for forward correlators. The first method discussed below, based on a proposal of Daniel Boer [D.B], can be applied to build ansätze for correlators defined only with respect to a spin basis. We develop further an alternative method to construct ansätze for forward quark-quark correlators in both spin and light-cone helicity basis. This alternative method will be easily generalized to build ansätze for off-forward quark-quark correlators.
Applying the alternative method, we then provide ansätze for forward quark-quark helicity correlators for the cases when the helicity of the hadron is flipped or not flipped.
Finally we show which constraints on forward quark-quark correlation functions are implied by conservation of helicity.

6.1 Method to construct a general ansatz

In order to construct the most general ansatz for the DIS quark-quark correlation function

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4} \int d^4z\, e^{i\,k\cdot z}\, \langle P,S|\, \overline{\psi}_j(-\frac{z}{2})\, \psi_i(\frac{z}{2})\, |P,S\rangle, \tag{6.93}$$

we propose a method that has the advantage of easily taking into account all possible independent structures. We note that the quark-quark correlator in (6.93) is a 4×4 matrix in Dirac space, a scalar in Lorentz space and can only depend on the three independent external (axial) vectors that appear in the process

$$[\, k \qquad P \qquad S\,] \tag{6.94}$$

Let us thus consider all possible 4×4 matrices which are not products of other 4×4 matrices (since those will be generated automatically)

$$[\, \mathbb{1} \qquad \gamma^\mu \qquad \gamma_5\,] \tag{6.95}$$

and multiply these matrices with the three vectors $k,\ P$ and S, requiring the following two additional rules to be fulfilled

1. constraint derived from invariance under parity transformation (4.44)

2. linearity in S

We have already shown how the parity constraint arises.
Linearity in S follows from Lorentz invariance that demands the hadronic tensor $W^{\mu\nu}$ to be linear in the initial and final nucleon spinors $\bar{u}(P,S)$ and $u(P,S)$. Tensors constructed from the spinors are either spin independent ($\bar{u}(P,S)\gamma^\mu u(P,S) = 2P^\mu$) or linear in S^μ ($\bar{u}(P,S)\gamma^\mu\gamma_5 u(P,S) = 2S^\mu$). ([Jaf96b])

Combining together Dirac matrices and the three vectors defines a set of basic elements

$$\{\mathbb{1},\ \not{k},\ \not{P},\ \not{S}\gamma_5, \gamma_5(k\cdot S)\} \tag{6.96}$$

where we will treat the unit matrix separately, since multiplying any other matrix with it does not produce a new structure.

We note that only one pseudo-scalar product is listed in (6.96). As a matter of fact one should consider all possible scalar products which can be built from k, P and S. We will treat explicity the dependence on the scalar involving the spin vector S, while the pre-factors A_i, which multiply every structure, will depend on $k\cdot P$ and k^2.

The most general ansatz is obtained by writing down all possible products of elements of the above set which are in accordance with the rules and produce new structures. The maximum number of the products is limited by the observation that self-products do not result in new structures, since $\mathbb{1}^2 = \mathbb{1}$, $\not{k}^2 = k^2\,\mathbb{1}$, $\not{P}^2 = P^2\,\mathbb{1}$, $(\not{S}\gamma_5)^2 = -\mathbb{1}$ for pure states and $(\not{S}\gamma_5)^2 \propto \mathbb{1}$ for mixed states and $(\gamma_5(k\cdot S))^2$ is forbidden by requirement of linearity in S. Thus, the independent structures are obtained by products of 4 or less elements of the basic set.

According to this procedure the most general ansatz for the quark-quark correlation function in DIS contains the following structures (we order according to the number n of basic elements multiplied in a product)

- n=1:
$$\mathbb{1} \quad \not{k} \quad \not{P} \quad \not{S}\gamma_5 \quad \gamma_5(k\cdot S)$$
- n=2:
$$\not{k}\,\not{P} \quad \not{k}\,\not{S}\gamma_5 \quad \not{k}\gamma_5(k\cdot S) \quad \not{P}\,\not{S}\gamma_5 \quad \not{P}\gamma_5(k\cdot S)$$
- n=3:
$$\not{k}\,\not{P}\,\not{S}\gamma_5 \quad \not{k}\,\not{P}\gamma_5(k\cdot S)$$

(6.97)

The method results in producing 12 independent structures. All other products are forbidden by virtue of the parity constraint or by the requirement of linearity in S.

Not all of those structures fulfill the constraints we obtained implementing hermiticity and time-reversal transformations. We look for linear combinations, which obey the constraint from hermiticity and have a symmetry under time reversal. We use the identity

$$\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu} \tag{6.98}$$

which can be used to rewrite any structure $\not{a}\,\not{b}$ as

$$\not{a}\,\not{b} = a\cdot b - i\sigma^{\mu\nu}a_\mu b_\nu \tag{6.99}$$

from which we keep the $\sigma^{\mu\nu}a_\mu b_\nu$ term only, since the first one is proportional to $\mathbb{1}$. In addition we make use of a second identity

$$\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho = ie_{\sigma\mu\nu\rho}\gamma^\sigma + g^{\nu\rho}\gamma^5\gamma^\mu - g^{\mu\rho}\gamma^5\gamma^\nu + g^{\nu\rho}\gamma^5\gamma^\mu \tag{6.100}$$

which allows to re-express the structure $\gamma^5 \not{a} \not{b} \not{c}$ as

$$\gamma^5 \not{a} \not{b} \not{c} = i\epsilon_{\sigma\mu\nu\rho}\gamma^\sigma a^\mu b^\nu c^\rho + g^{\nu\rho} a_\nu b_\rho \gamma^5 \not{c} - g^{\mu\rho} b_\mu c_\rho \gamma^5 \not{a} + g^{\nu\rho} c_\mu a_\rho \gamma^5 \not{b} \tag{6.101}$$

from which we keep the $i\epsilon_{\sigma\mu\nu\rho}\gamma^\sigma a^\mu b^\nu c^\rho$ term only, since the other ones are either already taken into account.
These are the linear combinations looked for. Finally we end up with the ansatz

$$\begin{aligned} \Phi(k,P,S) &= m\,A_1\,\mathbb{1} + A_2\,\not{P} + A_3\,\not{k} + m\,A_6\,\not{S}\gamma_5 + (A_7/m)\,(k\cdot S)\,\not{P}\gamma_5 \\ &\quad + (A_8/m)\,(k\cdot S)\,\not{k}\gamma_5 + i\,A_9\,\sigma^{\mu\nu}\gamma_5\,S_\mu P_\nu \\ &\quad + i\,A_{10}\,\sigma^{\mu\nu}\gamma_5\,S_\mu k_\nu + i\,(A_{11}/m^2)\,(k\cdot S)\,\sigma^{\mu\nu}\gamma_5\,k_\mu P_\nu\,, \end{aligned} \tag{6.102}$$

where factors m and i have been added, such that all the amplitudes A_i are real and have the same mass dimension. If the constraint from time reversal invariance is not applicable there are the additional terms

$$\begin{aligned} \Phi(k,P,S) &= \ldots + (A_4/m)\,\sigma^{\mu\nu}P_\mu k_\nu + i\,A_5\,(k\cdot S)\gamma_5 \\ &\quad + (A_{12}/m)\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma\,. \end{aligned} \tag{6.103}$$

This ansatz has been extensively used by Mulders' group in Amsterdam to classify the forward PDFs and to derive relations between them. [JMR97b, JMR97a, BM98, MR01, Mul99, Mul97, MT96, Mula, BBHM00b]
By definition, the different parton distributions are obtained by tracing the forward quark-quark correlator (3.28) with different Dirac γ matrices (see (2.2)). If one traces the ansatz given in (6.102) and (6.103) with the various Dirac structures, one expresses the PDFs in terms of the amplitudes forming this ansatz and can thus set relations between them.

6.2 Alternative method to construct an ansatz

We introduce an alternative method that reproduces the ansatz given in (6.102) and in (6.103) for the forward quark-quark correlator in the spin basis (3.28) and is easily generalized to describe quark-quark correlators in an helicity basis.
With this alternative method the ansatz is written as a product of the partonic and the hadronic sector separately and this leaves us the freedom to choose special initial and final hadronic spin states. The hadronic sector is delineated by considering all possible independent spinorial products evaluated between initial and final hadronic spin states. In this way one can choose to build correlators between any initial or final spin state. We consider the case of hadronic spin states which are eigenstates of light-cone helicity.
From (3.28) it is clear that $\Phi_{i,j}(k,P,S)$ has to be a Lorentz scalar while it represents a $4\otimes 4$ matrix in Dirac partonic space; the most general ansatz is thus obtained through the following tensor product

$$\Phi_{i,j}(k,P,S) = \hat{\Gamma}^{\mu_1\cdots\mu_p}_{i,j} \otimes \;\; \bar{u}_\alpha(P,S)\,\Gamma_{\alpha\beta}{}^{\nu_1\cdots\nu_p}\,u_\beta(P,S)\;\; t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}(\,P,k\,)\,, \tag{6.104}$$

where $\hat{\Gamma}_{i,j}$ denote the 16 Fierz independent $4 \otimes 4$ partonic Dirac matrices and $\bar{u}_\alpha(P,S)\,\Gamma_{\alpha\beta} u_\beta(P,S)$ the independent hadronic spinor products; $t_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_p}(P,k)$ represent all possible independent tensors, constructed from the kinematical variables P, k, the metric tensor $g_{\alpha\beta}$ and the antisymmetric tensor $\epsilon_{\alpha\beta\rho\sigma}$, which have rank equal to the rank of the tensor formed by the partonic matrices and the spinorial products.
Moreover we require that the tensors $t_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_p}(P,k)$ contain at most one Levi-Civita symbol since tensors with more than one do not result in new structures. For instance let us consider the case of a tensor $t_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_p}(\ P,k\ )$ constructed from two Levi-Civita tensors. Not more than two (or less) indices of each Levi-Civita tensors can be contracted with the only two available vectors P and k. Thus some or all indices of the two Levi-Civita symbols have to be contracted between themselves; alternatively each antisymmetric tensor could have one or at most two indices contracted with a structure $\bar{u}(P,S)\,\sigma^{\alpha\beta}\,u(P,S)$ deriving from the hadronic sector or with a partonic $\sigma^{\mu\nu}$.
If the Levi-Civita tensors are contracted between themselves, the following relations reduces the product of two Levi-Civita tensors to the product of Kronecker symbols from which no new structure arise

$$\begin{aligned}
\epsilon_{\alpha\beta\mu\nu}\,\epsilon^{\alpha\beta\mu\nu} &= -4! \\
\epsilon_{\alpha\beta\mu\nu}\,\epsilon^{\omega\beta\mu\nu} &= -3!\delta^\omega_\alpha \qquad (6.105)\\
\epsilon_{\alpha\beta\mu\nu}\,\epsilon^{\omega\tau\mu\nu} &= -2!\delta^\omega_\alpha\delta^\tau_\beta + 2!\delta^\tau_\alpha\delta^\omega_\beta \\
\epsilon_{\alpha\beta\mu\nu}\,\epsilon^{\omega\tau\sigma\nu} = -\delta^\omega_\alpha\delta^\tau_\beta\delta^\sigma_\mu + \delta^\omega_\alpha\delta^\sigma_\beta\delta^\tau_\mu &+ \delta^\tau_\alpha\delta^\omega_\beta\delta^\sigma_\mu - \delta^\tau_\alpha\delta^\sigma_\beta\delta^\omega_\mu + \delta^\sigma_\alpha\delta^\omega_\beta\delta^\sigma_\mu + \delta^\sigma_\alpha\delta^\sigma_\beta\delta^\omega_\mu\,.
\end{aligned}$$

In case the Levi-Civita tensors are contracted with a σ matrix, from each contraction a $\gamma^5\sigma$ structure derives

$$\begin{aligned}
\sigma^{\mu\nu}\,\epsilon_{\alpha\beta\mu\rho} &= -i\gamma_5(\ \sigma_{\alpha\beta}\,\delta^\nu_\rho - \sigma_{\alpha\rho}\,\delta^\nu_\beta + \sigma_{\beta\rho}\,\delta^\nu_\alpha\ ) \\
\sigma^{\mu\nu}\,\epsilon_{\alpha\beta\mu\nu} &= -2i\gamma_5\sigma_{\alpha\beta}\,. \qquad (6.106)
\end{aligned}$$

If both the Levi-Civita symbols are contracted with a σ, then the two $\gamma^5\sigma$ reduce to σ structures which are already present in the ansatz. The remaining possibility, i.e. one Levi-Civita contracted with a σ and one with another Levi-Civita tensor, can be traced back to the two previously discussed.
Similarly the case of tensors $t_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_p}(\ P,k\ )$ with more than two Levi-Civita symbols can be excluded.
We remark also that in building the ansatz as indicated in (6.104) the kinematical variables k and P are taken into account in the tensors $t_{\mu_1 \cdots \mu_p \nu_1 \cdots \nu_p}(P,k)$, while the information concerning the spin of the hadron is entirely contained in the spinor products listed below

$$\bar{u}(P,S)\,u(P,S) = 2\,m$$

$$\bar{u}(P,S)\,\gamma^\alpha\,u(P,S) = 2\,P^\alpha$$

$$
\begin{aligned}
\bar{u}(P,S)\,\gamma_5\,u(P,S) &= P\cdot S = 0 \\
\bar{u}(P,S)\,\gamma^\alpha\gamma^5\,u(P,S) &= 2m\,S^\alpha \\
\bar{u}(P,S)\,\sigma^{\alpha\beta}\,u(P,S) &= -2\,\epsilon^{\alpha\beta\rho\sigma}P_\rho S_\sigma
\end{aligned}
\tag{6.107}
$$

Only two spinorial products are independent since applying Gordon identities the vector product can be obtained from the scalar one

$$
\bar{u}(P,S)\,u(P,S)\,P^\alpha = M\,\bar{u}(P,S)\,\gamma^\alpha\,u(P,S) \tag{6.108}
$$

and the axial spinor product from the tensor one when $\sigma^{\alpha\beta}$ is contracted with the vector P_β

$$
m\,\bar{u}(P,S)\,\gamma^\alpha\gamma_5\,u(P,S) = \bar{u}(P,S)\,i\,\sigma^{\alpha\beta}\,g_5 P_\beta u(P,S)\,. \tag{6.109}
$$

According to (6.104) we now write all possible products resulting from the multiplication of any 4×4 partonic Dirac matrix

$$
[\,\hat{\mathbb{1}} \qquad \hat{\gamma}^\mu \qquad \hat{\gamma}_5 \qquad \hat{\gamma^\mu}\hat{\gamma}_5 \qquad \sigma^{\hat{\mu}\nu}]\,, \tag{6.110}
$$

with the two independent spinorial products

$$
\bar{u}(P,S)\,u(P,S) \qquad \bar{u}(P,S)\,\sigma^{\alpha\beta}\,u(P,S) \tag{6.111}
$$

and contract the indices with tensors $t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}(\,P,k\,)$ of the appropriate rank to obtain a scalar

$$
\hat{\mathbb{1}}\otimes \quad \left\{ \begin{array}{rcl} \bar{u}(P,S)\,u(P,S) & \rightarrow & m a_1\,\mathbb{1} \\ \bar{u}(P,S)\sigma^{\alpha\beta}u(P,S)\epsilon_{\alpha\beta\rho\sigma}P^\rho k^\sigma & \rightarrow & 4m^2\,(k\cdot S) \end{array} \right.
$$

$$
\hat{\gamma}^5\otimes \quad \left\{ \begin{array}{rcl} \bar{u}(P,S)u(P,S) & \rightarrow & 2m\,\gamma^5 \\ \bar{u}(P,S)\sigma^{\alpha\beta}u(P,S)\epsilon_{\alpha\beta\rho\sigma}P^\rho k^\sigma & \rightarrow & 4m^2 a_5(k\cdot S)\gamma^5 \end{array} \right.
$$

$$
\hat{\gamma}^\mu\otimes \quad \bar{u}(P,S)u(P,S) \qquad \left\{ \begin{array}{rcl} P_\mu & \rightarrow & 2m\,a_2\,\not{P} \\ k_\mu & \rightarrow & 2m\,a_3\,\not{k} \end{array} \right.
$$

$$
\hat{\gamma}^\mu\otimes\bar{u}(P,S)\sigma^{\alpha\beta}u(P,S) \left\{ \begin{array}{rcl} g_{\mu\alpha}k_\beta & \rightarrow & 2\,a_{12}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma \\ \epsilon_{\mu\alpha\beta\rho}P^\rho & \rightarrow & -4m^2\,\not{S} \\ \epsilon_{\mu\alpha\beta\rho}k^\rho & \rightarrow & 4\,\not{P}(k\cdot S) - 4(P\cdot k)\,\not{S} \\ \epsilon_{\alpha\beta\rho\sigma}P^\rho k^\sigma P_\mu & \rightarrow & 4m^2\,\not{P}(k\cdot S) \\ \epsilon_{\alpha\beta\rho\sigma}P^\rho k^\sigma k_\mu & \rightarrow & 4m^2\,\not{k}(k\cdot S) \\ \epsilon_{\mu\alpha\rho\sigma}P^\rho k^\sigma k_\beta & \rightarrow & -2\,\not{P}(P\cdot k)(k\cdot S) + 2\,\not{k}(k\cdot S)m^2 \\ & \rightarrow & +2\,\not{S}(P\cdot k)^2 - 2\,\not{S}(k\cdot k)m^2 \end{array} \right.
$$

$$
\hat{\gamma}^\mu\hat{\gamma}^5\otimes \quad \bar{u}(P,S)u(P,S) \qquad \left\{ \begin{array}{rcl} P_\mu & \rightarrow & -2m\,\gamma^5\,\not{P} \\ k_\mu & \rightarrow & -2m\,\gamma^5\,\not{k} \end{array} \right.
$$

$$
\hat{\gamma}^{\mu}\hat{\gamma}^{5}\otimes\bar{u}(P,S)\sigma^{\alpha\beta}u(P,S)\left\{\begin{array}{rcl}
g_{\mu\alpha}k_{\beta} & \rightarrow & -2\,\gamma^{5}\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu}P^{\nu}k^{\rho}S^{\sigma}\\
\epsilon_{\mu\alpha\beta\rho}P^{\rho} & \rightarrow & 4m^{2}\,\gamma^{5}\,a_{6}\,\not{S}\\
\epsilon_{\mu\alpha\beta\rho}k^{\rho} & \rightarrow & -4a_{7}\,\not{P}\gamma^{5}(k\cdot S)+4a_{6}\,\not{S}\gamma^{5}(P\cdot k)\\
\epsilon_{\alpha\beta\rho\sigma}P^{\rho}k^{\sigma}P_{\mu} & \rightarrow & -4a_{7}m^{2}\,\gamma^{5}\,\not{P}(k\cdot S)\\
\epsilon_{\alpha\beta\rho\sigma}P^{\rho}k^{\sigma}k_{\mu} & \rightarrow & -4a_{8}m^{2}\,\gamma^{5}\,\not{k}(k\cdot S)\\
\epsilon_{\mu\alpha\rho\sigma}P^{\rho}k^{\sigma}k_{\beta} & \rightarrow & 2a_{7}\,\not{P}\gamma^{5}\,(P\cdot k)(k\cdot S)-\\
 & \rightarrow & 2a_{8}\,\not{k}\,\gamma^{5}\,(k\cdot S)m^{2}-2a_{6}\,\not{S}\gamma^{5}\,(P\cdot k)^{2}\\
 & \rightarrow & +2a_{6}\,\not{S}\gamma^{5}\,(k\cdot k)m^{2}
\end{array}\right.
$$

$$
\hat{\sigma}^{\mu\nu}\otimes\quad\bar{u}(P,S)u(P,S)\left\{\begin{array}{rcl}
P_{\mu}k_{\mu} & \rightarrow & 2m\,a_{4}\,\sigma^{\mu\nu}P_{\mu}k_{\nu}\\
\epsilon_{\mu\nu\rho\sigma}k^{\rho}P^{\sigma} & \rightarrow & -4mi\,\gamma^{5}\,\sigma^{\mu\nu}P_{\mu}k_{\nu}
\end{array}\right.
$$

$$
\hat{\sigma}^{\mu\nu}\otimes\bar{u}(P,S)\sigma^{\alpha\beta}u(P,S)\left\{\begin{array}{rcl}
g_{\mu\alpha}g_{\nu\beta} & \rightarrow & 4i\,a_{9}\,\sigma^{\mu\nu}\,P_{\mu}S_{\nu}\gamma_{5}\\
g_{\mu\alpha}P_{\nu}k_{\beta} & \rightarrow & 2i\,a_{9}\,\sigma^{\mu\nu}\,P_{\mu}S_{\nu}\gamma_{5}(k\cdot P)-\\
 & \rightarrow & 2i\,a_{10}\,\sigma^{\mu\nu}k_{\mu}S_{\nu}\gamma_{5}m^{2}\\
g_{\mu\alpha}k_{\nu}k_{\beta} & \rightarrow & 2ia_{9}\,\sigma^{\mu\nu}\gamma_{5}\,k_{\mu}P_{\nu}\,(k\cdot S)-\\
 & \rightarrow & 2ia_{10}\,\sigma^{\mu\nu}S_{\mu}P_{\nu}\gamma_{5}k.k+\\
 & \rightarrow & 2ia_{11}\,\sigma^{\mu\nu}S_{\mu}k_{\nu}\gamma_{5}(k\cdot P)\\
\epsilon_{\alpha\beta\rho\sigma}P_{\mu}k_{\nu}P^{\rho}k^{\sigma} & \rightarrow & 4m^{2}\,(k\cdot S)\sigma^{\mu\nu}P_{\mu}k_{\nu}\\
\epsilon_{\mu\nu\alpha\beta} & \rightarrow & 8\,\sigma^{\mu\nu}P_{\mu}S_{\nu}\\
g_{\mu\alpha}\epsilon_{\nu\beta\rho\sigma}P^{\rho}k^{\sigma} & \rightarrow & -2\sigma^{\mu\nu}P_{\mu}S_{\nu}(k\cdot P)+2m^{2}\sigma^{\mu\nu}k_{\mu}S_{\nu}\\
\epsilon_{\mu\alpha\beta\rho}P^{\rho}P_{\nu} & \rightarrow & 4m^{2}\,\sigma^{\mu\nu}k_{\mu}S_{\nu}\\
\epsilon_{\alpha\mu\beta\rho}P^{\rho}k_{\nu} & \rightarrow & 4m^{2}\,\sigma^{\mu\nu}S_{\mu}k_{\nu}+4\sigma^{\mu\nu}P_{\mu}S_{\nu}(k\cdot P)\\
\epsilon_{\mu\alpha\beta\rho}k^{\rho}P_{\nu} & \rightarrow & -4\,(k\cdot P)\,\sigma^{\mu\nu}S_{\mu}P_{\nu}\\
\epsilon_{\mu\alpha\beta\rho}k^{\rho}k_{\nu} & \rightarrow & 4\,\sigma^{\mu\nu}P_{\mu}k_{\nu}(k\cdot S)+4\,\sigma^{\mu\nu}k_{\mu}S_{\nu}(k\cdot P)\\
\epsilon_{\mu\nu\alpha\rho}P^{\rho}k_{\beta} & \rightarrow & 4\sigma^{\mu\nu}P_{\mu}S_{\nu}(k\cdot P)-4\sigma^{\mu\nu}k_{\mu}S_{\nu}m^{2}\\
\epsilon_{\mu\nu\alpha\rho}k^{\rho}k_{\beta} & \rightarrow & 4\sigma^{\mu\nu}k_{\mu}P_{\nu}(k\cdot S)+4\sigma^{\mu\nu}P_{\mu}S_{\nu}k.k-\\
 & \rightarrow & 4\sigma^{\mu\nu}k_{\mu}S_{\nu}(k\cdot P)
\end{array}\right.
\tag{6.112}
$$

The structures which in the previous list are not multiplied by a coefficient a_i (i $= 1,\ldots,12$), are forbidden by the parity constraint (4.44) reducing the list of independent structures to the 12 given in (6.102) and in (6.103). This guarantees that the two methods developed to construct the ansatz for the quark-quark correlators are completely equivalent. The advantage of the second method proposed is that it can be adapted to build the ansatz for the off-forward quark-quark correlators. Indeed in the off-forward case we will show that one needs to introduce the quark-quark correlators with respect to an helicity basis, rather that in the usual spin basis.

Starting from the forward case we present now the concept of quark-quark correlators in the helicity basis.

6.3 Forward quark-quark correlators in the helicity basis

The forward quark-quark correlator is defined in a spin basis as

$$\Phi_{ij}(k,P,S) = \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,k\cdot z}\;\langle P,S|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,S\rangle \tag{6.113}$$

where the target hadron, a spin $\frac{1}{2}$ target, characterized by the spin vector S^μ (see Appendix for the derivation)

$$S^\mu = \frac{\Lambda}{m}(P^\mu - \frac{m^2}{P^+}v'^\mu) + S_\perp^\mu\;, \tag{6.114}$$

can be either in a pure state or in an mixed state. Mixed states, likes beams or targets which are incoherent mixtures of spin states, are described by spin densities.
As Φ_{ij} can be either independent or linear dependent on S, in the target rest frame one can then expand it as [Jaf96b]

$$\Phi_{ij}(S) = \Phi_{ij_0} + \Phi_{ij_\Lambda}\Lambda + \Phi_{ij_{S^1}}S_\perp^1 + \Phi_{ij_{S^2}}S_\perp^2 \tag{6.115}$$

where the coefficients Φ_0, Φ_Λ, Φ_{S^1} and Φ_{S^2}, independent of the spin vector, multiply the components of S^μ, that in the rest frame is given as $S = (0, S_\perp^1, S_\perp^2, \Lambda)$, where Λ coincides with the third component of the spin S^3. We remark that this decomposition is valid only in the rest-frame of the particle.
We want now to transform the quark-quark correlator Φ from the spin basis to the helicity basis. We introduce a spin density matrix ρ, built up, for instance, from eigenstates of the light-cone helicity $\mid \Lambda >$

$$\rho = \sum_\Lambda p_\Lambda \mid \Lambda > < \Lambda \mid\,. \tag{6.116}$$

$$\rho_{\Lambda\Lambda'}(S) = \frac{1}{2m}\bar{u}(P,\Lambda')\,\frac{1}{2}(\mathbb{1} + \gamma_5\gamma_\mu S^\mu)\,u(P,\Lambda)\;. \tag{6.117}$$

Assuming that we consider only protons and not anti-protons and thus the two spinors $u(P,\Lambda)$ and $u(P,\Lambda')$ are the positive energy light-cone helicity eigenstates in the particle rest frame which in the standard representation can be written as [BD65]

$$\begin{aligned} u_+(P,\Lambda) &= \begin{pmatrix} \sqrt{2m} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \mathrm{f}or \quad \mathrm{s}pin \quad \frac{1}{2} \\ u_-(P,\Lambda) &= \begin{pmatrix} 0 \\ \sqrt{2m} \\ 0 \\ 0 \end{pmatrix} \quad \mathrm{f}or \quad \mathrm{s}pin \quad -\frac{1}{2}\,, \end{aligned} \tag{6.118}$$

one can obtain the exact expression of the density matrix $\rho_{\Lambda\Lambda'}$ that reads

$$\begin{aligned} \rho_{++}(S) &= 1/2\,(S^3+1) \\ \rho_{+-}(S) &= -1/2\,(-S^1+i\,S^2) \\ \rho_{-+}(S) &= 1/2\,(S^1+i\,S^2) \\ \rho_{++}(S) &= -1/2\,(S^3-1) \end{aligned} \tag{6.119}$$

By considering $\rho_{\Lambda\Lambda'}$

$$\rho_{\Lambda\Lambda'} = \frac{1}{2}(\mathbb{1} + \vec{\sigma}\cdot\vec{S})_{\Lambda\Lambda'} = \frac{1}{2}\begin{bmatrix} 1+\Lambda & S_\perp^1 - iS_\perp^2 \\ S_\perp^1 + iS_\perp^2 & 1-\Lambda \end{bmatrix}_{\Lambda\Lambda'} \tag{6.120}$$

we can then express the quark-quark correlation function as

$$\begin{aligned} \Phi_{ij}(S) &= \mathrm{T}r\,[\,\rho_{\Lambda\Lambda'}(S)\Phi_{\Lambda' i;\,\Lambda j}\,] = \sum_{\Lambda\Lambda'=\pm 1} \rho_{\Lambda\Lambda'}(S)\Phi_{\Lambda' i;\,\Lambda j} = \\ &= \frac{1}{2}(\Phi_{+i;\,+j} + \Phi_{-i;\,-j}) + \frac{1}{2}\Lambda\,(\Phi_{+i;\,+j} - \Phi_{-i;\,-j}) \\ &+ \frac{1}{2}S_\perp^1(\Phi_{+i;\,-j} + \Phi_{-i;\,+j}) - \frac{i}{2}S_\perp^2(\Phi_{+i;\,-j} - \Phi_{-i;\,+j})\,. \end{aligned} \tag{6.121}$$

$\Phi_{ij}(S)$ is replaced by a set of correlators $\Phi_{\Lambda' i;\,\Lambda j}$ evaluated between eigenstates of the hadron light-cone helicity in the rest frame of the hadron (see the definition of the hadron helicity eigenstates in the Appendix)

$$\Phi_{i\Lambda',j\Lambda} = \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,k\cdot z}\;\langle P,\Lambda'|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,\Lambda\rangle\,. \tag{6.122}$$

Since the indices Λ and Λ' in (6.122) can take values $+1$ or -1, the four quark-quark correlators form a 2×2 matrix in the hadron helicity space is

$$\Phi_{i\Lambda',j\Lambda} = \begin{bmatrix} \Phi_{i+,j+} & \Phi_{i-,j+} \\ \Phi_{i+,j-} & \Phi_{i-,j-} \end{bmatrix}, \tag{6.123}$$

where initial and final hadron helicities are equal for the diagonal components of the matrix, $\Phi_{i+,j+}$ and $\Phi_{i-,j-}$, while they are opposite for non-diagonal ones, $\Phi_{i+,j-}$ and $\Phi_{i-,j+}$.
The information contained in the quark-quark correlator (6.113), that explicity depends on the spin vector S, is now given by four quark-quark correlation functions describing all possible transitions between eigenstates of helicity, as helicity eigenstates form a complete basis.
Comparing (6.115) with (4.76) one can establish the following relations between the description of the quark-quark correlation function in a spin basis and the one in the helicity

formalism

$$\begin{aligned}
\Phi_{i+,j+} &= \Phi_{ij\,0} + \Phi_{ij\,\Lambda} \\
\Phi_{i-,j-} &= \Phi_{ij\,0} - \Phi_{ij\,\Lambda} \\
\Phi_{i+,j-} &= \Phi_{ij\,S^1} - i\Phi_{ij\,S^2} \\
\Phi_{i-,j+} &= \Phi_{ij\,S^1} + i\Phi_{ij\,S^2}
\end{aligned} \tag{6.124}$$

From now on we do not display the indices i, j. Furthermore, starting from the ansatz in (6.102) and (6.103) for the quark-quark correlator in the spin basis, it is possible to deduce how the four ansätze for the four correlators $\Phi_{i\Lambda',j\Lambda}$ look like in the rest frame of the hadron. We stress that the explicit expression of the four ansätze $\Phi_{i\Lambda',j\Lambda}$ are valid only in the rest frame since the linear decomposition in (6.115) was deduced according to the expression of the spin vector in the proton rest frame in (6.114). The correlators $\Phi_{\Lambda'\,\Lambda}$, which will be obtained, in the helicity basis are therefore frame-dependent.
The term Φ_0 of the correlator is given by

$$\Phi_0(k,P) = mA_1 + \gamma_\mu P^\mu A_2 + \gamma_\mu k^\mu A_3 + \sigma_{\mu\nu} P^\mu k^\nu A_4 \ , \tag{6.125}$$

The part of the quark-quark correlator, that appears in (6.115) multiplied by the third component of the spin $S^3 = \Lambda$, is

$$\begin{aligned}
\Phi_\Lambda(k,P) &= iA_5\gamma_5 k_3 + m\gamma_5 A_6\gamma_3 + \frac{A_7}{m}\gamma_\mu P^\mu\gamma_5 k_3 + \frac{A_8}{m}\gamma_\mu k^\mu \gamma_5 k_3 \\
&\quad \frac{A_9}{m}\sigma_{\mu 3}P^\mu\gamma_5 + \frac{A_{10}}{m}\sigma_{\mu 3}k^\mu\gamma_5 + \frac{A_{11}}{m} i\sigma_{\mu\nu}P^\mu k^\nu\gamma_5 k_3 \\
&\quad + \frac{A_{12}}{m}\epsilon_{\mu\nu\rho 3}\gamma^\mu P^\nu k^\rho \ .
\end{aligned} \tag{6.126}$$

The term Φ_{S^1} of the helicity correlators is

$$\begin{aligned}
\Phi_{S^1}(k,P) &= iA_5\gamma_5 k_1 + m\gamma_5 A_6\gamma_1 + \frac{A_7}{m}\gamma_\mu P^\mu\gamma_5 k_1 + \frac{A_8}{m}\gamma_\mu k^\mu \gamma_5 k_1 \\
&\quad \frac{A_9}{m}\sigma_{\mu 1}P^\mu\gamma_5 + \frac{A_{10}}{m}\sigma_{\mu 1}k^\mu\gamma_5 + \frac{A_{11}}{m} i\sigma_{\mu\nu}P^\mu k^\nu\gamma_5 k_1 \\
&\quad + \frac{A_{12}}{m}\epsilon_{\mu\nu\rho 1}\gamma^\mu P^\nu k^\rho \ .
\end{aligned} \tag{6.127}$$

The analogous $\Phi_{S^2}(k,P)$ can be obtained substituting the transverse direction 2 wherever 1 appears.
Note also that the correlators Φ_Λ, Φ_{S^1} and Φ_{S^2} contain only structures multiplied by the γ_5 matrix. In fact under parity transformation the spin quark-quark correlator $\Phi_{ij}(S)$ is parity even and the spin vector S^μ, whose components multiply the correlators Φ_Λ, Φ_{S^1} and Φ_{S^2}, is, by definition, an axial vector.
The matrix $\Phi_{\Lambda',\Lambda}$ being a unitary ($A^\dagger = A^{-1}$) 2×2 matrix with determinant 1, it belongs to the 2-dimensional representation of the $SU(2)$ group of three-dimensional rotation,

whose infinitesimal generators are represented by the Pauli matrices

$$\Phi_{\Lambda'\Lambda} = \Phi_0\, \mathbb{1}_{\Lambda'\Lambda} + \Phi_\Lambda\, \sigma^3_{\Lambda'\Lambda} + \Phi_1\, \sigma^1_{\Lambda'\Lambda} + \Phi_2\, \sigma^2_{\Lambda'\Lambda} = \begin{bmatrix} \Phi_0 + \Phi_\Lambda & \Phi_1 - i\Phi_2 \\ \Phi_1 + i\Phi_2 & \Phi_0 - \Phi_\Lambda \end{bmatrix} \quad (6.128)$$

The components of this matrix have not definite transformation properties under rotations in 3 dimensions.
The diagonal entries are given by the linear combination of a scalar and the third component of a vector, while the off-diagonal components are given by a linear combination of the two transverse components of a vector.
The diagonal components of the matrix $\Phi_{i\Lambda',j\Lambda}$ can thus be written as

$$\begin{aligned} \Phi_{++}(k,P) \;=\; & mA_1 + \gamma_\mu P^\mu A_2 + \gamma_\mu k^\mu A_3 + \sigma_{\mu\nu} P^\mu k^\nu A_4 + iA_5\gamma_5 k_3 + \\ & m\gamma_5 A_6 \gamma_3 + \frac{A_7}{m}\gamma_\mu P^\mu \gamma_5 k_3 + \frac{A_8}{m}\gamma_\mu k^\mu \gamma_5 k_3 + \frac{A_9}{m}\sigma_{\mu 3} P^\mu \gamma_5 \\ & + \frac{A_{10}}{m}\sigma_{\mu 3} k^\mu \gamma_5 + \frac{A_{11}}{m} i\sigma_{\mu\nu} P^\mu k^\nu \gamma_5 k_3 + \frac{A_{12}}{m}\epsilon_{\mu\nu\rho 3}\gamma^\mu P^\nu k^\rho \ , \end{aligned} \quad (6.129)$$

while the off-diagonal components are

$$\begin{aligned} \Phi_{+-}(k,P) \;=\; & iA_5\gamma_5 (\, k_1 - ik_2\,) + m\gamma_5 A_6 (\, \gamma_1 - i\gamma_2\,) + \frac{A_7}{m}\gamma_\mu P^\mu \gamma_5\, (\, k_1 - ik_2\,) \\ & + \frac{A_8}{m}\gamma_\mu k^\mu \gamma_5\, (\, k_1 - ik_2\,) + \frac{A_9}{m}(\, \sigma_{\mu 1} - i\sigma_{\mu 2}\,)\, P^\mu \gamma_5 + \\ & \frac{A_{10}}{m}(\, \sigma_{\mu 1} - i\sigma_{\mu 2}\,) k^\mu \gamma_5 + \frac{A_{11}}{m} i\sigma_{\mu\nu} P^\mu k^\nu \gamma_5 (\, k_1 - ik_2\,) + \\ & \frac{A_{12}}{m}(\, \epsilon_{\mu\nu\rho 1} - i\epsilon_{\mu\nu\rho 2}\,)\gamma^\mu P^\nu k^\rho \ . \end{aligned} \quad (6.130)$$

While the explicit form of the helicity correlators depend on the specific frame of reference one chooses for the definition of the spin, the fact that the diagonal components, Φ_{++} and Φ_{--}, transform like a linear combination of a scalar and the third component of a vector and the non-diagonal entries, Φ_{+-} and Φ_{-+}, like a linear combination of the transverse components of a vector under Lorentz transformations does not depend on the particular frame of reference but it is instead a general property of the quark-quark correlators in the helicity basis.
The reason for this behavior lies in the choice of quantization axis. When one defines helicity hadron states as spinors whose spin can be aligned or anti-aligned with the particle four-momentum P^μ and fixes the vector P^μ to be along the z axis one performs also a specific choice of the quantization axis. As a consequence the operator of the spin along the z axis, the Pauli matrix σ^3, is diagonal while the operators of the spin along x and y, the Pauli matrices σ^1 and σ^2, respectively, contain non-zero off-diagonal components and they are then not-diagonalizable in the specific helicity basis chosen.

The choice of quantization axis fixes thus the Pauli matrices and consequently the definition of the spin density matrix used to transform the spin quark-quark correlator to the helicity basis. In (6.120) the operator of the spin along the z axis, σ^3, is diagonal and the operators of the spin along the x and y axis are not diagonal. A priori one could have chosen a basis where the spin operator along the x or y axis are diagonal, obtaining in this case a different set of relations (6.125).

In passing we note that, as already remarked, DIS does not allow for flipping of the hadron helicity and the off-diagonal components cannot be accessed in a DIS process unless the target particle has a transverse component of the spin and in the process there is a chiral-odd partner for the chiral-odd off-diagonal correlator, like for instance in semi-inclusive DIS. When writing the quark-quark correlator in the spin basis as in (6.115), the off-diagonal correlators appear multiplied by the transverse components of the spin vector $S_\perp^\mu$ and they do not contribute in case the target particle is in an eigenstate of helicity. Nevertheless even if in a particular process some informations about internal hadronic structure cannot be accessed, we stress that the quark-quark correlators must store up the complete knowledge about the hadron.

For further illustration we consider again the probabilistic interpretation of quark-quark correlator which states that the soft part of the DIS cross section $d\sigma$ can be parameterized by the matrix elements of the quark-quark correlator

$$d\sigma \propto \sum_X < P, S \mid \psi_+(0) \mid X > < X \mid {\psi_+}^\dagger(0) \mid P, S > \delta(P_X^+ - (1-x)\, P^+) \quad (6.131)$$

If in (6.131) we introduce the spin density matrix ρ,

$$\rho_{\Lambda'\Lambda} = < \Lambda' \mid S >< S \mid \Lambda > , \quad (6.132)$$

we can re-write it as

$$\begin{aligned} d\sigma &\propto \sum_X \sum_{\Lambda',\Lambda} < S \mid \Lambda' >< \Lambda' \mid \psi_+(0) \mid X >< X \mid {\psi_+}^\dagger(0) \mid \Lambda > \\ &\quad < \Lambda \mid S > \delta(P_X^+ - (1-x)\, P^+) \\ &\propto \sum_{\Lambda',\Lambda} < S \mid \Lambda' >< \Lambda' \mid \psi_+(0)\, {\psi_+}^\dagger(0) \mid \Lambda >< \Lambda \mid S > \delta(P_X^+ - (1-x)\, P^+) \\ &\propto \sum_{\Lambda',\Lambda} \rho_{\Lambda'\Lambda} \Phi_{\Lambda'\Lambda} \\ &\propto Tr[\, \rho_{\Lambda'\Lambda} \Phi_{\Lambda'\Lambda} \,]. \end{aligned} \quad (6.133)$$

The spin density matrix formalism links the quark-quark correlators $\Phi_{i\Lambda',j\Lambda}$, which represent transitions amplitudes between eigenstates of helicity, with experimental measured cross sections, expressed in terms of the target spin, where people have usually to handle with ensembles of large number of identical particles, which represent impure states.

This reasoning was applied to our objects of interest, the quark-quark correlators, in order to transform them from the spin basis to the helicity basis. Through the density matrix

formalism one shows that it provides the same kind of information about internal hadronic structure to describe it either using explicitly the spin vector dependence in the spin quark-quark correlator $\Phi_{i,j}(P,k,S)$ or alternatively determining the set of helicity correlators $\Phi_{i\Lambda',j\Lambda}$.
The forward quark-quark spin correlator, obtained as a trace of the helicity correlators matrix, which depend on the frame, and of the spin density matrix. The forward quark-quark spin correlator is thus an average of the helicity correlators weighted by the spin density matrix. We constructed the spin density matrix from the light-cone helicity eigenstates in the rest frame of the hadron but a different choice of the basis, in which one builds the spin density matrix, would have not changed the forward quark-quark spin correlator.

6.4 Ansätze for forward quark-quark correlators in helicity basis

The alternative method developed to build the ansatz for the quark-quark correlator in the spin basis can be quite easily exported to determine the ansätze for the quark-quark correlators $\Phi_{i\Lambda',j\Lambda}(k,P)$ in the light-cone helicity basis

$$\Phi_{i\Lambda',j\Lambda}(k,P) = \frac{1}{(2\pi)^4}\int d^4z\; e^{i\,k\cdot z}\;\langle P,\Lambda'|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,\Lambda\rangle\;. \tag{6.134}$$

The correlators $\Phi_{i\Lambda',j\Lambda}(k,P)$ are 4×4 partonic Dirac matrices $\hat{\Gamma}_{i,j}$. In addition the helicity correlators represent 2×2 matrices in the nucleon helicity space, labeled by Λ' and Λ. We define then the ansatz for the correlators (6.134) from the tensor product of

- the set of independent partonic Fierz structures

$$\{\;\hat{\mathbb{1}}\qquad\hat{\gamma}_5\qquad\hat{\gamma}^{\mu}\qquad\hat{\gamma}^{\mu}\hat{\gamma}_5\qquad\hat{\sigma}^{\mu\nu}\;\} \tag{6.135}$$

- the independent hadronic spinor products which provide the information about the spin degrees of freedom

$$\{\;u(P,\Lambda')\,u(P,\Lambda)\qquad u(P,\Lambda')\,\sigma^{\alpha\beta}\,u(P,\Lambda)\;\} \tag{6.136}$$

- a set of tensors which contain information about the kinematical variables k and P .

Altogether the ansatz reads

$$\Phi_{i\Lambda,j\Lambda}(k,P) = \hat{\Gamma}^{\mu_1\cdots\mu_p}_{i,j}\quad\otimes\quad\bar{u}_{\alpha}(P,\Lambda)\,\Gamma_{\alpha\beta}{}^{\nu_1\cdots\nu_p}\,u_{\beta}(P,\Lambda)\quad t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}(P,k)\;. \tag{6.137}$$

Since in building the ansatz as in (6.137) the spin dependence is kept explicitely in the spinorial products, one has the possibility to evaluate the matrix elements between any hadronic spin states, in particular between eigenstates of LC helicity by calculating different spinorial products. It is clear that the definition of the ansatz requires being able to write down the hadronic spinor products, which are computable with two different methods (refer to the Chapter ”‘Choice of spinors and evaluation of spinorial products”’):

- via a trace method, involving the calculation of traces only
- via an evaluation from explicit representation of spinors by which one obtains each spinor product separately component by component.

Let us regard the case of hadron helicity non-flip and the case of helicity flip separately.

6.4.1 Ansätze for the helicity non-flip case

In the helicity non-flip case one simply specializes the expression for the spinorial products given in (6.107) to the case of a spin vector corresponding to an eigenstate of LC helicity

$$S^\mu = \frac{\Lambda}{m}(P^\mu - \frac{m^2}{P^+}v'^\mu) \tag{6.138}$$

and the two independent spinorial currents become

$$\begin{aligned} \bar{u}(P,\Lambda)\, u(P,\Lambda) &= 2\,M \\ \bar{u}(P,\Lambda)\, \sigma^{\alpha\beta}\, u(P,\Lambda) &= -2\epsilon^{\alpha\beta\rho\sigma}\, P_\rho S_\sigma \end{aligned} \tag{6.139}$$

Structure by structure the contributions to the ansatz are

$$\hat{\mathbb{1}} \otimes \quad \bar{u}(P,\Lambda)u(P,\Lambda) \to a_1^{(k)} m\, \mathbb{1}$$

$$\hat{\mathbb{1}} \otimes \quad \bar{u}(P,\Lambda)\sigma^{\alpha\beta}u(P,\Lambda) \to a_2^{(\kappa)}\,(k\cdot S)$$

$$\hat{\gamma}^5 \otimes \quad \bar{u}(P,\Lambda)u(P,\Lambda) \to a_3^{(\kappa)}\,\gamma^5\, m$$

$$\hat{\gamma}^5 \otimes \quad \bar{u}(P,\Lambda)\sigma^{\alpha ta}u(P,\Lambda) \to a_4^{(\kappa)}\,\gamma^5\,(k\cdot S)$$

$$\hat{\gamma}^\mu \otimes \quad \bar{u}(P,\Lambda)u(P,\Lambda) \begin{cases} \to & a_5^{(\kappa)}\,\gamma_\mu\, P^\mu \\ \to & a_6^{(\kappa)}\,\gamma_\mu\, k^\mu \end{cases}$$

$$\hat{\gamma}^\mu \otimes\ \bar{u}(P,\Lambda)\sigma^{\alpha\beta}u(P,\Lambda) \begin{cases} \to & a_7^{(\kappa)}/m\,\gamma^\mu\,\epsilon_{\alpha\beta\mu\rho}S^\alpha k^\beta P^\rho \\ \to & a_8^{(\kappa)}\,\gamma_\mu\, S^\mu\, m \\ \to & a_9^{(\kappa)}\,\gamma_\mu\, P^\mu\,(k\cdot S)/m \\ \to & a_{10}^{(\kappa)}\,\gamma_\mu\, k^\mu\,(k\cdot S)/m \end{cases}$$

$$\hat{\gamma}^\mu\hat{\gamma}^5 \otimes \quad \bar{u}(P,\Lambda)u(P,\Lambda) \begin{cases} \to & a_{11}^{(\kappa)}\,\gamma^5\gamma_\mu P^\mu \\ \to & a_{12}^{(\kappa)}\,\gamma^5\gamma_\mu k^\mu \end{cases}$$

$$\hat{\gamma}^\mu\hat{\gamma}^5 \otimes\ \bar{u}(P,\Lambda)\sigma^{\alpha\beta}u(P,\Lambda) \begin{cases} \to & a_{13}^{(\kappa)}/m\,\gamma^\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\rho}S^\alpha k^\beta P^\rho \\ \to & a_{14}^{(\kappa)}\,\gamma_\mu\,\gamma^5\, S^\mu\, m \\ \to & a_{15}^{(\kappa)}\,\gamma_\mu\,\gamma^5\, P^\mu\,(k\cdot S)/m \\ \to & 4a_{16}^{(\kappa)}\,\gamma_\mu\,\gamma^5\, k^\mu\,(k\cdot S)/m \end{cases}$$

$$\hat{\sigma}^{\mu\nu} \otimes \quad \bar{u}(P,\Lambda)u(P,\Lambda) \begin{cases} \rightarrow & a_{17}^{(\kappa)}/m\,\sigma_{\mu\nu}P^{\mu}k^{\nu} \\ \rightarrow & a_{18}^{(\kappa)}/m\,\gamma^5\,\sigma_{\mu\nu}P^{\mu}k^{\nu} \end{cases}$$

$$\hat{\sigma}^{\mu\nu} \otimes \bar{u}(P,\Lambda)\sigma^{\alpha\beta}u(P,\Lambda) \begin{cases} \rightarrow & a_{19}^{(\kappa)}\,\sigma_{\mu\nu}P^{\mu}S^{\nu}\,\gamma^5 \\ \rightarrow & a_{20}^{(\kappa)}/m^2\,\sigma_{\mu\nu}P^{\mu}k^{\nu}\,\gamma^5\,(k\cdot S) \\ \rightarrow & a_{21}^{(\kappa)}\,\sigma_{\mu\nu}P^{\mu}S^{\nu} \\ \rightarrow & a_{22}^{(\kappa)}/m^2\,\sigma_{\mu\nu}P^{\mu}k^{\nu}\,(k\cdot S) \\ \rightarrow & a_{23}^{(\kappa)}\,\sigma_{\mu\nu}k^{\mu}S^{\nu}\,\gamma^5 \\ \rightarrow & a_{24}^{(\kappa)}\,\sigma_{\mu\nu}k^{\mu}S^{\nu} \end{cases} \tag{6.140}$$

We finally obtain the following ansatz for the diagonal correlators $\Phi_{+i\,;\,+j}$ and $\Phi_{-i\,;\,-j}$ with $\kappa = 1,4$

$$\begin{aligned} \Phi_{\Lambda\Lambda}(k,P,S) = {} & a_1^{(\kappa)} m\,\mathbb{1} + a_2^{(\kappa)}\,(k\cdot S) \,+\, i a_3^{(\kappa)}\,\gamma^5\,m \,+\, i a_4^{(\kappa)}\,\gamma^5\,(k\cdot S) + \\ & a_5^{(\kappa)}\,\gamma_\mu\,P^\mu \,+\, a_6^{(\kappa)}\,\gamma_\mu\,k^\mu \,+\, a_7^{(\kappa)}/m\,\gamma^\mu\,\epsilon_{\alpha\beta\mu\rho}S^\alpha k^\beta P^\rho \,+\, a_8^{(\kappa)}\,\gamma_\mu\,S^\mu\,m \\ & + a_9^{(\kappa)}\,\gamma_\mu\,P^\mu\,(k\cdot S)/m \,+\, a_{10}^{(\kappa)}\,\gamma_\mu\,k^\mu\,(k\cdot S)/m \,+ \\ & a_{11}^{(\kappa)}\,\gamma^5\gamma_\mu P^\mu \,+\, a_{12}^{(\kappa)}\,\gamma^5\gamma_\mu k^\mu \,+\, a_{13}^{(\kappa)}/m\,\gamma^\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\rho}S^\alpha k^\beta P^\rho \\ & + a_{14}^{(\kappa)}\,\gamma_\mu\,\gamma^5\,S^\mu\,m \,+\, a_{15}^{(\kappa)}\,\gamma_\mu\,\gamma^5\,P^\mu\,(k\cdot S)/m \\ + \quad & a_{16}^{(\kappa)}\,\gamma_\mu\,\gamma^5\,k^\mu\,(k\cdot S)/m \,+\, a_{17}^{(\kappa)}/m\,\sigma_{\mu\nu}P^\mu k^\nu \,+\, i a_{18}^{(\kappa)}/m\,\gamma^5\,\sigma_{\mu\nu}P^\mu k^\nu \,+ \\ & a_{19}^{(\kappa)}\,\sigma_{\mu\nu}P^\mu S^\nu\,\gamma^5 \,+\, a_{20}^{(\kappa)}/m^2\,\sigma_{\mu\nu}P^\mu k^\nu\,\gamma^5\,(k\cdot S) \,+\, a_{21}^{(\kappa)}\,\sigma_{\mu\nu}P^\mu S^\nu \\ & + a_{22}^{(\kappa)}/m^2\,\sigma_{\mu\nu}P^\mu k^\nu\,(k\cdot S) \,+\, a_{23}^{(\kappa)}\,\sigma_{\mu\nu}k^\mu S^\nu\,\gamma^5 \,+\, a_{24}^{(\kappa)}\,\sigma_{\mu\nu}k^\mu S^\nu\,. \end{aligned} \tag{6.141}$$

where the amplitudes $a_i^{(\kappa)}$ depend on the invariants $k\cdot P$ and k^2.
From now on we will indicate the four different helicity correlators $\Phi^{(\kappa)}$ with $\kappa = 1,2,3,4$ as

$$\begin{aligned} \Phi_{ij_{++}}(\bar{k},\bar{P},\Delta) &= \Phi_{ij}^{(1)}(\bar{k},\bar{P},\Delta) & \qquad \Phi_{ij_{+-}}(\bar{k},\bar{P},\Delta) &= \Phi_{ij}^{(2)}(\bar{k},\bar{P},\Delta) \\ \Phi_{ij_{-+}}(\bar{k},\bar{P},\Delta) &= \Phi_{ij}^{(3)}(\bar{k},\bar{P},\Delta) & \qquad \Phi_{ij_{--}}(\bar{k},\bar{P},\Delta) &= \Phi_{ij}^{(4)}(\bar{k},\bar{P},\Delta) \end{aligned} \tag{6.142}$$

We stress that in the quark-quark correlators (6.141) the dependence on the spin vector S is, in reality, a dependence on the vector v', which enters in the definition of the spin vector for the helicity state

$$S^\mu = \frac{\Lambda}{m}(P^\mu - \frac{m^2}{P^+}v'^\mu)\,. \tag{6.143}$$

Instead of the 2×24 independent amplitudes in (6.141) applying parity, hermiticity and time reversal reduces the number of independent amplitudes to the following set (see the Appendix for the explicit proof)

$$\begin{array}{rcllll} a^1_m &=& a^4_m & \mathrm{r}eal & m = 1,5,6,14,15,16 \\ a^1_m &=& -a^4_m & \mathrm{r}eal & m = 2,8,9,10,11,12 \\ a^1_m &=& a^4_m & \mathrm{i}maginary & m = 4,19,20,23 \\ a^1_m &=& -a^4_m & \mathrm{i}maginary & m = 3,18 \end{array} \tag{6.144}$$

If we want now to compare the diagonal helicity correlators in (6.141) with the one in (6.112), we have to remark that the helicity correlators as obtained in (6.112) are valid in the rest frame of the proton where the spin vector in usual coordinates equals $(0,0,0,\Lambda)$. Further assuming $\Lambda = 1$ for the correlator $\Phi_{+,+}$, we can conclude that through a renaming of the amplitudes a_i the correlators in (6.112) and (6.141) are equivalent.

6.4.2 Ansätze for the helicity flip case

In the helicity flip case it turns out that in order to calculate each spinor products via the trace method one has always to divide by the density $\bar{u}u$, which is obviously equal to zero. The density $\bar{u}u$ corresponds to the probability to find a spinor, carrying for instance helicity $-$, which spontaneously transforms to a spinor having helicity $+$ and this probability is clearly zero. By the way with a direct evaluation one succeeds in finding a finite solution for the spinor products; only one cannot cast the different components of each spinorial product in the form of a unique tensorial expression. For instance let us regard the case of the two independent forward spinorial currents $\bar{u}(P,\Lambda')\ u(P,\Lambda)$ and $\bar{u}(P,\Lambda')\ \sigma^{\alpha\beta}\ u(P,\Lambda)$ with $\Lambda' = -1$ and $\Lambda = 1$

$$\begin{array}{rcl} \bar{u}(P,-)\ u(P,+) &=& 0 \\ \bar{u}(P,-)\ \sigma^{+1}\ u(P,+) &=& i2P^+ \\ \bar{u}(P,-)\ \sigma^{+2}\ u(P,+) &=& -2P^+ \\ \bar{u}(P,-)\ \sigma^{-i}\ u(P,+) &=& 0 \\ \bar{u}(P,-)\ \sigma^{12}\ u(P,+) &=& 0 \\ \bar{u}(P,-)\ \sigma^{+-}\ u(P,+) &=& 0\,. \end{array} \tag{6.145}$$

That the different components of each spinor product cannot be written in terms of a unique tensor is due to the fact that the spinors $\bar{u}(P,-)$ and $u(P,+)$ carry also information about their phase and in general these phases are different.

When calculating spinor products between states of equal spin or helicity the phase contributions from each spinor cancel together and the net result is that the Lorentz behavior is given directly by the Lorentz behavior of the Γ structure in the spinor product ($\mathbb{1}$, γ^5, γ^μ, ect). The diagonal components in (6.103) do not carry explicity this phase information only because the phases of $\bar{u}$ and u cancel, being these two phases equal and the phase information for the diagonal spinorial products and therefore also for the correlators is

hidden. If the helicity of the initial and final spinor are different, then the result for the spinor product contains also the phase information carried by each spinor which do not cancel with each other.
In no way the results for the spinorial currents, obtained component by component, can be written in terms of a 4-dimensional Lorentz tensor and one needs introduce an auxiliary vector a defined as $a = [0, 0, \Lambda, i]$ such that

$$\begin{aligned} \bar{u}(P,\Lambda')\, u(P,\Lambda) &= 2m\,\delta_{\Lambda',\Lambda} = 0 \\ \bar{u}(P,\Lambda')\,\sigma^{\alpha\beta}u(P,\Lambda) &= i\Lambda\, 2m\,(S^\alpha\, a^\beta - S^\beta\, a^\alpha)\,\delta_{-\Lambda',\Lambda}\,, \end{aligned} \tag{6.146}$$

where

$$S^\alpha = \frac{\Lambda}{m}\,(P^\alpha - \frac{m^2}{P^+}v'^\alpha). \tag{6.147}$$

The off diagonal correlators $\Phi_{+i\,;\,-j}$ and $\Phi_{-i\,;\,+j}$ with $\kappa = 2, 3$ are

$$\hat{\mathbb{1}} \otimes\ \bar{u}(P,\Lambda')u(P,\Lambda) = 0$$

$$\hat{\mathbb{1}} \otimes \qquad \bar{u}(P,\Lambda')\sigma^{\alpha\beta}u(P,\Lambda) \rightarrow \frac{a_1^{(\kappa)}}{m}\ \epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda$$

$$\hat{\gamma}^5 \otimes\ \bar{u}(P,\Lambda')u(P,\Lambda) = 0$$

$$\hat{\gamma}^5 \otimes \qquad \bar{u}(P,\Lambda')\sigma^{\alpha\beta}u(P,\Lambda) \longrightarrow \frac{a_2^{(\kappa)}}{m}\,\gamma^5\,\epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda$$

$$\hat{\gamma}^\mu \otimes\ \bar{u}(P,\Lambda')u(P,\Lambda) = 0$$

$$\hat{\gamma}^\mu \otimes \bar{u}(P,\Lambda')\sigma^{\alpha\beta}u(P,\Lambda) \left\{ \begin{array}{ll} \rightarrow & a_3^{(\kappa)}/m^2\ P^\mu\gamma^\mu\,\epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda \\ \rightarrow & a_4^{(\kappa)}/m^2\ k^\mu\gamma^\mu\,\epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda \\ \rightarrow & a_5^{(\kappa)}\ S^\mu\gamma^\mu\,\Lambda\,(k\cdot a) \\ \rightarrow & a_6^{(\kappa)}\ a^\mu\gamma^\mu\,\Lambda\,(k\cdot S) \\ \rightarrow & a_7^{(\kappa)}/m^2\ \gamma_\mu\,\epsilon_{\alpha\beta\mu\sigma}S^\alpha k^\beta P^\sigma\,(k\cdot a)\,\Lambda \\ \rightarrow & a_8^{(\kappa)}/m^2\ \gamma_\mu\epsilon_{\alpha\beta\mu\sigma}a^\alpha k^\beta P^\sigma\,\Lambda\,(k\cdot S) \\ \rightarrow & a_9^{(\kappa)}\ \gamma_\mu\epsilon_{\alpha\beta\mu\sigma}a^\alpha S^\beta P^\sigma\,\Lambda \\ \rightarrow & a_{10}^{(\kappa)}\ \gamma_\mu\epsilon_{\alpha\beta\mu\sigma}a^\alpha S^\beta k^\sigma\,\Lambda \end{array} \right.$$

$$\hat{\gamma}^\mu\gamma^5 \otimes\ \bar{u}(P,\Lambda')u(P,\Lambda)\ \rightarrow \delta_{\Lambda',\Lambda} = 0$$

$$\hat{\gamma}^\mu\gamma^5 \otimes \bar{u}(P,\Lambda')\sigma^{\alpha\beta}u(P,\Lambda) \left\{ \begin{array}{ll} \rightarrow & a_{11}^{(\kappa)}/m^2\ P^\mu\gamma^\mu\,\gamma^5\,\epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda \\ \rightarrow & a_{12}^{(\kappa)}/m^2\ k^\mu\gamma^\mu\,\gamma^5\,\epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\,\Lambda \\ \rightarrow & a_{13}^{(\kappa)}\ S^\mu\gamma^\mu\,\gamma^5\,\Lambda\,(k\cdot a) \\ \rightarrow & a_{14}^{(\kappa)}\ a^\mu\gamma^\mu\,\gamma^5\,\Lambda\,(k\cdot S) \\ \rightarrow & a_{15}^{(\kappa)}/m^2\ \gamma_\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\sigma}S^\alpha k^\beta P^\sigma\,(k\cdot a)\,\Lambda \\ \rightarrow & a_{16}^{(\kappa)}/m^2\ \gamma_\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\sigma}a^\alpha k^\beta P^\sigma\,\Lambda\,(k\cdot S) \\ \rightarrow & a_{17}^{(\kappa)}\ \gamma_\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\sigma}a^\alpha S^\beta P^\sigma\,\Lambda \\ \rightarrow & a_{18}^{(\kappa)}\ \gamma_\mu\,\gamma^5\,\epsilon_{\alpha\beta\mu\sigma}a^\alpha S^\beta k^\sigma\,\Lambda \end{array} \right.$$

$$\hat{\sigma}^{\mu\nu} \otimes \bar{u}(P,\Lambda')\sigma^{\alpha\beta}u(P,\Lambda) \begin{cases} \rightarrow & a_{19}^{(\kappa)}/m\, \sigma_{\mu\nu}P^\mu S^\nu\, \gamma^5\Lambda\, (k\cdot a) \\ \rightarrow & a_{20}^{(\kappa)}/m\, \sigma_{\mu\nu}P^\mu S^\nu\, \Lambda\, (k\cdot a) \\ \rightarrow & a_{21}^{(\kappa)}/m\, \sigma_{\mu\nu}P^\mu a^\nu\, \gamma^5\Lambda\, (k\cdot S) \\ \rightarrow & a_{22}^{(\kappa)}/m\, \sigma_{\mu\nu}P^\mu a^\nu\, \Lambda\, (k\cdot S) \\ \rightarrow & a_{23}^{(\kappa)}\, \sigma_{\mu\nu}k^\mu S^\nu\, \gamma^5\Lambda\, (k\cdot a) \\ \rightarrow & a_{24}^{(\kappa)}/m\, \sigma_{\mu\nu}k^\mu S^\nu\, \Lambda\, (k\cdot a) \\ \rightarrow & a_{25}^{(\kappa)}/m\, \sigma_{\mu\nu}k^\mu a^\nu\, \gamma^5\Lambda\, (k\cdot S) \\ \rightarrow & a_{26}^{(\kappa)}/m\, \sigma_{\mu\nu}k^\mu a^\nu\, \Lambda\, (k\cdot S) \\ \rightarrow & a_{27}^{(\kappa)}\, \sigma_{\mu\nu}S^\mu a^\nu\, \gamma^5\Lambda\, m \\ \rightarrow & a_{28}^{(\kappa)}\, \sigma_{\mu\nu}S^\mu a^\nu\, \Lambda\, m \\ \rightarrow & a_{29}^{(\kappa)}/m^3\, \sigma_{\mu\nu}k^\mu P^\nu\, \epsilon_{\alpha\beta\rho\sigma}a^\alpha S^\beta P^\rho k^\sigma\, \Lambda \end{cases} \tag{6.148}$$

One has now to apply parity, hermiticity and time reversal to reduce the number of independent amplitudes to the following (see the Appendix for the explicit proof).

$$\begin{aligned} a_m^2 &= a_m^3 \qquad \mathrm{r}eal \qquad m = 5, 11, 12, 14 \\ a_m^2 &= -a_m^3 \qquad \mathrm{r}eal \qquad m = 3, 4, 6, 13, 21 \\ a_m^2 &= a_m^3 \qquad \mathrm{i}maginary \qquad m = 25 \\ a_m^2 &= -a_m^3 \qquad \mathrm{i}maginary \qquad m = 1, 3, 4, 19, 23, 27. \end{aligned} \tag{6.149}$$

Notice that besides the kinematical vectors k and P, other two vectors, v' and a, are introduced to write the ansätze for the diagonal and off-diagonal forward correlators. Both these vectors occur in the hadronic sector of (6.137); v' enters in the definition of the helicity eigenstate vectors while the auxiliary vector a, used to write down the spinorial products in the helicity flip case, appears only in the off-diagonal correlators and carries the information about the difference in phase between the initial and final helicity states.

6.5 Constraints on forward quark-quark correlators in the helicity basis

We have shown the complete equivalence of the descriptions of the quark-quark correlators in the spin and in the helicity formalisms. We constructed ansätze for the correlators in the helicity basis as we did for the correlators in the spin basis.
Some constraints on forward quark-quark helicity correlators come from conservation of total angular momentum. Let us then examine the implications of total angular momentum conservation in DIS.
One can always choose a frame of reference where the two independent external vectors of DIS, P and q, can be collinear; this means that conservation of total angular momentum implies for DIS the conservation of the longitudinal component of angular momentum.

For a pure collinear process there is, in fact, no preferred transverse direction; rotational invariance around the collinear axis requires total helicity to be conserved (see Fig.3.7)

$$\Lambda + \lambda' = \Lambda' + \lambda \, . \tag{6.150}$$

We remark that although Λ and λ' are the incoming helicities, it is convenient to label the amplitude in the sequence: initial hadron, struck quark, final hadron, returned quark, as the correlator represents the u-channel elastic quark-hadron scattering.
Helicity conservation in (6.150) states then a link between the quark and nucleon helicity degrees of freedom.
Let us see now which constraints on the the forward quark-quark correlators $\Phi_{i\Lambda,j\Lambda'}$ can be obtained from (6.150).
As already said, the correlators $\Phi_{i\Lambda,j\Lambda'}$ in (6.123) carry two kinds of indices, one concerning the helicity of the hadron; the other index indicates that these objects are 4×4 matrices in the Dirac space of the quarks. These matrices provide information about the process the quark extracted and re-inserted in the hadron undergoes in order to produce the final hadronic state from the initial one.
For the diagonal components of matrix $\Phi_{i\Lambda,j\Lambda'}$ the hadron helicity is not flipped and, by virtue of helicity conservation, the quark helicity is as well not flipped $\lambda = \lambda'$. In order to build an ansatz for $\Phi_{i+,j+}$ and $\Phi_{i-,j-}$ we will then take into account only Dirac structures which conserve parton helicity.
On the other hand in the off-diagonal correlators the hadron helicity is flipped and the rule in (6.150) implies also flipping of the quark helicity. The off-diagonal ansätze will therefore contain only Dirac structures which flip the quark helicity.
In the following we will develop a method to list all possible Dirac structures according to chirality properties. The different 4×4 Dirac matrices specify if during the process the quark chirality is flipped; chirality of the quark is conserved or flipped according to the properties of these matrices under projection with the chiral projectors $\mathcal{P}_{R/L}$. Chiral even structures conserve the chirality of the quark, while chiral odd ones flip it.
Helicity conservation rule states a selection rule for the parton helicities and not for the parton chiralities. Nevertheless the relation between chiralities and helicities of a spinor field is known and therefore we can derive a rule for the particles chirality from helicity conservation.
Chirality and helicity for the good LC components of a spinor field are equal. On the other hand, following Jaffe's argument [Jaf96b], bad light-cone components of Dirac fields haven't equal helicity and chirality but rather the helicity of the state is always the opposite of the chirality. Jaffe argues that Dirac equation for bad LC components of fields represent constraints independent of LC time, by which the bad components ψ_- of the fields are dependent fields, related to a good component ψ_+ and a transverse gluon. ψ_- can be regarded as a composite. Since the gluon carries helicity 1 but no chirality, angular momentum conservation for the composite requires that the helicity and chirality are equal for good components of fields and thus opposite for bad components.
As we know that:

- at twist 2 we have structures containing 2 good LC components, then
  - if a structure is chiral even it means also that the helicities of the initial and final states are equal
  - if a structure is chiral odd it means also that the helicities of the initial and final states are flipped
- at twist 3 we have structures containing 1 good LC component and 1 bad one, then
  - if a structure is chiral even it means also that the helicities of the initial and final states are flipped
  - if a structure is chiral odd it means also that the helicities of the initial and final states are equal
- at twist 4 we have structures containing 2 bad LC components, then
  - if a structure is chiral even it means also that the helicities of the initial and final states are equal
  - if a structure is chiral odd it means also that the helicities of the initial and final states are flipped,

then in the diagonal components of the matrix Φ there can be

- chiral even twist-2 structures
- chiral odd twist-3 structures
- chiral even twist-4 structures,

while in the off-diagonal components of the matrix Φ there can be

- chiral odd twist-2 structures
- chiral even twist-3 structures
- chiral odd twist-4 structures.

The most general form of the matrix $\Phi_{i\Lambda',j\Lambda}$ if $\vec{k}_\perp = 0$ at twist 2 and twist 4

$$\Phi_{\Lambda' i,\Lambda j}(P,k) = \left[\begin{array}{c|c} (\mathrm{c}hiral \quad \mathrm{e}ven)_{ij} & (\mathrm{c}hiral \quad \mathrm{o}dd)_{ij} \\ \hline (\mathrm{c}hiral \quad \mathrm{o}dd)_{ij} & (\mathrm{c}hiral \quad \mathrm{e}ven)_{ij} \end{array}\right] , \tag{6.151}$$

and for twist 3

$$\Phi_{\Lambda' i,\Lambda j}(P,k) = \left[\begin{array}{c|c} (\mathrm{c}hiral \quad \mathrm{o}dd)_{ij} & (\mathrm{c}hiral \quad \mathrm{e}ven)_{ij} \\ \hline (\mathrm{c}hiral \quad \mathrm{e}ven)_{ij} & (\mathrm{c}hiral \quad \mathrm{o}dd)_{ij} \end{array}\right] .$$

(6.152)

In order to implement these constraints, we must be able to recognize chiral even and chiral odd Dirac structures. Following Jaffe's convention [Jaf96b] an operator is chiral even if bracketed between fields it connects states of equal initial and final chirality; on the other hand an operator is chiral odd if it connects states of different initial and final chirality.
Let us then consider

$$\bar{\psi} A \psi = \psi^{\dagger} (\gamma^0 A) \psi \tag{6.153}$$

In order to take into account chirality properties of the structure, what one has to investigate is the density $\psi^{\dagger}\psi$. The density actually counts the fields chiralities. Then for instance for a chiral odd operator $(\gamma^0 A)$

$$\bar{\psi} A \psi = {\psi_R}^{\dagger} (\gamma^0 A) \psi_L + {\psi_L}^{\dagger} (\gamma^0 A) \psi_R \tag{6.154}$$

For a chiral even operator $(\gamma^0 A)$

$$\bar{\psi} A \psi = {\psi_R}^{\dagger} (\gamma^0 A) \psi_R + {\psi_L}^{\dagger} (\gamma^0 A) \psi_L . \tag{6.155}$$

In order to recognize chiral even and chiral odd structures we work out a method that makes use of the Weyl or chiral representation of Dirac γ matrices, where the chirality properties of Dirac structures are evident.
We recall the explicit form of Dirac matrices in the the Weyl representation (refer also to the Appendix)

$$\gamma^0 = \rho^1 \otimes \mathbb{1} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}; \qquad \vec{\gamma} = -i\rho^2 \otimes \vec{\sigma} = \begin{pmatrix} 0 & -\vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \tag{6.156}$$

$$\gamma_5 = \rho^3 \otimes \mathbb{1} = i\,\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} . \tag{6.157}$$

From the block-diagonal form of the boost $S^{0i} \equiv \sigma^{0i}$ and rotation generators $S^{ij} \equiv \sigma^{ij}$ of the Dirac representations of Lorentz group it is clear that this representation is reducible. We can form two 2-dimensional representations by considering each block separately.

The four spinor $\psi = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$, solution of the Dirac equation, can then be written as

$$\psi \;=\; \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \tag{6.158}$$

where the two objects $\psi_R \;=\; \begin{pmatrix} R_a \\ R_b \end{pmatrix}$ and $\psi_L \;=\; \begin{pmatrix} L_c \\ L_d \end{pmatrix}$ are called left-handed and right-handed Weyl spinors. By setting the mass of the particle to zero $m \;=\; 0$ Dirac equations in terms of ψ_R and ψ_L actually decouple. [PS95]

Defining chiral projectors $\mathcal{P}_{R/L}$ as

$$\mathcal{P}_{R/L} = \frac{1}{2}\left(\mathbb{1} + \gamma_5\right) = \frac{1}{2}\begin{pmatrix} \mathbb{1} \pm \mathbb{1} & 0 \\ 0 & \mathbb{1} \mp \mathbb{1} \end{pmatrix} \tag{6.159}$$

with

$$\mathcal{P}_R = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \qquad \mathcal{P}_L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \tag{6.160}$$

it is evident that the left-handed and right-handed Weyl spinors correspond to states of definite chirality. Note that we will indicate right-handed components of the fields are indicated with R and left-handed components with L.

$$\mathcal{P}_R \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (a\,,\, b\,,\, 0\,,\, 0) \;, \qquad \mathcal{P}_L \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (0\,,\, 0\,,\, c\,,\, d) \;. \tag{6.161}$$

Furthermore after obtaining the light-cone components of Weyl γ matrices

$$\gamma^{\pm} = \frac{1}{\sqrt{2}}\left(\gamma^0 \pm \gamma^3\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \mathbb{1} \mp \sigma_z \\ \mathbb{1} \pm \sigma_z & 0 \end{pmatrix} \tag{6.162}$$

$$\gamma^+ = \sqrt{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^- = \sqrt{2}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{6.163}$$

with

$$(\gamma^+)^2 = (\gamma^-)^2 = 0 \;, \tag{6.164}$$

we can define two projectors $\mathcal{P}_{\pm}$

$$\mathcal{P}_+ = \frac{1}{2}\gamma^-\gamma^+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} , \qquad \mathcal{P}_- = \frac{1}{2}\gamma^+\gamma^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} , \tag{6.165}$$

which project out the so called “good” and “bad” components of Dirac fields

$$\mathcal{P}_+ \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (a\,,\, 0\,,\, 0\,,\, d) \;, \qquad \mathcal{P}_- \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (0\,,\, b\,,\, c\,,\, 0) \;. \tag{6.166}$$

The light-cone projections of the Dirac field, $\psi_+ \equiv \mathcal{P}_+\psi = \begin{pmatrix} + \\ 0 \\ 0 \\ + \end{pmatrix}$, in the context of light-cone quantization can be regarded as independent propagating degrees of freedom; the bad components, $\psi_- \equiv \mathcal{P}_-\psi = \begin{pmatrix} 0 \\ - \\ - \\ 0 \end{pmatrix}$, are dependent fields whose equations represent constraints. [Jaf96b]

The classification of quark spin states depend on the Dirac matrices which commute with $\mathcal{P}_\pm$.
One can show that γ^1, γ^2, γ^5 and the generator of spin-rotations along the z-direction, $S^{12} \equiv \sigma^{12}$ that measures the helicity of the particle, commute with $\mathcal{P}_\pm$.
This set of operators suggests two different maximal sets of commuting observables:

- in the helicity or chirality basis the operators γ^5 and S^{12} are diagonalized (Weyl representation)
- in the transversity basis the operators γ^1 and γ^2 are diagonalized

In the chirality basis both good and bad components of ψ carry chirality labels, the eigenvalues of $P_{R/L}$, and the four-vector in Weyl representation has the following interpretation

$$\psi = \begin{pmatrix} R_+ \\ R_- \\ L_- \\ L_+ \end{pmatrix} . \tag{6.167}$$

Good and bad components of ψ carry also helicity labels, the eigenvalues ± 1 of the generator of rotations S^{12}. Indicating with P the components of the spinor whose helicity eigenvalue is $+1$ and M those components whose helicity eigenvalue is -1, the spinor can be interpreted as

$$\psi = \begin{pmatrix} P_+ \\ M_- \\ P_- \\ M_+ \end{pmatrix} . \tag{6.168}$$

We see that helicity and chirality are identical for the good components of ψ but opposite for the bad components. In fact the bad light-cone quark components of the fields can be regarded as composites of good light-cone component of a quark field and a gluon field. [Ji95] Since the gluon carries helicity 1 but no chirality, angular momentum conservation requires that the good quark field components have negative helicity and therefore negative chirality.

Analogously it is possible to show that the different blocks of an operator represented by a 4×4 matrix have different symmetry properties. For instance, the R_+R_+ matrix element connects a good right-handed LC component of a four spinor with a good right-handed LC component of a four spinor, and so on

$$\left(\begin{array}{cc|cc} R_+R_+ & R_-R_+ & L_-R_+ & L_+R_+ \\ R_+R_- & R_-R_- & L_-R_- & L_+R_- \\ \hline R_+L_- & R_-L_- & L_-L_- & L_+L_- \\ R_+L_+ & R_-L_+ & L_-L_+ & L_+L_+ \end{array}\right) . \tag{6.169}$$

Comparing a given Dirac matrix with (6.169) one can immediately decide if this matrix has a definite chirality. If the matrix has non-zero entries in the diagonal blocks only, the structure is chiral even. Vice versa, if the only non-zero components appear in the off-diagonal blocks, the structure is chiral odd. Matrices showing non-zero components in both diagonal and off-diagonal blocks have no definite symmetry. For instance the operator $(\gamma^0\,\gamma^+)$ is block diagonal and therefore chiral even as it connects states with equal even chirality while the operator $(\gamma^0\,\mathbb{1})$, not block diagonal, it connects states with different chiralities and it is thus chiral odd.
From the matrix in (6.169) one can also get complete information about the twist of the structure counting the number of bad and good fields components indicated by the minus sign and by a plus sign, respectively. Each minus sign reduces the twist by one.

Helicity conservation is obviously violated in deep inelastic reactions in case the parton carries a transverse component of four-momentum $\vec{k}_\perp$, since in this case only the total angular momentum has to be conserved and then other structures have to be considered when building the matrix $\Phi_{i\Lambda,j\Lambda'}$

$$\Phi_{\Lambda' i,\Lambda j}(P,k,\vec{k}_\perp) = \left[\begin{array}{c|c} \text{c}hiral\ \text{e}ven\ (\vec{k}_\perp) + \text{c}hiral\ \text{o}dd\ (\vec{k}_\perp) & \text{c}hiral\ \text{e}ven\ (\vec{k}_\perp) + \text{c}hiral\ \text{o}dd\ (\vec{k}_\perp) \\ \hline \text{c}hiral\ \text{e}ven\ (\vec{k}_\perp) + \text{c}hiral\ \text{o}dd\ (\vec{k}_\perp) & \text{c}hiral\ \text{o}dd\ (\vec{k}_\perp) + \text{c}hiral\ \text{e}ven\ (\vec{k}_\perp) \end{array}\right] \tag{6.170}$$

The new structures must depend on the transverse component of quark four-momentum and have to be zero if $\vec{k}_\perp = 0$.

7 Off-forward quark-quark correlators

7.1 Off-forward quark-quark correlators in spin basis

In order to build the most general ansatz for the off-forward quark-quark correlation function in the spin basis,

$$\Phi_{ij}(k,P,S,k',P',S') = \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\,\langle P',S'|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,S\rangle\,, \tag{7.171}$$

one could in principle repeat a procedure similar to the one used in the paragraph (6.1) to write an ansatz for the forward spin correlator and consider all independent external vectors occurring in a non-forward hard process

$$\{\bar{k} \qquad \bar{P} \qquad \Delta \qquad S+S' \qquad S-S'\}\,, \tag{7.172}$$

where $\bar{k}$ is the average of the initial and final parton momenta, $\bar{P}$ the average of the initial and final hadron momenta, $\Delta = P' - P$ and $S+S'$ and $S-S'$ substitute the initial and final spin vectors S and S' in order to apply the hermiticity and time reversal constraints, respectively (4.40) and (4.49). Contrary to S and S', the linear combinations $S+S'$ and $S-S'$ have indeed a definite symmetry with respect to these constraints.
Furthermore combining together Dirac matrices and the five independent vectors in (7.172) defines a set of basic elements

$$\begin{aligned}&\{\mathbb{1},\quad \gamma_\mu\,\bar{k}^\mu,\quad \gamma_\mu\,\bar{P}^\mu,\quad \gamma_\mu\,\Delta^\mu,\quad \gamma_\mu(S'+S)^\mu\gamma_5,\quad \gamma_\mu(S'-S)^\mu\,\gamma_5,\\ &\bar{P}\cdot(S'+S)\,\gamma_5,\quad \bar{P}\cdot(S'-S)\,\gamma_5,\quad \bar{k}\cdot(S'+S)\,\gamma_5,\quad \bar{k}\cdot(S'-S)\,\gamma_5,\quad S\cdot S'\}\,.\end{aligned} \tag{7.173}$$

The most general ansatz is obtained by writing down all possible products of elements of the above set with Dirac structures, treating explicitly all invariants involving spin vectors and leaving the dependence on other invariants implicit in the amplitudes.
Since scattering amplitudes contain spinorial products $\bar{u}(P',S')\,\Gamma\,u(P,S)$ which are at most linear dependent on S and S', the ansatz for the correlators can be either independent or linear dependent on the spin vectors S and S'. i.e. we know the kind of dependence on the spin degrees of freedom S and S' we may have for the correlators.
On the other hand we do not have such a linearity constraint for the vectors $S+S'$ and $S'-S$.
Let us further examine the ansatz built from the set (7.172); it will necessarily contain the following self products

$$\begin{aligned}\gamma_5\,\bar{P}\cdot(S'\pm S)\,\gamma_5\,\bar{P}\cdot(S'\pm S) &= (\bar{P}\cdot S')^2 \pm 2(\bar{P}\cdot S')(\bar{P}\cdot S) + (\bar{P}\cdot S)^2\\ \gamma_5\,\bar{k}\cdot(S'\pm S)\,\gamma_5\,\bar{k}\cdot(S'\pm S) &= (\bar{k}\cdot S')^2 \pm 2(\bar{k}\cdot S')(\bar{k}\cdot S) + (\bar{k}\cdot S)^2\,.\end{aligned} \tag{7.174}$$

which both consist of terms in which the spin vectors S and S' contribute at first and second order. The first order contributions in the right side of the Equation (5.147) are allowed according to the requirement of linearity in S and S', while the second order ones are forbidden. That is enough to say that we cannot anymore predict which kind of terms in the spin vectors S and S' contribute in the ansatz if in order to implement the hermiticity constraint (3.40) in constructing the ansatz we start from the vectors $S + S'$ and $S' - S$. For this reason we have to develop a different method to build the ansatz for the off-forward quark-quark correlators.
The helicity formalism actually turns out to be helpful to write down an ansatz for the off-forward correlators.

7.2 Ansätze for the off-forward quark-quark correlators in the helicity basis.

The off-forward quark-quark correlators in the helicity basis

$$\Phi_{i\Lambda,\, j\Lambda'}{}^{\Gamma}(k,k',P,P') = \frac{1}{(2\pi)^4}\int d^4z\, e^{i\,k\cdot z}\,\langle P',\Lambda'|\,\overline{\psi}_j(-\frac{z}{2})\,\psi_i(\frac{z}{2})\,|P,\Lambda\rangle \qquad (7.175)$$

represent $4\otimes 4$ matrices in Dirac partonic space,labeled by the indices i and j and $2\otimes 2$ matrices in hadronic helicity space, labeled by the indices Λ' and Λ; the most general ansatz for these quark-quark correlators is thus obtained through the following tensor product

$$\Phi_{i,j\,\Lambda'\Lambda}(\,\bar{k},\bar{P},\Delta\,) = \hat{\Gamma}^{\mu_1\cdots\mu_p}_{i,j}\otimes\ \ \bar{u}_\alpha(P',\Lambda)\,\Gamma_{\alpha\beta}{}^{\nu_1\cdots\nu_p}\,u_\beta(P,\Lambda)\ \ t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}(\,\bar{P},\bar{k},\Delta\,)\,, \qquad (7.176)$$

where $\hat{\Gamma}_{i,j}$ denote the 16 Fierz independent $4\otimes 4$ partonic Dirac matrices and $\bar{u}_\alpha(P',\Lambda')\,\Gamma_{\alpha\beta}\,u_\beta(P,\Lambda)$ the independent hadronic spinor products; as for the forward correlators

$$t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p} = t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}(\bar{P},\bar{k},\Delta) \qquad (7.177)$$

represent all possible independent tensors constructed from the kinematical variables $\bar{P}$, $\bar{k}$, Δ, the metric tensor $g_{\alpha\beta}$ and the antisymmetric tensor $\epsilon_{\alpha\beta\rho\sigma}$; the tensors $t_{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}$ must have rank equal to the rank of the tensor formed by the partonic matrices and the spinorial products.
Gordon identities and the relations (7.178), deriving from Dirac equations of motion, allow to determine the independent spinor products. [Die01]
From

$$\begin{aligned} 0 &= \bar{u}(P',\Lambda')\,u(P,\Lambda)\,\Delta^\alpha + \bar{u}(P',\Lambda')\,i\,\sigma^{\alpha\beta}\,2\bar{P}_\beta\,u(P,\Lambda) \\ 0 &= \bar{u}(P',\Lambda')\,\gamma_5\,u(P,\Lambda)\,2\bar{P}^\alpha + \bar{u}(P',\Lambda')\,i\,\sigma^{\alpha\beta}\,\gamma_5\,\Delta_\beta\,u(P,\Lambda) \end{aligned} \qquad (7.178)$$

we learn that the tensorial spinor product having one index contracted with the hadronic average momentum reduces to the scalar spinor product, while the pseudo-tensorial spinor product contracted with the Δ vector is proportional to the pseudo-scalar product.

The Gordon identities

$$2\,m\,\bar{u}(P',\Lambda')\,\gamma^{\alpha}\gamma_5\,u(P,\Lambda) = \bar{u}(P',\Lambda')\,\Delta^{\alpha}\gamma_5\,u(P,\Lambda) + 2\,\bar{u}(P',\Lambda')i\,\sigma^{\alpha\beta}\,\gamma_5\,\bar{P}_{\beta}\,u(P,\Lambda) \tag{7.179}$$

and

$$2\,m\,\bar{u}(P',\Lambda')\,\gamma^{\alpha}\,u(P,\Lambda) = \bar{u}(P',\Lambda')\,u(P,\Lambda)(\,2\bar{P}^{\alpha}\,) + \bar{u}(P',\Lambda')i\,\sigma^{\alpha\beta}\,\Delta_{\beta}\,u(P,\Lambda) \tag{7.180}$$

show that the vector and axial-vector currents are not independent since they can be obtained from the scalar and tensor ones.
We finally remark that the pseudo-tensor current $\bar{u}(P',\Lambda')\ i\,\sigma^{\alpha\beta}\,\gamma_5\,u(P,\Lambda)$ does not represent any new independent spinor product. Indeed the Dirac matrices $\sigma_{\alpha\beta}\gamma^5$ do not appear in the Fierz list of independent Dirac structures as they can be traced back to the Dirac σ matrices thanks to the following identity

$$i\,\sigma^{\alpha\beta} = \frac{1}{2}\,\epsilon^{\alpha\beta\rho\sigma}\,\sigma_{\rho\sigma}\gamma^5. \tag{7.181}$$

According to the rule stated in (7.176) in order to write down the most general ansatz for the off-forward helicity quark-quark correlators we thus take into account the following three independent spinorial products

$$[\ \bar{u}(P,\Lambda')\ u(P,\Lambda) \qquad \bar{u}(P,\Lambda')\ \gamma^5\ u(P,\Lambda) \qquad \bar{u}(P,\Lambda')\ \sigma^{\alpha\beta}\ u(P,\Lambda)\ ]. \tag{7.182}$$

These spinor currents have different expressions for the helicity non-flip and for the helicity flip case and their expressions are quoted in the Appendix.
As already said in the forward case one cannot calculate the spinor products in the helicity flip case via a trace method since they have different results for different components. In the off-forward situation this becomes possible because the scalar spinor product, by which one divides in the trace method to obtain the different spinor products, is also for the helicity flip case different from zero. What one gets for each spinorial product is a Lorentz tensor multiplied by a phase factor given in terms of the transverse components of the Δ vector (refer to the Appendix)

$$\eta = \frac{\Delta^1 + i\Delta^2}{|\boldsymbol{\Delta}_{\perp}|}\,. \tag{7.183}$$

All scalar products of the independent kinematical variables $\bar{K}$, $\bar{P}$ and Δ will be implicitly inserted in the ansatz as we assume that the amplitudes $d_m^{(\kappa)}$ will implicitly contain the dependence on all these scalar products

$$d_m^{(k)} = d_m^{(k)}(\bar{k}\cdot\bar{P}, \quad \bar{k}^2, \quad \bar{k}\cdot\Delta, \quad t)\ . \tag{7.184}$$

Note that $\bar{P}\cdot\Delta = 0$ and $\bar{P}^2 = \bar{m}^2 = \frac{m^2+t/4}{1-\xi^2}$ is fixed.
In the ansatz we will indicate explicity the scalar products formed by $\bar{K}$, $\bar{P}$ and Δ with the vector v' which occurs in the definition of the helicity eigenstates vectors and therefore as any spin degree of freedom appears in the hadronic sector of the ansatz (7.176).

For the helicity flip case the presented ansatz has to be multiplied by the additional factor which reflects the difference in phase of the initial and final hadronic spins. In particular we have to multiply by a factor $-\eta$ in (7.183) the off-diagonal correlator $\Phi_{i-,j+}$ and by a factor η^* the off-diagonal correlator $\Phi_{i+,j-}$ (refer to the Appendix).
We will indicate the four different helicity correlators $\tilde{\Phi}_{i\Lambda',j\Lambda}$ with an upper index $\kappa = 1,2,3,4$ such that

$$\begin{aligned}
\Phi_{ij_{++}}(\bar{k},\bar{P},\Delta) &= \Phi^{(1)}_{ij}(\bar{k},\bar{P},\Delta) & \Phi_{ij_{+-}}(\bar{k},\bar{P},\Delta) &= \Phi^{(2)}_{ij}(\bar{k},\bar{P},\Delta)\\
\Phi_{ij_{-+}}(\bar{k},\bar{P},\Delta) &= \Phi^{(3)}_{ij}(\bar{k},\bar{P},\Delta) & \Phi_{ij_{--}}(\bar{k},\bar{P},\Delta) &= \Phi^{(4)}_{ij}(\bar{k},\bar{P},\Delta)
\end{aligned} \tag{7.185}$$

The method results in producing the following ansatz

$$\begin{aligned}
\tilde{\Phi}^{(\kappa)}_{ij}(\,\bar{k},\bar{P},\Delta\,) \;=&\\
& \mathbb{1}\, m\, d^{(\kappa)}_1\\
+&\quad \Lambda\, m\, d^{(\kappa)}_2\\
+&\quad (\bar{k}\cdot v')/\bar{P}^+\, m\, d^{(\kappa)}_3\\
+&\quad (\bar{k}\cdot v')/\bar{P}^+\, m\,\Lambda\, d^{(\kappa)}_4\\
+&\quad (\Delta\cdot v')/\bar{P}^+\, m\, d^{(\kappa)}_5\\
+&\quad (\Delta\cdot v')/\bar{P}^+\,\Lambda\, m\, d^{(\kappa)}_6\\
+&\quad (\epsilon_{\alpha\beta\rho\sigma}v'^{\alpha}\Delta^{\beta}\bar{P}^{\rho}\bar{k}^{\sigma})/\bar{P}^+\, 1/m\, d^{(\kappa)}_7\\
+&\quad (\epsilon_{\alpha\beta\rho\sigma}v'^{\alpha}\Delta^{\beta}\bar{P}^{\rho}\bar{k}^{\sigma})/\bar{P}^+\,\Lambda\, 1/m\, d^{(\kappa)}_8\\
+&\quad \gamma_\mu\,\bar{P}^\mu\, d^{(\kappa)}_9\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,\Lambda\, d^{(\kappa)}_{10}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\, d^{(\kappa)}_{11}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\, d^{(\kappa)}_{12}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\Delta\cdot v')/\bar{P}^+\, d^{(\kappa)}_{13}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\, d^{(\kappa)}_{14}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^{\alpha}\Delta^{\beta}\bar{P}^{\rho}\bar{k}^{\sigma})/\bar{P}^+\, 1/m^2\, d^{(\kappa)}_{15}\\
+&\quad \gamma_\mu\,\bar{P}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^{\alpha}\Delta^{\beta}\bar{P}^{\rho}\bar{k}^{\sigma})/\bar{P}^+\,\Lambda\, 1/m^2\, d^{(\kappa)}_{16}\\
+&\quad \gamma_\mu\,\bar{k}^\mu\, d^{(\kappa)}_{17}\\
+&\quad \gamma_\mu\,\bar{k}^\mu\,\Lambda\, d^{(\kappa)}_{18}\\
+&\quad \gamma_\mu\,\bar{k}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\, d^{(\kappa)}_{19}\\
+&\quad \gamma_\mu\,\bar{k}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\, d^{(\kappa)}_{20}\\
+&\quad \gamma_\mu\,\bar{k}^\mu\,(\Delta\cdot v')/\bar{P}^+\, d^{(\kappa)}_{21}
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \gamma_\mu\,\bar{k}^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d_{22}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\bar{k}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,d_{23}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\bar{k}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,1/m^2\,d_{24}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,d_{25}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,\Lambda\,d_{26}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,d_{27}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d_{28}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\Delta\cdot v')/\bar{P}^+\,d_{29}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d_{30}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,d_{31}^{(\kappa)} \\
&+ \quad \gamma_\mu\,\Delta^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,1/m^2\,d_{32}^{(\kappa)} \\
&+ \quad \gamma_\mu\,v'^\mu\,m\,d_{33}^{(\kappa)} \\
&+ \quad \gamma_\mu\,v'^\mu\,m\,\Lambda\,d_{34}^{(\kappa)} \\
&+ \quad \gamma^5\,m\,d_{35}^{(\kappa)} \\
&+ \quad \gamma^5\,\Lambda\,m\,d_{36}^{(\kappa)} \\
&+ \quad \gamma^5\,m\,(\bar{k}\cdot v')/\bar{P}^+\,d_{37}^{(\kappa)} \\
&+ \quad \gamma^5\,m\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d_{38}^{(\kappa)} \\
&+ \quad \gamma^5\,m\,(\Delta\cdot v')/\bar{P}^+\,d_{39}^{(\kappa)} \\
&+ \quad \gamma^5\,m\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d_{40}^{(\kappa)} \\
&+ \quad \gamma^5\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m\,d_{41}^{(\kappa)} \\
&+ \quad \gamma^5\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m\,\Lambda\,d_{42}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,d_{43}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,\Lambda\,d_{44}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,d_{45}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d_{46}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,(\Delta\cdot v')/\bar{P}^+\,d_{47}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d_{48}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{P}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,d_{49}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma^\mu\,\bar{P}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,\Lambda\,d_{50}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,d_{51}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,\Lambda\,d_{52}^{(\kappa)} \\
&+ \quad \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\;d_{53}^{(\kappa)}
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{54} \\
+ \quad & \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\Delta\cdot v')/\bar{P}^+\,d^{(\kappa)}_{55} \\
+ \quad & \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{56} \\
+ \quad & \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,d^{(\kappa)}_{57} \\
+ \quad & \gamma^5\,\gamma_\mu\,\bar{k}^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,1/m^2\,d^{(\kappa)}_{58} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,d^{(\kappa)}_{59} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,\Lambda\,d^{(\kappa)}_{60} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,d^{(\kappa)}_{61} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{62} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\Delta\cdot v')/\bar{P}^+\,d^{(\kappa)}_{63} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{64} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,1/m^2\,d^{(\kappa)}_{65} \\
+ \quad & \gamma^5\,\gamma_\mu\,\Delta^\mu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,1/m^2\,d^{(\kappa)}_{66} \\
+ \quad & \gamma^5\gamma_\mu\,m\,v'^\mu\,d^{(\kappa)}_{67} \\
+ \quad & \gamma^5\gamma_\mu\,v'^\mu\,m\,\Lambda\,d^{(\kappa)}_{68} \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,A^{(k)}_{69}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,\Lambda\,A^{(k)}_{70}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\bar{k}\cdot v')/\bar{P}^+\,A^{(k)}_{71}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,A^{(k)}_{72}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\Delta\cdot v')/\bar{P}^+\,A^{(k)}_{73}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,A^{(k)}_{74}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,A^{(k)}_{75}/m^3 \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\bar{k}^\nu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,A^{(k)}_{76}/m^3 \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,A^{(k)}_{77}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,\Lambda\,A^{(k)}_{78}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\bar{k}\cdot v')/\bar{P}^+\,d^{(\kappa)}_{79}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\bar{k}\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{80}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\Delta\cdot v')/\bar{P}^+\,d^{(\kappa)}_{81}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\Delta\cdot v')/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{82}/m \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,d^{(\kappa)}_{83}/m^3 \\
+ \quad & \sigma_{\mu\nu}\bar{P}^\mu\Delta^\nu\,(\epsilon_{\alpha\beta\rho\sigma}v'^\alpha\Delta^\beta\bar{P}^\rho\bar{k}^\sigma)/\bar{P}^+\,\Lambda\,d^{(\kappa)}_{84}/m^3 \\
+ \quad & \sigma_{\mu\nu}\Delta^\mu\bar{k}^\nu\,d^{(\kappa)}_{85}/m
\end{aligned}
$$

$$
\begin{array}{ll}
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, \Lambda \, d_{86}^{(\kappa)}/m \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, d_{87}^{(\kappa)}/m \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, \Lambda \, d_{88}^{(\kappa)}/m \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\Delta\cdot v')/\bar{P}^+ \, d_{89}^{(\kappa)}/m \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\Delta\cdot v')/\bar{P}^+ \, \Lambda \, d_{90}^{(\kappa)}/m \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\epsilon_{\alpha\beta\rho\sigma} v'^\alpha \Delta^\beta \bar{P}^\rho \bar{k}^\sigma)/\bar{P}^+ \, d_{91}^{(\kappa)}/m^3 \\
+ & \sigma_{\mu\nu}\Delta^\mu \bar{k}^\nu \, (\epsilon_{\alpha\beta\rho\sigma} v'^\alpha \Delta^\beta \bar{P}^\rho \bar{k}^\sigma)/\bar{P}^+ \, \Lambda \, d_{92}^{(\kappa)}/m^3 \\
+ & \sigma_{\mu\nu}\bar{P}^\mu v'^\nu \, d_{93}^{(\kappa)} \\
+ & \sigma_{\mu\nu}\bar{P}^\mu v'^\nu \, \Lambda \, d_{94}^{(\kappa)} \\
+ & \sigma_{\mu\nu}\bar{k}^\mu v'^\nu \, d_{95}^{(\kappa)} \\
+ & \sigma_{\mu\nu}\bar{k}^\mu v'^\nu \, \Lambda \, d_{96}^{(\kappa)} \\
+ & \sigma_{\mu\nu}\Delta^\mu v'^\nu \, d_{97}^{(\kappa)} \\
+ & \sigma_{\mu\nu}\Delta^\mu v'^\nu \, \Lambda \, d_{98}^{(\kappa)} \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, d_{99}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, \Lambda \, d_{100}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, d_{101}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, \Lambda \, d_{102}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, (\Delta\cdot v')/\bar{P}^+ \, d_{103}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \bar{k}^\nu \, (\Delta\cdot v')/\bar{P}^+ \, \Lambda \, d_{104}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, d_{105}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, \Lambda \, d_{106}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, d_{107}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, \Lambda \, d_{108}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, (\Delta\cdot v')/\bar{P}^+ \, d_{109}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu \Delta^\nu \, (\Delta\cdot v')/\bar{P}^+ \, \Lambda \, d_{110}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{\Delta}^\mu \bar{k}^\nu \, d_{111}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{\Delta}^\mu \bar{k}^\nu \, \Lambda \, d_{112}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{\Delta}^\mu \bar{k}^\nu \, (\bar{k}\cdot v')/\bar{P}^+ \, d_{113}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{\Delta}^\mu \bar{k}^\nu \, (\Delta\cdot v')/\bar{P}^+ \, d_{114}^{(\kappa)}/m \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu v'^\nu \, d_{115}^{(\kappa)} \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{P}^\mu v'^\nu \, \Lambda \, d_{116}^{(\kappa)} \\
+ & \gamma^5 \, \sigma_{\mu\nu}\bar{k}^\mu v'^\nu \, d_{117}^{(\kappa)}
\end{array}
$$

$$
\begin{aligned}
&+ \quad \gamma^5\, \sigma_{\mu\nu} \bar{k}^\mu v'^\nu\, \Lambda\, d^{(\kappa)}_{118} \\
&+ \quad \gamma^5\, \sigma_{\mu\nu} \Delta^\mu v'^\nu\, d^{(\kappa)}_{119} \\
&+ \quad \gamma^5\, \sigma_{\mu\nu} \Delta^\mu v'^\nu\, \Lambda\, d^{(\kappa)}_{120} \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, d^{(\kappa)}_{121}/m^2 \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{122}/m^2 \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, (\bar{k} \cdot v')/\bar{P}^+\, d^{(\kappa)}_{123}/m^2 \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, (\Delta \cdot v')/\bar{P}^+\, d^{(\kappa)}_{124}/m^2 \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \bar{k}^\beta \bar{P}^\rho\, d^{(\kappa)}_{125}/\bar{P}^+ \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \bar{k}^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{126}/\bar{P}^+ \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{P}^\rho\, d^{(\kappa)}_{127}/\bar{P}^+ \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{128}/\bar{P}^+ \\
&+ \quad \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{k}^\rho\, d^{(\kappa)}_{129}/\bar{P}^+ \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, d^{(\kappa)}_{130}/m^2 \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{131}/m^2 \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, (\bar{k} \cdot v')/\bar{P}^+\, d^{(\kappa)}_{132}/m^2 \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} \Delta^\alpha \bar{k}^\beta \bar{P}^\rho\, (\Delta \cdot v')/\bar{P}^+\, d^{(\kappa)}_{133}/m^2 \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \bar{k}^\beta \bar{P}^\rho\, d^{(\kappa)}_{134}/\bar{P}^+ \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \bar{k}^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{135}/\bar{P}^+ \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{P}^\rho\, d^{(\kappa)}_{136}/\bar{P}^+ \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{P}^\rho\, \Lambda\, d^{(\kappa)}_{137}/\bar{P}^+ \\
&+ \quad \gamma^5\, \gamma_\mu\, \epsilon_{\alpha\beta\mu\rho} v'^\alpha \Delta^\beta \bar{k}^\rho\, d^{(\kappa)}_{138}/\bar{P}^+\, .
\end{aligned} \tag{7.186}
$$

If we assume $\Delta = 0$, namely we take the forward limit, we are lead to the ansatz given in (6.141) for the forward helicity correlators. That the ansatz given has a correct limit in the forward case provides a proof of the self consistence of the method applied to build the ansätze.
Imposing parity, hermiticity and time reversal constraints (see Appendix for the derivation) reduces further the number of independent amplitudes in the ansätze. Instead of the expected $138 \times 4 = 552$ independent amplitudes combining the constraints from parity, hermiticity and time reversal invariance results in the following independent amplitudes

- for the diagonal amplitudes:

$$
d^1_m = d^4_m \qquad \mathrm{r}eal \qquad m \;=\; 1,3,9,11,17,19,29,33,40,44,46,49, \\
52,54,57,64,68,83,91
$$

$$
\begin{array}{llll}
d_m^1 = -d_m^4 & \mathrm{r}eal & m & = 2, 4, 10, 12, 18, 20, 30, 34, 39, 43, 45, 50, \\
 & & & \quad 51, 53, 58, 63, 67, 84, 92 \\
d_m^1 = d_m^4 & \mathrm{i}maginary & m & = 8, 16, 24, 73, 77, 79, 85, 87, 97, 100, 102, \\
 & & & \quad 110, 116, 118, 122, 124, 128, 130, 132, \\
 & & & \quad 136, 138 \\
d_m^1 = -d_m^4 & \mathrm{i}maginary & m & = 7, 15, 23, 74, 78, 80, 86, 88, 98, 99, 101, \\
 & & & \quad 109, 114, 115, 117, 121, 123, 127, 129, \\
 & & & \quad 131, 137
\end{array}
\tag{7.187}
$$

- for the off-diagonal amplitudes:

$$
\begin{array}{llll}
d_m^2 = d_m^3 & \mathrm{r}eal & m & = 1, 3, 9, 11, 17, 19, 29, 33, 41, 48, 56, 60, \\
 & & & \quad 62, 76 \\
d_m^2 = -d_m^3 & \mathrm{r}eal & m & = 2, 4, 10, 12, 18, 20, 30, 34, 42, 47, 55, \\
 & & & \quad 59, 61, 75 \\
d_m^2 = d_m^3 & \mathrm{i}maginary & m & = 40, 50, 58, 73, 77, 79, 85, 87, 97, \\
 & & & \quad 104, 106, 108, 112, 120, 124, 126, 130, \\
 & & & \quad 132, 136, 138 \\
d_m^2 = -d_m^3 & \mathrm{i}maginary & m & = 39, 49, 57, 74, 78, 80, 86, 88, 98, 103, \\
 & & & \quad 105, 107, 111, 113, 119, 125, 131, 137\ .
\end{array}
\tag{7.188}
$$

Applying constraint derived from hermiticity, parity and time reversal we reduced the number of independent amplitudes in the ansätze. Notice that the requirement coming from parity fixes relations between the two diagonal correlators and between the two off-diagonal correlators. The helicity non-flip correlators $\Phi_{i+,j+}$ and $\Phi_{i-,j-}$ can be expressed in terms of the 38 real amplitudes and 42 imaginary amplitudes. The helicity flip correlators $\Phi_{i-,j+}$ and $\Phi_{i+,j-}$ are given by 28 real amplitudes and 38 imaginary ones. In total the number of independent amplitudes is reduced to 146.

8 Definition of distribution functions

A parton distribution, whether forward or off-forward, arises from removal of a parton from a nucleon by a hard probe and its subsequent return to form the nucleon ground state.
In hard scattering processes forward and non-forward distribution functions link the quark and gluon lines to hadrons in the initial or final state and are defined as Dirac projections of twist-two bilocal, hadronic matrix elements of fields operators, taken along a light-cone direction in position space, which is related to the direction of the momentum of the nucleon in the infinite momentum frame.
The transition from hadrons to quarks and gluons is described in terms of distribution functions and the cross sections of these processes are written in terms of observables that are directly derived from distribution functions when convoluted with the perturbative hard part of the reaction.
One of the most important property of distribution functions is the independence from the particular reaction where a given distribution is measured. This property, called *universality*, guarantees that any distribution function, evaluated in a particular experiment, can be used without change to predict the results of another hard process.
Factorization theorems assure that distribution functions fulfill universality for some of the hard processes [CF99], whereas for others factorization is used as a plausible assumption.

8.1 Definition of forward distribution functions

In the light-cone gauge let us consider quark-quark matrix elements between states of equal initial and final momenta. DIS distribution functions are obtained from the integrals over dk^- and $d^2\vec{\mathbf{k}}_\mathbf{T}$ of the Dirac projections of the quark correlation functions,

$$\Phi^{[\Gamma]}{}_{ij}(x_B) = \frac{1}{2}\int dk^-\ d^2\vec{\mathbf{k}}_\mathbf{T}\ \mathrm{T}r(\Gamma\ \Phi(k,P,S))|_{k^+=x_BP^+}, \tag{8.189}$$

The projections depend on the fractional momentum $x_B = k^+/P^+$, on $\vec{\mathbf{k}}_T$ and on the hadron momentum P, i.e. they depend on P^+ and M.
The different projections can therefore be ordered according to the power of M/P^+ multiplied for a function depending only on x_B and $\vec{k}_T^2$ and each factor produces a suppression in the cross section of the order M/Q. According to Jaffe's definition of "effective twist", any projection is given a twist t related to the power $(M/P^+)^{t-2}$ that appears in the projection itself. [Jaf96b]
In a particular hard process only some of the Dirac projections enter in the cross section. At leading order in $1/Q$ the cross section for unpolarized DIS is written in terms of few structure functions

$$2F_1(x_B)\ =\ F_2(x_B)/x_B\ =\ \sum_a e_a^2 f_1^a(x_B) \tag{8.190}$$

where the unpolarized quark distribution function f_1 is obtained from Φ as

$$\begin{aligned} f_1(x_B) &= \frac{1}{2}\int dk^-\, d^2\vec{\mathbf{k}}_{\mathbf{T}}\, \mathrm{T}r(\Phi\gamma^+)|_{k^+=x_B P^+} \\ &= \int \frac{dz^-}{4\pi}\, e^{iP^+z^-}\, \langle P,S|\,\overline{\psi}_j(0)\,\gamma^+\,\psi_i(z)\,|P,S\rangle|_{\mathbf{z}^+=\mathbf{z_T}=\mathbf{0}} \end{aligned} \tag{8.191}$$

The case of unpolarized quarks inside protons requires the proton matrix element of the plus component of a flavor-diagonal bi-local quark field operator (summed over colour). For polarized DIS one introduces a polarized distribution function

$$\begin{aligned} \Lambda g_1(x_B) &= \frac{1}{2}\int dk^-\, d^2\vec{\mathbf{k}}_{\mathbf{T}}\, \mathrm{T}r(\Phi\gamma^+\gamma_5)|_{k^+=x_B P^+} \\ &= \int \frac{dz^-}{4\pi}\, e^{iP^+z^-}\, \langle P,S|\,\overline{\psi}_j(-\frac{z}{2})\,\gamma^+\gamma_5\,\psi_i(\frac{z}{2})\,|P,S\rangle|_{z^+=\mathbf{z_T}=\mathbf{0}} \end{aligned} \tag{8.192}$$

that is related to the probability of finding a longitudinally polarized quark in a polarized hadron and contributes at leading order in the DIS cross section as the structure function $G_1 = \sum_q e_q^2\, g_1^q(x_B)$.
Apart from the unpolarized and polarized chiral even distribution functions, f_1 and g_1, there is also one forward chirally odd twist-two proton distribution function, known as $\delta q(x_B) = h_1(x_B)$ or transversity distribution, that changes the helicity of the active parton, constructed from the operator $\bar{\psi}_q\,\sigma^{+i}\gamma_5\,\psi_q$ [HJ98] (the latin indices denote always the transverse directions in light-cone coordinates)

$$\begin{aligned} h_1(x_B) &= \frac{1}{2}\int dk^-\, d^2\vec{\mathbf{k}}_T\, \mathrm{T}r(\Phi\sigma^{+i}\gamma_5)|_{k^+=x_B P^+} \\ &= \int \frac{dz^-}{4\pi}\, e^{ip^+z^-}\, \langle P,S|\,\overline{\psi}_j(-\frac{z}{2})\,\sigma^{+i}\gamma_5\,\psi_i(\frac{z}{2})\,|P,S\rangle|_{z^+=\mathbf{z}_\perp=\mathbf{0}} \end{aligned} \tag{8.193}$$

In case we consider a forward reaction where no transverse spin contribution is available, the transversity distribution is not accessible. There is in fact no coupling of quark-quark operators to the Dirac structure $\sigma^{+i}\gamma_5$ if the target is in an eigenstate of helicity. In a transversity basis h_1 gets an easy probabilistic interpretation: it is the probability to find a quark polarized along the transverse polarization of the nucleon minus the probability to find the quark polarized in the opposite direction.
In case the integration over $\vec{\mathbf{k}}_T$ is not performed, one is sensitive to transverse separation between quarks

$$\Phi^{[\Gamma]}_{ij}(x_B,\vec{\mathbf{k}}_T) = \frac{1}{2}\int dk^-\ \mathrm{T}r(\Gamma\ \Phi(k,P,S)|_{k^+=x_B P^+,\vec{\mathbf{k}}_{\mathbf{T}}}$$

$$= \int \frac{dz^- \, d^2\vec{\mathbf{z}}_{\mathbf{T}}}{2(2\pi)^3} \, e^{i\,k\cdot z} \, \langle P,S|\,\overline{\psi}_j(-\frac{z}{2})\,\Gamma\,\psi_i(\frac{z}{2})\,|P,S\rangle|_{z^+=0} \tag{8.194}$$

DIS is not sensitive to transverse momentum distribution functions; the observables related to these distributions are measured, for instance, in semi-inclusive hard processes. The study of transverse momentum dependent distribution functions is essential for the comprehension of the structure of hadrons. $\vec{\mathbf{k}}_T$ dependent distribution functions, whose study was extensively done by many groups, for instance by the group of Piet Mulders in Amsterdam, give access to aspects of hadrons structure which cannot be investigated by $\vec{\mathbf{k}}_T$ independent distribution functions.
Aspects of hadronic internal structures, accessible if also $\vec{\mathbf{k}}_T$-dependent distribution functions are investigated, can be reached in off-forward reactions thanks to the presence of the transverse component $\mathbf{\Delta}_T$ of the vector Δ.

8.2 Definition of skewed distribution functions

Skewed parton distributions are defined from the off-forward matrix elements of quark and gluon operators and are the non-perturbative input for Compton scattering in deep virtual region of small $-t$ but large Q^2 and s. Factorization of the process into hard and soft physics garantuees that SPD, like distribution functions, are universal. [CF99], [JO98], [Rad97a].
At the leading order, one needs to consider only the matrix elements of bilinear operators at two different points on the light-cone, integrated over transverse momenta of partons.
For a collinear process rotational invariance requires that the helicity should be conserved but in case a non-zero transverse momentum is present, helicity conservation does not necessarily hold. Compared to forward processes like DIS additional distribution functions then appear.

8.2.1 The unpolarized skewed quark distribution

In the following we will consider only quark SPDs. Let us consider the case of unpolarized quarks inside protons. Thus, we investigate the proton matrix element of the plus component of a flavor-diagonal bi-local quark field operator (summed over colour). Following Xi [Ji97b], we define the SPDs $H^q(x,\xi;t)$ and $E^q(x,\xi;t)$ for a quark of flavor q by

$$\mathcal{H}^q_{\Lambda'\Lambda} \equiv \frac{1}{2\sqrt{1-\xi^2}} \int \frac{d\,z^-}{2\pi} \, e^{i\,x\,\bar{P}^+z^-} \, \langle P',\Lambda'|\,\bar{\psi}_q(-z/2)\,\gamma^+\,\psi_q(z/2)\,|P,\Lambda\rangle\,|_{z^+=0,\mathbf{z}_\perp=0}$$

$$= \frac{\bar{u}(P',\Lambda')\gamma^+u(P,\Lambda)}{2\bar{P}^+\sqrt{1-\xi^2}} \, H^q(x,\xi;t) + \frac{\bar{u}(P',\Lambda')i\sigma^{+\alpha}\Delta_\alpha u(P,\Lambda)}{4m\,\bar{P}^+\sqrt{1-\xi^2}} \, E^q(x,\xi;t)$$

(8.195)

The link operator normally needed to render the definition gauge-invariant does not appear because we choose the gauge $A^+ = 0$, which together with an integration path along the minus direction reduces the link operator to unity. Evaluating $\mathcal{H}^q_{\Lambda'\Lambda}$ for both proton helicity flip and non-flip, one obtains the usual SPDs for quark flavor q, H^q and E^q from the following set of two equations

$$\begin{aligned} \mathcal{H}^q_{++} = \quad \mathcal{H}^q_{--} \quad &= H^q - \frac{\xi^2}{1-\xi^2}\, E^q \,, \\ \mathcal{H}^q_{-+} = -(\mathcal{H}^q_{+-})^* \quad &= \eta\, \frac{\sqrt{t_0 - t}}{2m}\, \frac{1}{\sqrt{1-\xi^2}}\, E^q \end{aligned} \tag{8.196}$$

with t_0 defined as

$$-t_0 = \frac{4\xi^2 m^2}{1-\xi^2} \,, \tag{8.197}$$

and the phase η as before given by

$$\eta = \frac{\Delta^1 + i\Delta^2}{|\mathbf{\Delta}_\perp|} \,. \tag{8.198}$$

In a general reference frame Δ^α in Equation (8.195) is to be replaced with $\Delta^\alpha - (\Delta^+/\bar{P}^+)\, \bar{P}^\alpha$. Note that the factors in (8.196), which are spinorial products, are frame-dependent and here calculated in the parameterization (B.255). (refer to the Chapter "Choice of spinors and evaluation of spinorial products").
Evaluating $\mathcal{H}^q_{\lambda'\lambda}$ for both proton helicity flip and non-flip, one obtains the usual SPDs for quark flavor q, H^q and E^q. $\mathcal{H}_{\lambda\lambda'}$ and $\widetilde{\mathcal{H}}_{\lambda\lambda'}$ provide linear combinations of the SPDs H, E and $\widetilde{H}$, $\widetilde{E}$, respectively. Evaluating $\mathcal{H}_{\Lambda'\Lambda}$ for both proton helicity flip and non-flip, one then obtains from Eq. (8.196) (or (8.200) in the polarized case) the SPDs H ($\widetilde{H}$) and E ($\widetilde{E}$) separately for each quark flavor q or gluons.

8.2.2 The polarized skewed quark distribution

The polarized skewed quark distributions, $\widetilde{H}^q(\bar{x}, \xi; t)$ and $\widetilde{E}^q(\bar{x}, \xi; t)$, are defined by the Fourier transform of the axial vector matrix element

$$\begin{aligned} \widetilde{\mathcal{H}}^q_{\Lambda'\Lambda} \;&\equiv\; \frac{1}{2\sqrt{1-\xi^2}} \int \frac{d\,z^-}{2\pi}\, e^{i\,x\,\bar{P}^+ z^-}\, \langle P', \Lambda'|\, \bar{\psi}_q(-\frac{z}{2})\, \gamma^+\gamma_5\, \psi_q(\frac{z}{2})\, |P, \Lambda\rangle\, |_{z^+=0, \mathbf{z}_\perp=0} \\ &= \frac{\bar{u}(P', \Lambda')\gamma^+\gamma_5 u(P, \Lambda)}{2\bar{p}^+\sqrt{1-\xi^2}}\, \widetilde{H}^q(x, \xi; t) + \frac{\bar{u}(P', \Lambda')\Delta^+\gamma_5 u(P, \Lambda)}{4m\, \bar{P}^+\sqrt{1-\xi^2}}\, \widetilde{E}^q(x, \xi; t) \,. \end{aligned}$$

(8.199)

For the different proton helicity combinations we now find

$$
\begin{aligned}
\widetilde{\mathcal{H}}^q_{++} = -\,\widetilde{\mathcal{H}}^q_{--} &= \widetilde{H}^q - \frac{\xi^2}{1-\xi^2}\,\widetilde{E}^q\,, \\
\widetilde{\mathcal{H}}^q_{-+} = \quad (\widetilde{\mathcal{H}}^q_{+-})^* &= \eta\,\frac{\sqrt{t_0-t}}{2m}\,\frac{\xi}{\sqrt{1-\xi^2}}\,\widetilde{E}^q\,.
\end{aligned}
\tag{8.200}
$$

By solving (8.200) $\widetilde{H}^q(\bar{x},\xi;t)$ and $\widetilde{E}^q(\bar{x},\xi;t)$ are obtained.

8.2.3 Parton helicity changing distributions

There are also twist-two skewed distributions that change the helicity of the active parton [HJ98]. The corresponding quark distributions are constructed from the operator $\bar{\psi}_q\,\sigma^{+i}\gamma_5\,\psi_q$, and one of them becomes the ordinary quark transversity distribution $\delta q(x)$ in the forward limit.
In case we consider a forward reaction where no transverse spin contribution is available, the transversity distribution is not accessible. Nevertheless DVCS transverse components of hadronic momenta $\mathbf{\Delta}_\perp$ make it possible to have a flip of the hadron helicity and this produces a distribution that resembles ordinary transversity distribution.

$$
\begin{aligned}
\mathcal{G}^{q\ \ i}_{\Lambda'\Lambda} &\equiv \frac{1}{2\sqrt{1-\xi^2}}\int\frac{d\,z^-}{2\pi}\,e^{i\,x\,\bar{P}^+z^-}\,\langle P',\Lambda'|\,\bar{\psi}_q(-\frac{z}{2})\,\sigma^{+i}\,\psi_q(\frac{z}{2})\,|P,\Lambda\rangle\,|_{z^+=0,\mathbf{z}_\perp=0} \\
&= \bar{u}(P',\Lambda')\sigma^{+i}u(P,\Lambda)H^q_T(x,\xi;t) + \bar{u}(P',\Lambda')\gamma^{[+}\Delta^{i]}u(P,\Lambda)E^q(x,\xi;t)
\end{aligned}
\tag{8.201}
$$

By considering the implications of parity and time reversal invariance properties on the number of independent amplitudes in a off-forward process like DVCS, Hoodboy and Ji [HJ98] argue that there should be two parton helicity changing skewed distributions (8.201).
In a recent paper [Die01] Diehl affirms that by implementing correctly the constraint coming from time reversal invariance the number of independent parton helicity changing skewed distributions is fixed to 4, $H^q_T(x,\xi;t)$, $\widetilde{E}^q_T(x,\xi;t)$ $H^q_T(x,\xi;t)$ and $\widetilde{E}^q_T(x,\xi;t)$ such that (8.201) reads

$$
\begin{aligned}
\mathcal{G}^{q\ \ i}_{\Lambda'\Lambda} &\equiv \frac{1}{2\sqrt{1-\xi^2}}\int\frac{d\,z^-}{2\pi}\,e^{i\,x\,\bar{P}^+z^-}\,\langle P',\Lambda'|\,\bar{\psi}_q(-\frac{z}{2})\,\sigma^{+i}\,\psi_q(\frac{z}{2})\,|P,\Lambda\rangle\,|_{z^+=0,\mathbf{z}_\perp=0} \\
&= \bar{u}(P',\Lambda')\sigma^{+i}u(P,\Lambda)H^q_T(x,\xi;t) + \bar{u}(P',\Lambda')\gamma^{[+}\Delta^{i]}/mu(P,\Lambda)\widetilde{E}^q_T(x,\xi;t) \\
&= \bar{u}(P',\Lambda')\bar{P}^{[+}\Delta^{i]}/m^2u(P,\Lambda)H^q_T(x,\xi;t) + \bar{u}(P',\Lambda')\gamma^{[+}\bar{P}^{i]}u(P,\Lambda)\widetilde{E}^q_T(x,\xi;t)\,.
\end{aligned}
\tag{8.202}
$$

In the Equations (8.195),(8.196) and (8.201) the first term survives in the forward limit (helicity non-flip case), the second term is, on the contrary, an amplitude that decouples but does not vanish in the forward DIS limit. In Equation (8.202) only the first term which multiplies the skewed distribution $H_T^q(x, \xi; t)$ survives if $\Delta = 0$.
The forward limit represents indeed a kinematical check of the correct behavior of the SPDs.

8.2.4 Interpretation of SPDs

We do not have a probabilistic interpretation for SPDs as they are interference amplitudes. Nevertheless we can interpret them as follows.
The matrix elements in (8.195),(8.196) and (8.201) are different from zero for $-1 \leq x \leq 1$. In the region $\xi \leq x \leq 1$ a quark is at the beginning emitted and one quark is at the end re-absorbed.
If one reinterprets a quark with negative momentum fraction as an anti-quark with positive fraction, one finds that in the region $-1 \leq x \leq -\xi$ the matrix elements in (8.195),(8.196) and (8.201) describe emission and reabsorption of an anti-quark. In the region $-\xi \leq x \leq \xi$ the proton with momentum P emits a quark-anti-quark pair and is left as a proton with momentum $P' = P + \Delta$.
We remark in passing that one can define distributions $H^{\bar{q}}(x, \xi; t) \equiv -H^q(-x, \xi; t)$ and $E^{\bar{q}}(x, \xi; t) \equiv -E^q(-x, \xi; t)$, which in the region $\xi < \bar{x} < 1$ describe the emission and reabsorption of anti-quarks and may thus be called “skewed anti-quark distributions”, which will not be discussed in the following.

9 Twist-analysis of quark-quark correlators

One of the main goals of this work consists in developing a *twist-analysis* of off-forward quark-quark correlators following and generalizing the method worked out for the forward correlators by Mulders' group in Amsterdam [JMR97b, JMR97a, BM98, MR01, Mul99, Mul97, MT96, Mula, BBHM00b].
Carrying out a twist-analysis consists in tracing the correlation functions with different Dirac structures. We analyze the Dirac content of the correlation functions by tracing them with various Dirac matrices, different Dirac structures probing different properties of the hadrons. Furthermore the different traces correspond to the different distribution functions and the twist-analysis determines which distribution functions occur in a process at the different orders in $1/P^+$ or $1/\bar{P}^+$ for the off-forward case, where P^+ or $\bar{P}^+$ scale with the hard scale in the process. Twist 2 distribution functions are obtained by projecting out the ansätze for the correlators with the Dirac matrices γ^+, $\gamma^+\gamma_5$ and $i\,\sigma^{+i}\gamma_5$. We access sub-leading order in $1/P^+$ or in $1/\bar{P}^+$ distribution functions by tracing our ansätze with the Dirac matrices $\mathbb{1}$, γ^i, $\gamma^i\gamma_5$, $i\,\sigma^{ij}\gamma_5$, and $i\,\sigma^{+-}\gamma_5$. Finally twist 4 distribution functions are obtained by tracing the ansätze with $i\gamma_5$, γ^-, $\gamma^-\gamma_5$ and $i\,\sigma^{i-}\gamma_5$.
The advantage of having built ansätze for the quark-quark correlators is that by tracing them with the different Γ Dirac matrices, we gain the different quark-quark distributions functions in terms of some of the amplitudes occurring in the ansätze and we are thus able to predict the dependence of the quark-quark distribution functions upon the different fundamental structures in terms of which we constructed the ansatz for the correlators.
We will consider only Dirac projections, also called *profile functions*, which do not depend on the transverse momentum of the quarks $\vec{\mathbf{k}}_T$ or $\vec{\bar{\mathbf{k}}}_T$, integrating over them as indicated in (9.203) and in (9.204), respectively.
In principle having an ansatz for the quark-quark correlators one could extract the forward and off-forward profile functions which depend additionally on the transverse momentum $\vec{\mathbf{k}}_T$ or $\vec{\bar{\mathbf{k}}}_T$, respectively. In this case these distributions are also sensitive to the difference in the transverse distance between quarks.
Investigating $\vec{\mathbf{k}}_T$-dependent distribution functions or $\vec{\bar{\mathbf{k}}}_T$-dependent skewed distributions is beyond the scope of this work and therefore we will not pursue it. On one hand the investigation of $\vec{\mathbf{k}}_T$-depending ordinary parton distributions has been extensively carried out by many groups theoretically and experimental investigations are currently under way. On the other hand for the non-forward processes no formalism for the systematic study of $\vec{\mathbf{k}}_T$-dependence has been attempted yet and an experimental program on $\vec{\mathbf{k}}_T$ effects seems far beyond present abilities. In the foreseeable future we will be happy to acquire some knowledge on SPDs depending on $(x, \xi, t; Q^2)$, not to speak of additional $\vec{\mathbf{k}}_T$-dependence .

9.1 Dirac projections of the quark-quark correlators

In writing the cross sections of hard processes we need forward or off-forward quark-quark correlation functions, integrated over the transverse and minus components of the quark momentum and traced with one Γ Dirac matrix

$$
\begin{aligned}
\Phi^{[\Gamma]}(x) &= \frac{1}{2}\int d^2\vec{\mathbf{k}}_T\, dk^-\, Tr[\Phi\,\Gamma] \\
&= \int \frac{d\,z^-}{4\pi}\, e^{i\,k\cdot z}\langle P,\Lambda'|\,\overline{\psi}_j(-\frac{z}{2})\,\Gamma\,\psi_i(\frac{z}{2})\,|P,\Lambda\rangle|_{z^+=z_T=0}
\end{aligned}
\tag{9.203}
$$

$$
\begin{aligned}
\tilde{\Phi}^{[\Gamma]}(x,\xi;t) &= \frac{1}{2}\int d^2\vec{\bar{\mathbf{k}}}_T\, dk^-\, Tr[\tilde{\Phi}\,\Gamma] \\
&= \int \frac{d\,z^-}{4\pi}\, e^{i\,k\cdot z}\langle P',\Lambda'|\,\overline{\psi}_j(-\frac{z}{2})\,\Gamma\,\psi_i(\frac{z}{2})\,|P,\Lambda\rangle|_{z^+=z_T=0}\,.
\end{aligned}
\tag{9.204}
$$

These traces give thus the quark-quark distribution functions, which occur in the soft parts of many hard processes.

Note that in (9.203) and in (9.204) one integrates over $d\,k^-\,d^2\,\vec{\mathbf{k}}_T$ or $d\,\bar{k}^-\,d^2\,\vec{\bar{\mathbf{k}}}_T$, respectively. In order to have covariant integration variables, one performs a change of variables so that the integral over $d\,k^-\,d^2\,\vec{\mathbf{k}}_T$ can be rewritten as follows [Mul97]

$$
\Phi^{[\Gamma]}(x) = \int d\,\sigma\, d\,\tau\,\theta(x\,\sigma - x^2\,m^2 - \tau)\,\frac{Tr[\Phi\,\Gamma]}{4P^+}\,, \tag{9.205}
$$

where we have introduced the two integration variables $\sigma = k\cdot P$ and $\tau = k^2$, while the integral over $d\,\bar{k}^-\,d^2\,\vec{\bar{\mathbf{k}}}_T$ becomes

$$
\tilde{\Phi}^{[\Gamma]}(x,\xi;t) = \int d\,\tilde{\sigma}\, d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tilde{\tau})\,\frac{Tr[\tilde{\Phi}\,\Gamma]}{4\bar{P}^+}\,, \tag{9.206}
$$

where the new variables of integration are $\tilde{\sigma} = 2\bar{P}\cdot\bar{k}$ and $\tilde{\tau} = \bar{k}^2$.

9.2 Leading order Dirac projections of the forward quark-quark correlators

We take into account the leading order projections of the forward helicity correlators. We trace the helicity non-flip quark-quark correlators in (6.141) with the matrices γ^+, $\gamma^+\gamma_5$ and the helicity flip forward correlator (6.148) with the matrix $i\,\sigma^{+i}\gamma_5$.

9.2.1 Unpolarized ordinary parton distribution

Substituting the ansatz (6.141) for the forward quark-quark correlators in (9.205) and tracing it with the matrices γ^+ we obtain the unpolarized ordinary parton distribution $f_1(x)$ as

$$
\begin{aligned}
f_1(x) &= \Phi_{++}^{[\gamma^+]}(x) = \Phi_{--}^{[\gamma^+]}(x) = \\
&\int d\,\sigma\, d\,\tau\, \theta(x\,\sigma - x^2\,m^2 - \tau)\,[\frac{k\cdot v'}{\bar{P}^+}\,a_9^{(1)} + x\,a_6^{(1)} + x\,\frac{k\cdot v'}{\bar{P}^+}\,a_{10} + (a_5^{(1)} + a_8^{(1)})] \\
&\int d\,\sigma\, d\,\tau\, \theta(x\,\sigma - x^2\,m^2 - \tau)\,[x\,(a_9^{(1)} + a_6^{(1)}) + x^2\,a_{10}^{(1)} + (a_5^{(1)} + a_8^{(1)})]\,,
\end{aligned}
\tag{9.207}
$$

where $(\bar{k}\cdot v')/(\bar{P}^+) = x$. The distribution function $f_1(x)$ is thus expressed in terms of the amplitudes $a_i^{(1)}$ occurring in the ansatz (6.141).
Helicity conservation implies that tracing the ansatz (6.148) for the helicity flip correlators with the γ^+ gives identically zero. The Dirac structure γ^+ indeed does not flip the parton helicity $\lambda = \lambda'$ and as a consequence of the helicity conservation rule (6.150), characteristic for the forward processes, which we report here

$$
\Lambda + \lambda' = \Lambda' + \lambda\,,
\tag{9.208}
$$

the hadron helicity as well cannot be flipped and then the γ^+ trace of the correlators $\Phi_{i+,j-}$ and $\Phi_{i-,j+}$ is zero

$$
\Phi_{+-}^{[\gamma^+]}(x) = -\Phi_{+-}^{[\gamma^+]}(x)^* = 0\;.
\tag{9.209}
$$

9.2.2 Polarized ordinary parton distribution

By tracing Φ_{++} and Φ_{--} with $\gamma^+\gamma^5$ and integrating over $d\,\sigma\,d\,\tau$, we get the expression of the leading order distribution function $g_1(x)$ in terms of the amplitudes $a_m^{(k)}$ characterizing the ansätze (6.141)

$$
\begin{aligned}
g_1(x) &= \Phi_{++}^{[\gamma^+]}(x) = -\Phi_{--}^{[\gamma^+\gamma_5]}(x) = \\
&\int d\,\sigma\, d\,\tau\, \theta(x\,\sigma - x^2\,m^2 - \tau)\,[\frac{k\cdot v'}{\bar{P}^+}\,a_{15}^{(1)} + x\,a_{12}^{(1)} + x\,\frac{k\cdot v'}{\bar{P}^+}\,a_{16} + (a_{11}^{(1)} + a_{14}^{(1)})] \\
&\int d\,\sigma\, d\,\tau\, \theta(x\,\sigma - x^2\,m^2 - \tau)\,[x\,(a_{12}^{(1)} + a_{15}^{(1)}) + x^2\,a_{16}^{(1)} + (a_{11}^{(1)} + a_{14}^{(1)})]\,,
\end{aligned}
\tag{9.210}
$$

Because of helicity conservation tracing the correlators $\Phi_{i+,j-}$ and $\Phi_{i-,j+}$ with $\gamma^+\gamma_5$ also gives zero

$$
\Phi_{+-}^{[\gamma^+\gamma_5]}(x) = \Phi_{+-}^{[\gamma^+\gamma_5]}(x)^* = 0\;,
\tag{9.211}
$$

as one can check by substituting the ansätze (6.148) for $\Phi_{i+,j-}$ and $\Phi_{i-,j+}$ directly in (9.205) and tracing with the Dirac structure $\gamma^+\gamma_5$.

9.2.3 Transversity parton distribution

For $\vec{\mathbf{k}}_T$-integrated profile functions conservation of helicity (6.150) implies that the trace of the helicity non flip correlators $\Phi_{i+,j+}$ and $\Phi_{i+,j+}$ with the matrix $i\,\sigma^{i-}\gamma_5$ is identically zero. Indeed

$$\Phi_{++}^{[i\,\sigma^{i-}\gamma_5]}(x) = \Phi_{--}^{[i\,\sigma^{i-}\gamma_5]}(x) = 0\,. \tag{9.212}$$

As we have already remarked, transversity distribution function is not accessible if the hadron is in an eigenstate of helicity and the helicity correlators we built up just describe the soft part of a hard process in which initial and final hadron states are eigenstates of hadron helicity. One needs a transverse component of the hadron spin vector which couples to the partonic transverse momentum $\vec{\mathbf{k}}_T$ in order to be sensitive to transverse separation between quarks. We know that helicity flip correlators link hadronic states carrying opposite helicity and a transverse spin state can always be represented as linear combinations of helicity eigenstates. Therefore also the matrix elements of non-diagonal spin states, which are transverse spin states, can be re-expressed as an appropriate linear combination of matrix elements evaluated between eigenstates of helicity. It follows that the trace of the helicity flip correlators (6.148) with the matrix $i\,\sigma^{i-}\gamma_5$ cannot not be zero and indeed implementing the ansätze (6.148) for $\Phi_{i+,j-}$ and $\Phi_{i-,j+}$ in (9.205) results in the following

$$\begin{aligned} h_1(x) &= \Phi_{+-}^{[i\,\sigma^{i-}\gamma_5]}(x) = -\Phi_{-+}^{[i\,\sigma^{i-}\gamma_5]}(x) = \\ h_1(x) &= \int d\sigma\, d\tau\, \theta(x\,\sigma - x^2\,m^2 - \tau)\,[\vec{\mathbf{k}}_T\, k\cdot a\, a_{21}^{(3)} + \epsilon^{+i\rho\sigma} P_\rho k_\sigma\, k\cdot a\, i a_{27}^{(3)}], \end{aligned} \tag{9.213}$$

As announced previously (see Eq. (6.151) at twist 2 we expect to produce chiral odd distributions functions as $h_1(x)$ from the helicity flip correlators, namely off-diagonal correlators. On the other hand the distribution functions as $f_1(x)$ and $g_1(x)$, obtained by tracing helicity non-flip, namely diagonal, correlators, have to be chiral even. In the off-forward case this restriction is not anymore valid since helicity conservation is violated as in off-forward hard process a transverse component of the vector Δ, the momentum transfer of the process, is always available.

9.3 Leading order Dirac projections of the off-forward quark-quark correlators

We analyze now the leading order Dirac projections of the off-forward quark-quark correlators given in (7.186), namely we trace the correlators with the Dirac matrices γ^+, $\gamma^+\gamma_5$ and $i\,\sigma^{+i}\gamma_5$.

9.3.1 Unpolarized skewed parton distributions

Substituting the explicit expression of the ansatz (7.186) for the helicity non-flip correlators, $\tilde{\Phi}_{++}$ or $\tilde{\Phi}_{--}$, in (9.206) and tracing with γ^+ we find that

$$\tilde{\Phi}_{++}^{[\gamma^+]}(x,\xi;t) = \tilde{\Phi}_{--}^{[\gamma^+]}(x,\xi;t) =$$
$$\int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\sigma - x^2\,m^2 - \tau)\,[-2\xi\frac{\Delta\cdot v'}{\bar{P}^+}\,(d_{29}^{(1)} + d_{30}^{(1)}) + \frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{11}^{(1)} + d_{12}^{(1)})$$
$$+x\,(d_{17}^{(1)} + d_{18}^{(1)}) + x\,\frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{19}^{(1)} + d_{20}^{(1)}) + (d_9^{(1)} + d_{10}^{(1)})]\,. \tag{9.214}$$

On the other hand from (8.196) we also know that

$$\mathcal{H}_{++} = \frac{\tilde{\Phi}_{++}^{[\gamma^+]}}{\sqrt{1-\xi^2}} = \mathcal{H}_{--} = \frac{\tilde{\Phi}_{--}^{[\gamma^+]}}{\sqrt{1-\xi^2}} = H^q - \frac{\xi^2}{1-\xi^2}\,E^q\,, \tag{9.215}$$

and therefore

$$\begin{aligned}\mathcal{H}_{++} \;&=\; \mathcal{H}_{--} = \frac{1}{\sqrt{1-\xi^2}}\,[-2\xi\frac{\Delta\cdot v'}{\bar{P}^+}\,(d_{29}^{(1)} + d_{30}^{(1)}) + \frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{11}^{(1)} + d_{12}^{(1)}) \\ &+\; x\,(d_{17}^{(1)} + d_{18}^{(1)}) + x\,\frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{19}^{(1)} + d_{20}^{(1)}) + (d_9^{(1)} + d_{10}^{(1)})]\,.\end{aligned} \tag{9.216}$$

$\Delta\cdot v'$ and $\bar{k}\cdot v'$ are given as

$$\begin{aligned}\Delta\cdot v' \;&=\; -2\,\xi\,\bar{P}^+ \\ \bar{k}\cdot v' \;&=\; x\,\bar{P}^+\,.\end{aligned} \tag{9.217}$$

and thus (9.216) becomes

$$\begin{aligned}\mathcal{H}_{++} \;&=\; \mathcal{H}_{--} = \frac{1}{\sqrt{1-\xi^2}}\,[\,4\xi^2\,(d_{29}^{(1)} + d_{30}^{(1)}) + x\,(d_{11}^{(1)} + d_{12}^{(1)} \\ &\qquad +d_{17}^{(1)} + d_{18}^{(1)}) + x^2\,(d_{19}^{(1)} + d_{20}^{(1)}) + (d_9^{(1)} + d_{10}^{(1)})\,]\,.\end{aligned} \tag{9.218}$$

We remark in passing that by virtue of the relations (7.187) between $d_m^{(1)}$ and $d_m^{(4)}$ we can indeed verify that the trace $\mathcal{H}_{++}$ of Φ_{++} with the matrix γ^+ is equal to the trace $\mathcal{H}_{--}$ of Φ_{--} with the same Dirac matrix.
We can further insert the ansatz (6.148) for the off-diagonal correlators Φ_{-+} or Φ_{+-} in (9.206) obtaining

$$\tilde{\Phi}_{-+}^{[\gamma^+]}(x,\xi;t) = -(\tilde{\Phi}_{+-}^{[\gamma^+]}(x,\xi;t))^*$$
$$\int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tau)\,(-\eta)\,[-2\xi\frac{\Delta\cdot v'}{\bar{P}^+}\,(d_{29}^{(3)} + d_{30}^{(3)}) + \frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{11}^{(3)} + d_{12}^{(3)})$$
$$+x\,(d_{17}^{(3)} + d_{18}^{(3)}) + x\,\frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{19}^{(3)} + d_{20}^{(3)}) + (d_9^{(3)} + d_{10}^{(3)})]\,. \tag{9.219}$$

where η is defined as

$$\eta = \frac{\Delta^1 + i\Delta^2}{|\mathbf{\Delta}_\perp|}\,. \tag{9.220}$$

In the helicity flip case the trace of the correlators with the Dirac matrix γ^+ in (8.196) gives

$$\begin{aligned}
\mathcal{H}_{-+} &= -(\mathcal{H}_{+-})^* = \eta\,\frac{\sqrt{t_0-t}}{2m}\frac{1}{\sqrt{1-\xi^2}}\,E^q \\
&= \frac{-\eta}{\sqrt{1-\xi^2}}\,[-2\xi\frac{\Delta\cdot v'}{\bar{P}^+}\,(d_{29}^{(3)}+d_{30}^{(3)}) \\
&+ \frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{11}^{(3)}+d_{12}^{(3)}) + x\,(d_{17}^{(3)}+d_{18}^{(3)}) \\
&+ x\,\frac{\bar{k}\cdot v'}{\bar{P}^+}\,(d_{19}^{(3)}+d_{20}^{(3)}) + (d_{9}^{(3)}+d_{10}^{(3)})]\,.
\end{aligned} \tag{9.221}$$

We have now a set of two equations for the two unknown skewed distribution functions H^q and E^q, which is solvable

$$\begin{aligned}
H^q - \frac{\xi^2}{1-\xi^2}\,E^q &= \frac{1}{\sqrt{1-\xi^2}}\,[4\xi^2\,(d_{29}^{(1)}+d_{30}^{(1)}) \\
&+ x\,(d_{11}^{(1)}+d_{12}^{(1)}+d_{17}^{(1)}+d_{18}^{(1)}) \\
&+ x^2\,(d_{19}^{(1)}+d_{20}^{(1)}) + (d_{9}^{(1)}+d_{10}^{(1)})]
\end{aligned} \tag{9.222}$$

$$\begin{aligned}
\eta\,\frac{\sqrt{t_0-t}}{2m}\frac{1}{\sqrt{1-\xi^2}}\,E^q &= -\frac{\eta}{\sqrt{1-\xi^2}}\,[4\xi^2\,(d_{29}^{(3)}+d_{30}^{(3)}) \\
&+ x\,(d_{11}^{(3)}+d_{12}^{(3)}+d_{17}^{(3)}+d_{18}^{(3)}) \\
&+ x^2\,(d_{19}^{(3)}+d_{20}^{(3)}) + (d_{9}^{(3)}+d_{10}^{(3)})]\,.
\end{aligned} \tag{9.223}$$

and gives the two unpolarized skewed parton distribution functions H^q and E^q as

$$\begin{aligned}
H^q &= \frac{1}{\sqrt{1-\xi^2}}\,[A^{(1)}(x,\xi;t) - \frac{2m\,\xi^2}{\sqrt{1-\xi^2}\,\sqrt{t_0-t}}\,A^{(3)}(x,\xi;t)] \\
E^q &= -\frac{2\,m}{\sqrt{t_0-t}}\,A^{(3)}(x,\xi;t)\,,
\end{aligned} \tag{9.224}$$

where we have introduced the function $A^{(1)}(x,\xi;t)$ and $A^{(3)}(x,\xi;t)$ defined as

$$\begin{aligned}
A^{(1)}(x,\xi;t) &= \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tau)\,[4\xi^2\,(d_{29}^{(1)}+d_{30}^{(1)}) \\
&+ x\,(d_{11}^{(1)}+d_{12}^{(1)}+d_{17}^{(1)}+d_{18}^{(1)}) + x^2\,(d_{19}^{(1)}+d_{20}^{(1)}) + (d_{9}^{(1)}+d_{10}^{(1)})]
\end{aligned} \tag{9.225}$$

and

$$
\begin{aligned}
A^{(3)}(x,\xi;t) \;&=\; \int d\,\tilde\sigma\,d\,\tilde\tau\,\theta(x\,\tilde\sigma - x^2\,m^2-\tau)\,[4\xi^2\,(d_{29}^{(3)}+d_{30}^{(3)}) \\
&+\; x\,(d_{11}^{(3)}+d_{12}^{(3)}+d_{17}^{(3)}+d_{18}^{(3)})+x^2\,(d_{19}^{(3)}+d_{20}^{(3)})+(d_9^{(3)}+d_{10}^{(3)})]
\end{aligned}
\tag{9.226}
$$

respectively. As expected, the skewed parton distributions E^q decouples from the set of equations in the forward limit and the H^q function leads to the the ordinary parton distribution f_1^q

$$
\begin{aligned}
&\lim_{\xi\to 0, t\to 0} H^q(x,\xi;t) = A^{(1)}(x,\xi=0;t=0) = \\
&\quad (d_9^{(1)}+d_{10}^{(1)})+x\,(d_{11}^{(1)}+d_{12}^{(1)}+d_{17}^{(1)}+d_{18}^{(1)})+x^2\,(d_{19}^{(1)}+d_{20}^{(1)})
\end{aligned}
\tag{9.227}
$$

By an appropriate renaming of the amplitudes the unpolarized skewed parton distribution $H^q(x,\xi;t)$ in the limit $\xi\to 0$ and $t\to 0$ is equal to the unpolarized parton distribution $f_1(x)$ expressed in (9.207).

9.3.2 Polarized skewed parton distributions

By tracing the helicity non-flip quark-quark correlators (7.186), Φ_{++} or Φ_{--}, with $\gamma^+\gamma_5$ we have

$$
\begin{aligned}
&\tilde\Phi_{++}^{[\gamma^+\gamma_5]}(x,\xi;t) = -\tilde\Phi_{--}^{[\gamma^+\gamma_5]}(x,\xi;t) = \\
&\quad\int d\,\tilde\sigma\,d\,\tilde\tau\,\theta(x\,\tilde\sigma - x^2\,m^2-\tau)[-2\xi\,\frac{\Delta\cdot v'}{\bar P^+}\,(d_{63}^{(1)}+d_{64}^{(1)})+\,\frac{\bar k\cdot v'}{\bar P^+}\,(d_{45}^{(1)}+d_{46}^{(1)}) \\
&\quad +x\,(d_{51}^{(1)}+d_{52}^{(1)})+(d_{43}^{(1)}+d_{44}^{(1)})+x\,\frac{\bar k\cdot v'}{\bar P^+}(\,d_{53}^{(1)}+d_{54}^{(1)}\,)]\,.
\end{aligned}
\tag{9.228}
$$

From (8.200) we also know that

$$
\widetilde{\mathcal{H}}_{++} = \frac{\tilde\Phi_{++}^{[\gamma^+\gamma_5]}}{\sqrt{1-\xi^2}} = -\widetilde{\mathcal{H}}_{--} = \frac{\tilde\Phi_{--}^{[\gamma^+\gamma_5]}}{\sqrt{1-\xi^2}} = \tilde H^q - \frac{\xi^2}{1-\xi^2}\,\tilde E^q\,,
\tag{9.229}
$$

Furthermore the trace of the helicity flip correlators with $\gamma^+\gamma_5$ reads

$$
\begin{aligned}
&\tilde\Phi_{-+}^{[\gamma^+\gamma_5]}(x,\xi;t) = (\tilde\Phi_{+-}^{[\gamma^+\gamma_5]}(x,\xi;t))^* = \\
&\quad\int d\,\tilde\sigma\,d\,\tilde\tau\,\theta(x\,\tilde\sigma - x^2\,m^2-\tau)(-\eta)\,[-2\xi\,\frac{\bar k\cdot v'}{\bar P^+}\,(d_{61}^{(3)}+d_{62}^{(3)}\,) \\
&\quad +x\,\frac{\Delta\cdot v'}{\bar P^+}\,(d_{55}^{(3)}+d_{56}^{(3)})+\,\frac{\Delta\cdot v'}{\bar P^+}\,(d_{47}^{(3)}+d_{48}^{(3)})-2\xi\,(d_{59}^{(3)}+d_{60}^{(3)}))\,]\,,
\end{aligned}
\tag{9.230}
$$

and from (8.200)

$$
\widetilde{\mathcal{H}}_{-+} = \quad (\widetilde{\mathcal{H}}_{+-}^q)^* = \eta\,\frac{\sqrt{t_0-t}}{2m}\frac{\xi}{\sqrt{1-\xi^2}}\,\tilde E^q\,.
\tag{9.231}
$$

We obtain a set of two equations in the two unknown functions $\tilde{H}^q$ and $\tilde{E}^q$ which give

$$\begin{aligned}\tilde{E}^q &= -\frac{2\,m}{\xi\,\sqrt{t_0-t}}\,B^{(3)}(x,\xi;t) \\ \tilde{H}^q &= \frac{1}{\sqrt{1-\xi^2}}\,[B^{(1)}(x,\xi;t) - \frac{2m\,\xi}{\sqrt{1-\xi^2}\,\sqrt{t_0-t}}\,B^{(3)}(x,\xi;t)]\,,\end{aligned} \tag{9.232}$$

where we have introduced the function $B^{(1)}(x,\xi;t)$ and $B^{(3)}(x,\xi;t)$ defined as

$$\begin{aligned}B^{(1)}(x,\xi;t) &= \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma}-x^2\,m^2-\tau)\,[4\xi^2\,(d^{(1)}_{63}+d^{(1)}_{64})+(d^{(1)}_{43}+d^{(1)}_{44}) \\ &+ x\,(d^{(1)}_{45}+d^{(1)}_{46}+d^{(1)}_{51}+d^{(1)}_{52})+x^2\,(d^{(1)}_{53}+d^{(1)}_{54})]\end{aligned} \tag{9.233}$$

and

$$\begin{aligned}B^{(3)}(x,\xi;t) &= \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma}-x^2\,m^2-\tau)\,[4\xi^2\,(-2\,\xi\,x\,d^{(3)}_{61}+d^{(3)}_{62} \\ &+d^{(3)}_{56}+d^{(3)}_{55}-2\,\xi\,(d^{(3)}_{47}+d^{(3)}_{48}+d^{(3)}_{59}+d^{(3)}_{60})]\,,\end{aligned} \tag{9.234}$$

respectively. Note that $B^{(3)}(x,\xi;t)$ is identically equal to zero in the forward limit and $\tilde{H}^q$ coincides with $g_1(x)$ in the forward limit.

9.3.3 Parton helicity changing distributions

Since the number of independent parton helicity changing SPDs was subject to debate in the literature, it is important that the present twist-analysis offers the opportunity of an independent and unambiguous check of this number, which only relays on the general principles used to construct the ansätze.
Tracing the ansätze (7.186) with the matrix $i\,\sigma^{i+}\gamma_5$ we obtain

$$\begin{aligned}\tilde{\Phi}^{[i\,\sigma^{i+}\gamma_5]}_{++}(x,\xi;t) &= C^{(1)\,i}+D^{(1)\,i} \\ \tilde{\Phi}^{[i\,\sigma^{i+}\gamma_5]}_{--}(x,\xi;t) &= C^{(1)\,i}-D^{(1)\,i} \\ \tilde{\Phi}^{[i\,\sigma^{i+}\gamma_5]}_{-+}(x,\xi;t) &= C^{(3)\,i}+D^{(3)\,i} \\ \tilde{\Phi}^{[i\,\sigma^{i+}\gamma_5]}_{+-}(x,\xi;t) &= C^{(3)\,i}-D^{(3)\,i}\end{aligned} \tag{9.235}$$

where $C^{(1)}$, expressed in terms of the amplitudes occurring in the ansatz (7.186), reads

$$\begin{aligned}C^{(1)\,i} &= \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma}-x^2\,m^2-\tau) \\ &\{\Delta^i\,[2\,\xi(d^{(1)}_{83}+d^{(1)}_{84})\,(-x\,\bar{P}\cdot\bar{k})+2\,\xi\,(d^{(1)}_{91}+d^{(1)}_{92})\,(x\,\bar{P}\cdot\bar{k}-\bar{k}^2)] \\ &+1/P^+\,\epsilon^{+i\rho\sigma}\Delta_\rho\bar{P}_\sigma\,(-d^{(1)}_{77}-d^{(1)}_{78}+d^{(1)}_{85}+d^{(1)}_{86})+x\,(d^{(1)}_{87}+d^{(1)}_{88}-d^{(1)}_{79}-d^{(1)}_{80})]\}\,,\end{aligned} \tag{9.236}$$

$D^{(1)\,i}$ reads

$$D^{(1)\,i} = \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tau)\left\{\Delta^i\,[2\,y\,(-x\,d^{(1)}_{114} + i\,d^{(1)}_{109} + i\,d^{(1)}_{110})]\right\} \tag{9.237}$$

and $C^{(3\,i)}$ can be written as

$$C^{(3)\,i} = \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tau)$$
$$\{\Delta^i\,[2\,\xi\,(d^{(3)}_{75} - d^{(3)}_{76})(x^2 - \bar{P}\cdot\bar{k}\,x^2 - \bar{P}\cdot\bar{k}\,x - \bar{k}^2\,x)]$$
$$+1/P^+\,\epsilon^{\rho\sigma+i}\Delta_\rho\bar{P}_\sigma[x\,(d^{(3)}_{80} - d^{(3)}_{79}) + x^2\,(d^{(3)}_{87} - d^{(3)}_{88}) + d^{(3)}_{78} - d^{(3)}_{77} - d^{(3)}_{86} + d^{(3)}_{85}]\} \tag{9.238}$$

and $D^{(3)\,i}$ in terms of the amplitudes $d^{(3)}_m$ is

$$D^{(3)\,i} = \int d\,\tilde{\sigma}\,d\,\tilde{\tau}\,\theta(x\,\tilde{\sigma} - x^2\,m^2 - \tau)$$
$$[\Delta^i\,(i\,x\,(d^{(3)}_{111} - d^{(3)}_{112} - d^{(3)}_{107} + d^{(3)}_{108}) - i\,d^{(3)}_{105} + i\,d^{(3)}_{106}) + x^2\,d^{(3)}_{113})] \tag{9.239}$$

We observe that only two tensor structures, Δ^i and $\epsilon^{\rho\sigma+i}\Delta_\rho\bar{P}_\sigma$, occur in $C^{(1)\,i}$, $D^{(1)\,i}$, $C^{(3)\,i}$ and $D^{(3)\,i}$. The fact that the Eq. (9.235) constitutes a set of four linearly independent equations unambiguously signals that one can define four independent SPDs from it. This situation is different from the projections with γ^+ and $\gamma^+\gamma_5$ where only two independent equations arise from the different hadron helicity combinations.
From (9.235) we have freedom of choice for the definition of SPDs, like for the elastic form factors which some define as F_1,F_2 and others as G_E, G_M. In the following we adopt the choice of definitions for the parton helicity changing SPDs as given by Diehl. From Equation (8.202) follows

$$\mathcal{G}^{q}_{\Lambda'\Lambda}{}^{i} = \frac{\tilde{\Phi}^{[i\sigma^{i+}\gamma_5]}_{\Lambda'\Lambda}}{\sqrt{1-\xi^2}} \tag{9.240}$$

and inserting the results for the spinorial products in the average frame, defined in the Appendix, one obtains up to a phase ($-\eta$ for $\mathcal{G}^q_{-+}$ and η^* for $\mathcal{G}^q_{+-}$)

$$\begin{aligned}
\mathcal{G}^{q}_{++}{}^{i} \;=\;& H^q_T\,\frac{\epsilon^{+j\rho\sigma}\,\bar{P}_\rho\Delta_\sigma\,\xi\,\sqrt{1-\xi^2}}{(1+\xi)\,m} \\
&-\widetilde{H}^q_T\,2m\,\epsilon^{+j\rho\sigma}\,\bar{P}_\rho\Delta_\sigma\,\sqrt{-1+\xi^2} \\
&+E^q_T\,\frac{2\,(\bar{P}^+\Delta^j i\,\xi - \epsilon^{+j\rho\sigma}\,\bar{P}_\rho\Delta_\sigma)}{\sqrt{-1+\xi^2}} \\
&+\widetilde{E}^q_T\,\frac{-i\,\bar{P}^+\,\Delta^j + \xi\,\epsilon^{+j\rho\sigma}\,\bar{P}_\rho\Delta_\sigma}{\sqrt{1-\xi^2}}
\end{aligned} \tag{9.241}$$

$$
\begin{aligned}
\mathcal{G}^{q}_{--}{}^{i} \;\; = \;\; & H^q_T \, \frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \, \xi \, \sqrt{1-\xi^2}}{(1+\xi)\, m} \\
& - \widetilde{H}^q_T \, 2m \, \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \, \sqrt{-1+\xi^2} \\
& + E^q_T \, \frac{2\,(-\bar{P}^+ \Delta^j i\, \xi - \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma)}{\sqrt{-1+\xi^2}} \\
& + \widetilde{E}^q_T \, \frac{i\, \bar{P}^+ \, \Delta^j + \xi \, \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{1-\xi^2}}
\end{aligned}
\tag{9.242}
$$

$$
\begin{aligned}
\mathcal{G}^{q}_{-+}{}^{i} \;\; = \;\; & -2\, H^q_T \, \frac{-i \bar{P}^+ \Delta^j + \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \\
& - \widetilde{H}^q_T \, \frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \, (4m^2 \xi^2 - \xi^2\, t + t)}{\sqrt{1-\xi^2}\sqrt{t_0 - t}} \\
& + E^q_T \, 4m\xi^2 \, \frac{-\bar{P}^+ \Delta^j i - \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{-1+\xi^2}\, \sqrt{t_0 - t}} \\
& - \widetilde{E}^q_T \, 2 \, \frac{m\xi(-i\, \bar{P}^+ \, \Delta^j - \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{-1+\xi^2}\sqrt{t_0 - t}}
\end{aligned}
\tag{9.243}
$$

$$
\begin{aligned}
\mathcal{G}^{q}_{+-}{}^{i} \;\; = \;\; & -2\, H^q_T \, \frac{i \bar{P}^+ \Delta^j + \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{t_0 - t}} \\
& - \widetilde{H}^q_T \frac{\epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma \, (4m^2 \xi^2 - \xi^2\, t + t)}{\sqrt{1-\xi^2}\sqrt{t_0 - t}} \\
& + E^q_T \, 4m\xi^2 \, \frac{\bar{P}^+ \Delta^j i - \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{-1+\xi^2}\, \sqrt{t_0 - t}} \\
& - \widetilde{E}^q_T \, 2 \, \frac{m\xi(i\, \bar{P}^+ \, \Delta^j - \epsilon^{+j\rho\sigma} \bar{P}_\rho \Delta_\sigma}{\sqrt{-1+\xi^2}\sqrt{t_0 - t}}
\end{aligned}
\tag{9.244}
$$

We have shown that the number of independent helicity changing skewed parton distributions is four as claimed in a recent paper by Markus Diehl.
Note that the two methods applied by Diehl and by us, respectively, to deduce the number of helicity changing skewed parton distributions are completely independent. On one hand starting from very general principles we wrote the most general ansatz which can describe off-forward quark-quark correlation functions and then we traced this ansatz with different Dirac matrices. On the other hand Diehl reached the same conclusion by considering directly the skewed parton distributions and implementing correctly for them the time reversal constraint. [Die01]

10 Conclusion

In this work we presented a detailed analysis of forward and off-forward quark-quark correlation functions. We stressed the physical significance of quark-quark correlators in parametrising the long-distance physics, not describable by perturbative QCD.
We provided a definition of forward and off-forward quark-quark correlators with respect to a spin basis and to a light-cone helicity basis, clarifying the relation between the two representations. Moreover constraints on quark-quark correlation functions were obtained from the transformation properties of the fundamental fields of QCD occurring in the definition of the correlators. In particular, the constraints obtained for the off-forward correlators were not yet in literature.
A further step of the work consisted in developing a method to construct ansätze for both forward and off-forward quark-quark correlation functions in spin and light-cone helicity basis. We provided motivations to build ansätze for the off-forward correlators in an helicity basis. We stress that the method used to construct the ansatz in the helicity basis is new. Ansätze for the forward quark-quark correlation functions were modelled in both spin basis and helicity basis, while those for the off-forward case were built in the helicity basis. The quark-quark correlators can be expressed in terms of tensorial structures formed by the independent vectors and the Dirac matrices. The constraints previously obtained for the correlators were implemented to reduce the number of independent amplitudes multiplying these tensorial structures in the ansätze.
Finally we projected out the leading order SPDs, i.e. we expressed the unpolarised, polarised and parton helicity flip distributions in terms of the amplitudes entering in the ansätze we wrote for the forward and off-forward case. The formalism of twist-analysis here adopted allowed to conclude that the number of independent parton helicity changing distributions is four in agreement with Diehl's argument. We want to stress that the result about the number of these independent functions was obtained by Diehl in a completely different way and this is a confirmation of both methods used to approach the problem. On one hand we represented matrix elements of non-local quark-quark operators in terms of tensorial structures built from the involved momenta on the basis of general properties of invariance. Then we traced the correlators with different Dirac matrices and we could read off which of these structures contribute to each SPD. On the other hand Diehl's approach was to count the number of independent helicity amplitudes occurring in DVCS cross section on the basis of time reversal and parity invariance which these amplitudes have to fulfil.
We worked out a powerful method of analysis which in the present was applied completely to the leading twist level. The same method can be implemented to investigate twist 3 and twist 4 skewed distribution functions. The analysis of twist 3 and twist 4 skewed distribution functions is beyond the scope of this work but it may be extremely useful. For instance one could expect that useful relations between leading and next to leading order skewed distributions could emerge as suggested by similar experience in the forward case. In this sense the present work represents a valuable starting point for further investigations.

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Appendix

A Light-cone components of a vector

We define the light-cone components of a vector, which will be used extensively through the whole thesis.
In terms of the four light-cone components, an arbitrary four-vector $a^\mu = (\, a^0, a^1, a^2, a^3 \,)$ can be rewritten as

$$a^\mu = [\, a^+, a^-, \vec{a}_\perp \,] \tag{A.245}$$

where

$$\begin{aligned} a^\pm &= \frac{1}{\sqrt{2}} (\, a^0 \pm a^3 \,) \\ \vec{a}^\perp &= (\, a^1, a^2 \,). \end{aligned} \tag{A.246}$$

In this basis the metric $g_{\mu\nu}$ has non-zero components $g_{+-} = g_{-+} = 1$ and $g_{ij} = -\delta_{ij}$, where the indices i and j can be either 1 or 2.

B Kinematics and scalar products

B.1 Useful transformation

We define Lorentz transformation between different frames of reference which leave the "$+$"-component unchanged i.e. involving a parameter b^+ and a two-dimensional vector $\vec{b}$ such that

$$a^\mu = [\, a^+ \,,\, a^- \,,\, \vec{a} \,] \qquad \longrightarrow \qquad \tilde{a}^\mu = \left[\, a^+ \,,\, a^- - \frac{\vec{a}\cdot\vec{b}}{b^+} + \frac{a^+\,\vec{b}^2}{2\,(b^+)^2} \,,\, \vec{a} - \frac{a^+}{b^+}\,\vec{b} \,\right] \tag{B.247}$$

with

$$\begin{aligned} \tilde{a}^2 &= 2\,a^+a^- - 2\,a^+\frac{\vec{a}\cdot\vec{b}}{b^+} + 2\,a^+\,\frac{a^+\,\vec{b}^2}{2(b^+)^2} - \vec{a}^2 + 2\,\frac{a^+}{b^+}\,\vec{a}\cdot\vec{b} - \left(\frac{a^+}{b^+}\right)^2 \vec{b}^2 \\ &= 2\,a^+a^- - \vec{a}^2 = a^2 \,. \end{aligned} \tag{B.248}$$

B.2 Frames of reference

We discuss two different frames of reference in which the description of DVCS reaction is usually worked out.

B.2.1 "Skewness frame"

In the frame of reference which we call "skewness frame", one chooses the momentum of the incoming proton P and of the virtual photon q to be collinear [RW01]

$$\begin{aligned}
P^\mu &= \left[P^+ \, , \, \frac{M^2}{2\,P^+} \, , \, \vec{0}_\perp \right] \\
\Delta^\mu &= \left[-\zeta\, P^+ \, , \, \frac{\zeta M^2 + \vec{\Delta}_\perp^2}{2\,P^+(1-\zeta)} \, , \, \vec{\Delta}_\perp \right] \\
q^\mu &= \left[-x_N\, P^+ \, , \, \frac{Q^2}{2\,x_N\,P^+} \, , \, \vec{0}_\perp \right] \\
P'^\mu &= P^\mu + \Delta^\mu = \left[(1-\zeta)\,P^+ \, , \, \frac{M^2 + \vec{\Delta}_\perp^2}{2\,P^+\,(1-\zeta)} \, , \, \vec{\Delta}_\perp \right]
\end{aligned} \tag{B.249}$$

Partons in the incoming proton have momenta

$$k_i^\mu = \left[X_i\, P^+ \, , \, \frac{k_i^2 + \vec{k}_{i\,\perp}^2}{2\,X_i\,P^+} \, , \, \vec{k}_{i\,\perp} \right] \tag{B.250}$$

and partons in the outgoing proton

$$\begin{aligned}
k_i'^{\,\mu} &= k_i^\mu \qquad \text{for} \quad i \neq j \\
k_j'^{\,\mu} &= k_j^\mu + \Delta^\mu = \left[(X_i - \zeta)\,P^+ \, , \, \frac{k_j'^{\,2} + (\vec{k}_{j\,\perp} + \vec{\Delta}_\perp)^2}{2\,(X_i-\zeta)\,P^+} \, , \, \vec{k}_{j\,\perp} + \vec{\Delta}_\perp \right] \qquad \textit{active}
\end{aligned} \tag{B.251}$$

A transverse boost with $\vec{b} = \vec{\Delta}_\perp$ and $b^+ = (1-\zeta)\,p^+$ leads to a frame, where the outgoing proton has no transverse momentum components

$$P'^{\,\mu} \quad \longrightarrow \quad \breve{P}'^{\,\mu} = \left[(1-\zeta)\,P^+ \, , \, \frac{M^2}{2\,(1-\zeta)\,P^+} \, , \, \vec{0}_\perp \right] \tag{B.252}$$

and

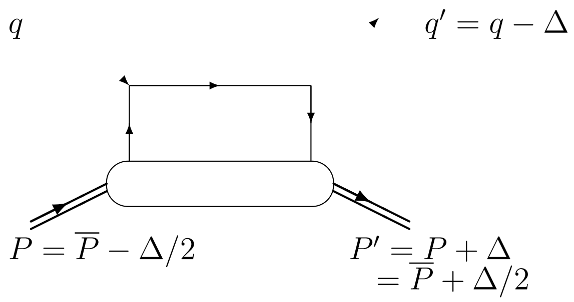
$$\begin{aligned}
k_i' \;\longrightarrow\; \breve{k}_i' &= \left[X_i\,P^+ \, , \, \frac{k_i'^{\,2} + (\vec{k}_{i\,\perp} - \frac{X_i}{1-\zeta}\vec{\Delta}_\perp)^2}{2\,X_i\,P^+} \, , \, \vec{k}_{i\,\perp} - \frac{X_i}{1-\zeta}\vec{\Delta}_\perp \right] \qquad \text{for} \quad i \neq j \\
k_j' \;\longrightarrow\; \breve{k}_j' &= \left[(X_j-\zeta)\,P^+ \, , \, \frac{k_j'^{\,2} + (\vec{k}_{j\,\perp} + \frac{1-X_j}{1-\zeta}\vec{\Delta}_\perp)^2}{2\,(X_j-\zeta)\,P^+} \, , \, \vec{k}_{j\,\perp} + \frac{1-X_j}{1-\zeta}\vec{\Delta}_\perp \right]
\end{aligned} \tag{B.253}$$

such that the arguments for the outgoing proton wave function read

$$\psi^{out}\left(\frac{X_i}{1-\zeta}, \vec{k}_{i\,\perp} - \frac{X_i}{1-\zeta}\vec{\Delta}_\perp ; \; \frac{X_j-\zeta}{1-\zeta}, \vec{k}_{j\,\perp} + \frac{1-X_j}{1-\zeta}\vec{\Delta}_\perp \right) \tag{B.254}$$

B.2.2 "Average frame"

Close to Ji's conventions [Ji98b] we choose a frame where the longitudinal direction is defined by the proton average momentum



$$\boxed{\begin{array}{c} P^2 = P'^2 = M^2 \\ q^2 = -Q^2,\ q'^2 = 0 \end{array}}$$

$$\Downarrow$$

$$P \cdot \Delta = -\frac{\Delta^2}{2}$$

$$q \cdot \Delta = \frac{\Delta^2 - Q^2}{2}$$

One defines the proton average momentum as:

$$\overline{P} = (P + P')/2 \quad \text{such that} \quad \begin{cases} P = \overline{P} - \Delta/2 \\ \text{and} \\ P' = \overline{P} + \Delta/2 \end{cases} \tag{B.255}$$

and the following light-like vectors:

$$v^\mu = \left[\ 1\ ,\ 0\ ,\ \vec{0}_\perp\ \right]\ ; \qquad v'^\mu = \left[\ 0\ ,\ 1\ ,\ \vec{0}_\perp\ \right]\ . \tag{B.256}$$

Partons in the incoming proton have momenta

$$k_i^\mu = \left[\ X_i\, P^+\ ,\ \frac{k_i^2 + \vec{k}_{i\perp}^2}{2\, X_i\, P^+}\ ,\ \vec{k}_{i\perp}\ \right] \tag{B.257}$$

and partons in the outgoing proton

$$\begin{aligned} k_i'^\mu &= k_i^\mu \qquad \text{for} \quad i \neq j \\ k_j'^\mu &= k_j^\mu + \Delta^\mu = \left[\ (X_i - \zeta)\, P^+\ ,\ \frac{k_j'^2 + (\vec{k}_{j\perp} + \vec{\Delta}_\perp)^2}{2\,(X_i - \zeta)\, P^+}\ ,\ \vec{k}_{j\perp} + \vec{\Delta}_\perp\ \right] \qquad \mathit{active} \end{aligned} \tag{B.258}$$

The Sudakov decomposition of the external vectors reads: (choose $\overline{p}$ and q collinear)

$$\overline{P}^\mu \ = \ \left[\ \overline{P}^+\ ,\ \frac{\overline{M}^2}{2\overline{P}^+}\ ,\ \vec{0}_\perp\ \right] \tag{B.259}$$

$$q^\mu = \left[-x_N \overline{P}^+ , \frac{Q^2}{2\, x_N \overline{P}^+} , \vec{0}_\perp \right] \qquad \boxed{x_N = -\frac{q^+}{\overline{P}^+}} \tag{B.260}$$

$$\Delta^\mu = \left[-2\xi \overline{P}^+ , \frac{\xi \overline{M}^2}{\overline{P}^+} , \vec{\Delta}_\perp \right] \qquad \boxed{\xi = -\frac{\Delta^+}{2\, \overline{P}^+}} \tag{B.261}$$

with

$$\overline{M}^2 = \overline{P}^2 = (P + \Delta/2)^2 = M^2 + P \cdot \Delta + \Delta^2/4 = M^2 - \Delta^2/4 \tag{B.262}$$

and

$$\overline{P} \cdot q = \frac{Q^2}{2\, x_N} - \frac{x_N\, \overline{M}^2}{2} \quad \Longrightarrow \quad x_N = \left(-\overline{P} \cdot q + \sqrt{(\overline{P} \cdot q)^2 + Q^2 \overline{M}^2} \right) / \overline{M}^2 \tag{B.263}$$

Note that

$$\lim_{\overline{M}^2 \to 0} x_N = \frac{Q^2}{2\, \overline{P} \cdot q} \qquad \text{(de l'Hospital)}$$

the component Δ^- is determined by

$$\begin{aligned} \overline{P} \cdot \Delta &= (P + \Delta/2) \cdot \Delta = P \cdot \Delta + \Delta^2/2 = 0 \\ &= \overline{P}^+ \Delta^- + \frac{\overline{M}^2}{2\overline{P}^+} \Delta^+ \\ &\Longrightarrow \quad \Delta^- = -\frac{\overline{M}^2}{2(\overline{P}^+)^2} \Delta^+ = \frac{\xi\, \overline{M}^2}{\overline{P}^+} \end{aligned} \tag{B.264}$$

The Mandelstam variable t reads

$$t = \Delta^2 = -4\xi^2 \overline{M}^2 - \vec{\Delta}_\perp^2 \tag{B.265}$$

from which we obtain (insert (B.265) in (B.262))

$$\overline{M}^2 (1 - \xi^2) = M^2 + \vec{\Delta}_\perp^2/4 \tag{B.266}$$

or (insert (B.262) in (B.265))

$$\Delta^2 = \frac{-4\, \xi^2 M^2 - \vec{\Delta}_\perp^2}{1 - \xi^2} \,. \tag{B.267}$$

The momentum of the real photon is

$$q'^\mu = (q - \Delta)^\mu = \left[(2\xi - x_N)\, \overline{P}^+ , \frac{Q^2 - 2\, x_N \xi \overline{M}^2}{2 x_N \overline{P}^+} , -\vec{\Delta}_\perp \right] . \tag{B.268}$$

We also show the explicit form of the incoming and outgoing proton momenta

$$\begin{aligned} P^{\mu} &= (\overline{P}-\Delta/2)^{\mu} = \left[(1+\xi)\,\overline{P}^{+}\;,\; \frac{M^2+\vec{\Delta}_{\perp}^2/4}{2(1+\xi)\overline{P}^{+}}\;,\; -\vec{\Delta}_{\perp}/2 \right] \\ P'^{\mu} &= (\overline{P}+\Delta/2)^{\mu} = \left[(1-\xi)\,\overline{P}^{+}\;,\; \frac{M^2+\vec{\Delta}_{\perp}^2/4}{2(1-\xi)\overline{P}^{+}}\;,\; \vec{\Delta}_{\perp}/2 \right] \end{aligned} \tag{B.269}$$

Note that the minus components can also be written as

$$P^{-} = \frac{\overline{M}^2\,(1-\xi)}{2\overline{P}^{+}} \qquad\qquad P'^{-} = \frac{\overline{M}^2\,(1+\xi)}{2\overline{P}^{+}} \tag{B.270}$$

B.3 Complete set of Mandelstam variables and Lorentz invariants

The Mandelstam variables for DVCS are the following

$$\begin{aligned} s &= (P+q)^2 = (P'+q')^2 = m^2 + 2P'\cdot q' \\ t &= (P'-P)^2 = \Delta^2 \\ u &= (P-q')^2 = (q-P')^2 = m^2 - 2P'\cdot q'\;. \end{aligned} \tag{B.271}$$

(B.272)

For later use we calculate also some Lorentz invariants of the process in the "average frame".

$$\begin{aligned} P\cdot P' &= m^2 - t/2 \\ P\cdot P' &= \bar{m}^2 - t/4 \\ S\cdot S' &= \lambda\lambda'\,(-\frac{t}{2m^2} \;-\frac{3\xi^2+1}{1-\xi^2}) \\ \bar{P}\cdot(S'+S) &= \frac{\lambda'+\lambda}{2m}P'\cdot P - \frac{\lambda m}{2}\frac{1-\xi}{1+\xi} - \frac{\lambda' m}{2}\frac{1+\xi}{1-\xi} \\ \bar{P}\cdot(S'+S) &= -\frac{\lambda t}{2m} - \frac{2m\lambda\xi^2}{1-\xi^2} \qquad \text{if} \qquad \lambda=\lambda' \\ \bar{P}\cdot(S'+S) &= \frac{2\lambda\xi m}{1-\xi^2} \qquad \text{if} \qquad \lambda=-\lambda' \\ \bar{P}\cdot(S'-S) &= \frac{\lambda'-\lambda}{2m}P'\cdot P + \frac{\lambda m}{2}\frac{1-\xi}{1+\xi} - \frac{\lambda' m}{2}\frac{1+\xi}{1-\xi} \\ \bar{P}\cdot(S'-S) &= -\frac{2m\lambda\xi}{1-\xi^2} \qquad \text{if} \qquad \lambda=\lambda' \\ \bar{P}\cdot(S'-S) &= \frac{\lambda t}{2m} + \frac{2m\lambda\xi^2}{1-\xi^2} \qquad \text{if} \qquad \lambda=-\lambda' \\ \bar{k}\cdot(S'+S) &= \frac{\bar{k}}{m}\cdot(\,\lambda'P'+\lambda P\,) - \frac{\lambda' m\bar{x}}{1-\xi} - \frac{\lambda m\bar{x}}{1+\xi} \end{aligned}$$

$$
\begin{aligned}
\bar{k}\cdot(S'+S) &= \frac{2\lambda\bar{k}\cdot\bar{P}}{m}-\frac{2m\lambda\bar{x}}{1-\xi^2} \qquad \text{if} \qquad \lambda=\lambda' \\
\bar{P}\cdot(S'+S) &= -\frac{\lambda\bar{k}\cdot\Delta}{m}+\frac{2m\lambda\bar{x}\xi}{1-\xi^2} \qquad \text{if} \qquad \lambda=-\lambda' \\
\bar{k}\cdot(S'-S) &= \frac{\bar{k}}{m}\cdot(\ \lambda' P'-\lambda P\ )-\frac{\lambda' m\bar{x}}{1-\xi}+\frac{\lambda m\bar{x}}{1+\xi} \\
\bar{k}\cdot(S'-S) &= \frac{\lambda\bar{k}\cdot\Delta}{m}-\frac{2m\lambda\bar{x}\xi}{1-\xi^2} \qquad \text{if} \qquad \lambda=\lambda' \\
\bar{k}\cdot(S'-S) &= \frac{-2\lambda\bar{P}\cdot\bar{k}}{m}+\frac{2m\lambda\bar{x}}{1-\xi^2} \qquad \text{if} \qquad \lambda=-\lambda' \\
\Delta\cdot(S'+S) &= -\bar{P}(S'-S) \\
\Delta\cdot(S'-S) &= -\bar{P}(S'+S)
\end{aligned}
\tag{B.273}
$$

C Constraints on the quark-quark correlators

C.1 Constraints on the forward quark-quark correlators in the helicity basis

C.1.1 Constraint from parity

The parity invariance constraint in (4.45), implemented for the helicity ansätze, reads

$$\Phi_{\Lambda' i\,;\,\Lambda j}(k,P) = \gamma_0\, \Phi_{-\Lambda' i\,;\,-\Lambda j}(\tilde{k},\tilde{P})\,\gamma_0 \tag{C.274}$$

or

$$\begin{array}{ll} \Phi^{(1)}_{ij}(k,P) = \gamma_0\, \Phi^{(4)}_{ij}(\tilde{k},\tilde{P})\,\gamma_0 & \Phi^{(2)}_{ij}(k,P) = \gamma_0\, \Phi^{(3)}_{ij}(\tilde{k},\tilde{P})\,\gamma_0 \\ \Phi^{(3)}_{ij}(k,P) = \gamma_0\, \Phi^{(2)}_{ij}(\tilde{k},\tilde{P})\,\gamma_0 & \Phi^{(4)}_{ij}(k,P) = \gamma_0\, \Phi^{(1)}_{ij}(\tilde{k},\tilde{P})\,\gamma_0 \end{array} \tag{C.275}$$

For the forward quark-quark correlators (6.141) and (6.148) in the helicity basis these constraints imply

$$\begin{aligned} a^{(1)}_m &= a^{(4)}_m\,, \quad \text{for} \quad m = 1,4,5,6,7,14,15,16,17,19,20,23 \\ a^{(1)}_m &= -a^{(4)}_m\,, \quad \text{for} \quad m = 2,3,8,9,10,11,12,13,18,21,22,24 \\ a^{(2)}_m &= a^{(3)}_m\,, \quad \text{for} \quad m = 2,5,6,11,12,15,16,17,18,20,22,24,26,28 \\ a^{(2)}_m &= -a^{(3)}_m\,, \quad \text{for} \quad m = 1,3,4,7,8,9,10,13,14,19,21,23,25,27,29. \end{aligned}$$

We report some of the proofs of the constraints on the amplitudes a_i of the forward correlators for the helicity non-flip case. Note that the vector S occurring in the following equations represents the spin vector (5.74) corresponding to light-cone helicity eigenstates. For the proofs we make use of the following relation $\gamma_0\gamma_\mu\gamma_0 = \gamma^\dagger_\mu = \tilde{\gamma}_\mu$.

$$\begin{aligned} a^{(n')}_1 &= a^{(n)}_1\,\gamma_0\gamma_0 \\ a^{(n')}_2\,(k\cdot S) &= -a^{(n)}_2\,\gamma_0\,(\tilde{k}\cdot\tilde{S})\,\gamma_0 = -a^{(n')}_2\,(k\cdot S) \\ a^{(n')}_3\,\gamma^5 &= a^{(n)}_3\,\gamma_0\gamma^5\gamma_0 = -a^{(n)}_3\,\gamma^5 \\ a^{(n')}_4\,\gamma^5\,(k\cdot S) &= a^{(n)}_4\,\gamma_0\gamma^5\,(\tilde{k}\cdot\tilde{S})\,\gamma_0 = a^{(n)}_4\,\gamma^5\,(k\cdot S) \\ a^{(n')}_5\,\not{P} &= a^{(n)}_5\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0 = a^{(n)}_5\,\tilde{P}^\mu\tilde{\gamma}_\mu = a^{(n)}_5\,\not{P} \\ a^{(n')}_7\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma &= a^{(n)}_7\,\gamma_0\left(\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{P}^\nu\tilde{k}^\rho S^\sigma\right)\gamma_0 = a^{(n)}_7\,\epsilon_{\mu\nu\rho\sigma}\tilde{\gamma}^\mu\tilde{P}^\nu\tilde{k}^\rho S^\sigma \\ &= a^{(n)}_7\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma \\ a^{(n')}_8\,\not{S} &= -a^{(n)}_8\,\tilde{S}^\mu\gamma_0\gamma_\mu\gamma_0 = -a^{(n)}_8\,\tilde{S}^\mu\tilde{\gamma}_\mu = -a^{(n)}_8\,\not{S} \\ a^{(n')}_9\,\not{P}(k\cdot S) &= -a^{(n)}_9\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,(k\cdot S) = -a^{(n)}_9\,\tilde{P}^\mu\tilde{\gamma}_\mu\,(k\cdot S) \\ &= -a^{(n)}_9\,\not{P}(k\cdot S) \end{aligned}$$

$$
\begin{aligned}
a_{13}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma\,\gamma^5 &= -a_{13}^{(n)}\,\gamma_0\left(\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{P}^\nu\tilde{k}^\rho S^\sigma\right)\gamma_0\,\gamma^5 \\
&= -a_{13}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\tilde{\gamma}^\mu\tilde{P}^\nu\tilde{k}^\rho S^\sigma\,\gamma^5 \\
&= a_{13}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma\,\gamma^5 \\
a_{15}^{(n')}\,\gamma_5\,\not{P}(k\cdot S) &= a_{15}^{(n)}\,\tilde{P}^\mu\,\gamma_0\gamma_5\gamma_\mu\gamma_0\,(k\cdot S) = -a_{15}^{(n)}\,\gamma_5\tilde{P}^\mu\tilde{\gamma}_\mu\,(k\cdot S) \\
&= -a_{15}^{(n)}\,\gamma_5,\ \not{P}(k\cdot S) \\
a_{17}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu &= a_{17}^{(n)}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 \\
&= a_{17}^{(n)}\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0)-(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\,\tilde{P}^\mu\tilde{k}^\nu \\
&= = a_{17}^{(n)}\,\frac{1}{2}\,[\,\tilde{\gamma}^\mu\tilde{\gamma}^\nu-\tilde{\gamma}^\nu\tilde{\gamma}^\mu]\,\tilde{P}^\mu\tilde{k}^\nu = a_{17}^{(n)}\,\sigma^{\mu\nu}\tilde{P}_\mu\,\tilde{k}_\nu \\
a_{18}^{(n')}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu &= a_{18}^{(n)}\,\gamma_0\gamma_5\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 = -a_{18}^{(n)}\,\gamma_5\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 \\
&= -a_{18}^{(n)}\,\tilde{P}^\mu\tilde{k}^\nu\,\gamma_5(i/2)\,[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0)-(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)] \\
&= -a_{18}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu
\end{aligned} \tag{C.276}
$$

C.1.2 Constraint from hermiticity

From the hermiticity constraint (4.48) implemented for the helicity quark-quark correlators

$$
\Phi_{\Lambda' i\,;\Lambda j}(k,P) = \gamma_0\,\Phi^\dagger_{\Lambda i\,;\Lambda' j}(k,P)\,\gamma_0 \tag{C.277}
$$

or

$$
\begin{aligned}
&\Phi^{(1)}_{ij}(k,P) = \gamma_0\,\Phi^{(1)\,\dagger}_{ij}(k,P)\,\gamma_0 \qquad && \Phi^{(2)}_{ij}(k,P) = \gamma_0\,\Phi^{(3)\,\dagger}_{ij}(k,P)\,\gamma_0 \\
&\Phi^{(3)}_{ij}(k,P) = \gamma_0\,\Phi^{(2)\,\dagger}_{ij}(k,P)\,\gamma_0 \qquad && \Phi^{(4)}_{ij}(k,P) = \gamma_0\,\Phi^{(4)\,\dagger}_{ij}(k,P)\,\gamma_0
\end{aligned} \tag{C.278}
$$

since $\gamma_5^\dagger = \gamma_5$ and $\gamma_0\gamma^{\mu\dagger}\gamma_0 = \gamma^\mu$ it follows that

$$
\begin{aligned}
a_m^{(1)} &= a_m^{(4)}\,, \quad \text{for} \quad m = 1,2,5,6,7,8,9,10,11,12,14,15,16,17,21,22,24 \\
a_m^{(1)} &= -a_m^{(4)}\,, \quad \text{for} \quad m = 3,4,13,18,19,20,23
\end{aligned} \tag{C.279}
$$

We report some of the proofs of the constraints on the amplitudes a_i of the forward correlators for the helicity non-flip case.

$$
\begin{aligned}
a_1^{(n')} &= a_1^{(n)*}\,\gamma_0\gamma_0 \\
a_2^{(n')}\,(k\cdot S) &= a_2^{(n)*}\,\gamma_0\,\gamma_0\,(k\cdot S) = a_2^{(n)*}\,(k\cdot S) \\
a_4^{(n')}\,\gamma_5\,(k\cdot S) &= a_4^{(n)*}\,\gamma_0\,(\gamma_5)^\dagger\,\gamma_0(k\cdot S) = -a_4^{(n)*}\,\gamma_5\,(k\cdot S) \\
&= -a_5^{(n)*}\,\gamma_a\,\gamma_5 = a_5^{(n)*}\,\gamma_5\,\gamma_a \\
a_5^{(n')}\,\not{P} &= a_5^{(n)*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0 P_\mu = a_5^{(n)*}\,\not{P}
\end{aligned}
$$

$$
\begin{aligned}
a_7^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma &= \gamma_0\left(a_7^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma\right)^\dagger\gamma_0 = a_7^{(n)^*}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\gamma^{\mu\dagger}\gamma_0 P^\nu k^\rho S^\sigma\\
&= a_7^{(n)^*}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma\\
a_9^{(n')}\,\gamma_5\,\not{k}\,(k\cdot S) &= a_9^{(n)*}\,\gamma_0\gamma^{\mu\dagger}\gamma_5^\dagger\gamma_0 k_\mu\,(k\cdot S) = -a_9^{(n)*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0\gamma_5 k_\mu\,(k\cdot S)\\
&= -a_9^{(n)*}\,\not{k}\gamma_5\,(k\cdot S) = a_9^{(n)*}\,\gamma_5\,\not{k}\,(k\cdot S)\\
a_{11}^{(n')}\,\not{P}\gamma^5 &= a_{11}^{(n)*}\,\gamma_0\gamma^{\mu\dagger}\,\gamma^5\,\gamma_0 P_\mu = a_2^{(n)*}\,\not{P}\\
a_{17}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu &= a_{17}^{(n)*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\\
&= a_{17}^{(n)*}\,(-i/2)[(\gamma_0\,\not{k}^\dagger\gamma_0)(\gamma_0\,\not{P}^\dagger\gamma_0)-(\gamma_0\,\not{P}^\dagger\gamma_0)(\gamma_0\,\not{k}^\dagger\gamma_0)]\\
&= a_{17}^{(n)*}\,(-i/2)[\,\not{k}\,\not{P}-\,\not{P}\not{k}] = a_{17}^{(n)*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\\
a_{18}^{(n')}\,\gamma_5\sigma^{\mu\nu}\,P_\mu k_\nu &= a_{18}^{(n)*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_5^\dagger\gamma_0 = -a_{18}^{(n)*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\gamma_5\\
&= -a_{18}^{(n)*}\,\sigma^{\mu\nu}P_\mu k_\nu\gamma_5 = -a_{18}^{(n)*}\,\gamma_5\sigma^{\mu\nu}P_\mu k_\nu
\end{aligned}
\tag{C.280}
$$

C.2 Constraints on off-forward quark-quark correlators in the helicity basis

C.2.1 Constraint from parity invariance

The parity invariance constraint in (4.45), implemented for the off-forward helicity ansätze, reads

$$
\tilde{\Phi}_{\Lambda' i\,;\,\Lambda j}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}_{-\Lambda' i\,;\,-\Lambda j}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})\,\gamma_0 \tag{C.281}
$$

or

$$
\begin{aligned}
&\tilde{\Phi}_{ij}^{(1)}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}_{ij}^{(4)}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})\,\gamma_0 \qquad && \tilde{\Phi}_{ij}^{(2)}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}_{ij}^{(3)}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})\,\gamma_0\\
&\tilde{\Phi}_{ij}^{(3)}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}_{ij}^{(2)}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})\,\gamma_0 && \tilde{\Phi}_{ij}^{(4)}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}_{ij}^{(1)}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})\,\gamma_0
\end{aligned}
\tag{C.282}
$$

from which the following relations between amplitudes on diagonal and non-diagonal components of the matrix $\tilde{\Phi}_{\Lambda'\Lambda}(\bar{P},\bar{k},\Delta)$ derive

$$
\begin{aligned}
d_m^{(1)} \;=\; d_m^{(4)}\,,\quad \text{for}\quad m = {}&1,3,5,8,9,11,13,16,17,19,21,24,25,27,29,32,33,\\
&36,38,39,40,41,43,46,48,49,52,54,56,57,60,62,64,65,68,69,\\
&71,73,76,77,79,81,84,85,87,89,92,93,95,97,100,102,104,106,\\
&108,110,112,116,118,120,122,126,128,130,131,133,134,135,137;
\end{aligned}
\tag{C.283}
$$

$$\begin{aligned} d_m^{(2)} &= d_m^{(3)}\,, \quad \text{for} \quad m = 1,3,5,8,9,11,13,16,17,19,21,24,25,27,29,32,33,\\ &\quad 36,38,39,40,41,43,46,48,49,52,54,56,57,60,62,64,65,68,69,\\ &\quad 71,73,76,77,79,81,84,85,87,89,92,93,95,97,100,102,104,106,\\ &\quad 108,110,112,116,118,120,122,126,128,130,131,133,134,135,137; \end{aligned} \tag{C.284}$$

$$\begin{aligned} d_m^{(1)} &= -d_m^{(4)}\,, \quad \text{for} \quad m = 2,4,6,7,10,12,14,15,18,19,22,23,26,28,30,31,34,\\ &\quad 36,38,40,43,44,46,48,51,52,54,56,59,60,62,64,67,68,69,71,73,75,\\ &\quad 76,79,81,83,84,87,89,91,92,95,97,99,100,102,104,106,108,110,112,\\ &\quad 114,115,116,118,120,122,124,125,126,128,130,132,136,138; \end{aligned} \tag{C.285}$$

$$\begin{aligned} d_m^{(2)} &= -d_m^{(3)}\,, \quad \text{for} \quad m = 2,4,6,7,10,12,14,15,18,19,22,23,26,28,30,31,34,\\ &\quad 36,38,40,43,44,46,48,51,52,54,56,59,60,62,64,67,68,69,71,73,75,\\ &\quad 76,79,81,83,84,87,89,91,92,95,97,99,100,102,104,106,108,110,112,\\ &\quad 114,115,116,118,120,122,124,125,126,128,130,132,136,138; \end{aligned} \tag{C.286}$$

since $\gamma_0\gamma_\mu\gamma_0 = \gamma_\mu^\dagger = \tilde{\gamma}_\mu$ and $\Lambda^{(n)\prime} = -\Lambda^{(n)}$

$$\begin{aligned}
d_1^{(n)} &= d_1^{(n')}\,\gamma_0\gamma_0 = d_1^{(n')}\\
d_2^{(n)}\Lambda^{(n)} &= \gamma_0 d_2^{(n')}\,\Lambda^{(n')}\gamma_0 = -d_2^{(n)}\,\Lambda^{(n)}\gamma_0\gamma_0 = -d_2^{(n)}\,\Lambda\\
d_3^{(n)}\,k\cdot v' &= d_3^{(n')}\,\tilde{k}\cdot\tilde{v}'\gamma_0\gamma_0 = d_3^{(n')}\,k\cdot v'\\
d_4^{(n)}\Lambda^{(n)}\,k\cdot v' &= \gamma_0 d_4^{(n')}\,\Lambda^{(n')}\gamma_0\,\tilde{k}\cdot\tilde{v}' = -d_4^{(n)}\,\Lambda^{(n)}\gamma_0\gamma_0\,k\cdot v'\\
&= -d_4^{(n)}\,\Lambda\,k\cdot v'\\
d_5^{(n)}\,\Delta\cdot v' &= d_5^{(n')}\,\tilde{\Delta}\cdot\tilde{v}'\gamma_0\gamma_0 = d_5^{(n')}\,\Delta\cdot v'\\
d_6^{(n)}\Lambda^{(n)}\,\Delta\cdot v' &= \gamma_0 d_6^{(n')}\,\Lambda^{(n')}\gamma_0\,\tilde{\Delta}\cdot\tilde{v}' = -d_6^{(n)}\,\Lambda^{(n)}\gamma_0\gamma_0\,\Delta\cdot v'\\
&= -d_6^{(n)}\,\Lambda\,\Delta\cdot v'\\
d_7^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_7^{(n')}\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\gamma_0\\
&= -d_7^{(n')}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
d_8^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_8^{(n')}\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\gamma_0\,\Lambda^{(n')}\\
&= d_8^{(n')}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)}
\end{aligned}$$

$$
\begin{aligned}
d_9^{(n)}\ \not{P} &= d_9^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0 = d_9^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu = d_9^{(n')}\ \not{P} \\
d_{10}^{(n)}\ \not{P}\Lambda &= d_{10}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\Lambda^{(n')} = -d_{10}^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu\Lambda^{(n)} \\
&= -d_{10}^{(n')}\ \not{P}\Lambda^{(n)} \\
d_{11}^{(n)}\ \not{P}k\cdot v' &= d_{11}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\tilde{k}\cdot\tilde{v}' = d_{11}^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu\,k\cdot v' \\
&= d_{11}^{(n')}\ \not{P}k\cdot v' \\
d_{12}^{(n)}\ \not{P}\Lambda\,k\cdot v' &= d_{12}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\Lambda^{(n')}\,\tilde{k}\cdot\tilde{v}' \\
&= -d_{12}^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu\Lambda^{(n)}\,k\cdot v' \\
&= -d_{12}^{(n')}\ \not{P}\Lambda^{(n)}\,k\cdot v' \\
d_{13}^{(n)}\ \not{P}\Delta\cdot v' &= d_{13}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\tilde{\Delta}\cdot\tilde{v}' = d_{13}^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu\,\Delta\cdot v' \\
&= d_{13}^{(n')}\ \not{P}\Delta\cdot v' \\
d_{14}^{(n)}\ \not{P}\Lambda\,\Delta\cdot v' &= d_{14}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\Lambda^{(n')}\,\tilde{\Delta}\cdot\tilde{v}' \\
&= -d_{14}^{(n')}\,\tilde{P}^\mu\tilde{\gamma}_\mu\Lambda^{(n)}\,\Delta\cdot v' \\
&= -d_{14}^{(n')}\ \not{P}\Lambda^{(n)}\,\Delta\cdot v' \\
d_{15}^{(n)}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{15}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{15}^{(n')}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{16}^{(n)}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{16}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= d_{16}^{(n')}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{25}^{(n)}\ \not{\Delta} &= d_{25}^{(n')}\,\tilde{\Delta}^\mu\gamma_0\gamma_\mu\gamma_0 = d_{25}^{(n')}\,\tilde{\Delta}^\mu\tilde{\gamma}_\mu = d_{25}^{(n')}\ \not{\Delta} \\
d_{26}^{(n)}\ \not{\Delta}\Lambda^{(n)} &= d_{26}^{(n')}\,\tilde{\Delta}^\mu\gamma_0\gamma_\mu\gamma_0\,\Lambda^{(n')} = -d_{26}^{(n')}\,\tilde{\Delta}^\mu\tilde{\gamma}_\mu\Lambda^{(n)} \\
&= -d_{26}^{(n')}\ \not{\Delta}\Lambda^{(n)} \\
d_{31}^{(n)}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{31}^{(n')}\,\tilde{\Delta}^\mu\gamma_0\gamma_\mu\gamma_0\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{31}^{(n')}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{32}^{(n)}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{32}^{(n')}\,\tilde{\Delta}^\mu\gamma_0\gamma_\mu\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= d_{32}^{(n')}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{35}^{(n)}\,\gamma_5 &= d_{35}^{(n')}\,\gamma_0\gamma_5\gamma_0 = -d_{35}^{(n')}\,\gamma_5 \\
d_{36}^{(n)}\,\gamma_5\Lambda^{(n)} &= d_{36}^{(n')}\,\gamma_0\gamma_5\gamma_0\,\Lambda^{(n')} = d_{36}^{(n')}\,\gamma_5\,\Lambda^{(n)} \\
d_{42}^{(n)}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{42}^{(n')}\,\gamma_0\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\gamma_0 \\
&= d_{42}^{(n')}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{43}^{(n)}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\Lambda^{(n)} &= d_{43}^{(n')}\,\gamma_0\,\gamma_5\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{43}^{(n')}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{44}^{(n)}\gamma_5\ \not{P} &= d_{44}^{(n')}\,\tilde{P}^\mu\gamma_0\gamma_5\gamma_\mu\gamma_0 = -d_{44}^{(n')}\,\gamma_5\tilde{P}^\mu\gamma_0\gamma_\mu\gamma_0 \\
&= -d_{44}^{(n')}\,\gamma_5\,\not{P}
\end{aligned}
$$

$$
\begin{aligned}
d_{45}^{(n)}\ \not{P}\gamma_5\,\Lambda^{(n)} &= d_{45}^{(n')}\,\tilde{P}^{\mu}\gamma_0\,\gamma_5\,\gamma_\mu\gamma_0\,\Lambda^{(n')} = d_{45}^{(n')}\,\tilde{P}^{\mu}\tilde{\gamma}_\mu\,\gamma_5\,\Lambda^{(n)} \\
&= d_{45}^{(n')}\ \not{P}\gamma_5\,\Lambda^{(n)} \\
d_{50}^{(n)}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5 &= d_{50}^{(n')}\,\tilde{P}^{\mu}\,\gamma_0\gamma_\mu\,\gamma_5\,\gamma_0\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= d_{50}^{(n')}\ \not{P}\gamma_5\ \epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{51}^{(n)}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5\,\Lambda^{(n)} &= d_{51}^{(n')}\,\tilde{P}^{\mu}\gamma_0\gamma_\mu\,\gamma_5\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= -d_{51}^{(n')}\ \not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5\,\Lambda^{(n)} \\
d_{60}^{(n)}\gamma_5\ \not{\Delta} &= d_{60}^{(n')}\,\tilde{\Delta}^{\mu}\gamma_0\gamma_5\gamma_\mu\gamma_0 = -d_{60}^{(n')}\,\gamma_5\tilde{\Delta}^{\mu}\gamma_0\gamma_\mu\gamma_0 \\
&= -d_{60}^{(n')}\,\gamma_5\ \not{\Delta} \\
d_{61}^{(n)}\ \not{\Delta}\gamma_5\,\Lambda^{(n)} &= d_{61}^{(n')}\,\tilde{\Delta}^{\mu}\gamma_0\,\gamma_5\,\gamma_\mu\gamma_0\,\Lambda^{(n')} = d_{61}^{(n')}\,\tilde{\Delta}^{\mu}\tilde{\gamma}_\mu\,\gamma_5\,\Lambda^{(n)} \\
&= d_{61}^{(n')}\ \not{\Delta}\gamma_5\,\Lambda^{(n)} \\
d_{66}^{(n)}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5 &= d_{66}^{(n')}\,\tilde{\Delta}^{\mu}\,\gamma_0\gamma_\mu\,\gamma_5\,\gamma_0\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= d_{66}^{(n')}\ \not{\Delta}\gamma_5\ \epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{67}^{(n)}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5\,\Lambda &= d_{67}^{(n')}\,\tilde{\Delta}^{\mu}\gamma_0\gamma_\mu\,\gamma_5\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= -d_{67}^{(n')}\ \not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\gamma_5\,\Lambda^{(n)} \\
d_{69}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu &= d_{69}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 \\
&= d_{69}^{(n')}\,\tilde{P}^{\mu}\tilde{k}^{\nu}\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)] = d_{69}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu \\
d_{70}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\Lambda^{(n)} &= d_{70}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0\,\Lambda^{(n')} \\
&= -d_{70}^{(n')}\,\tilde{P}^{\mu}\tilde{k}^{\nu}\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)\,\Lambda^{(n')} \\
&= -d_{70}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\Lambda^{(n)} \\
&= d_{70}^{(n')}\,\tilde{P}^{\mu}\tilde{k}^{\nu}\,\gamma_5(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\Lambda^{(n)} \\
&= d_{70}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu\,\Lambda^{(n)} \\
d_{75}^{(n)}\,\sigma^{\mu\nu}P_\mu k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{75}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{75}^{(n')}\,\tilde{P}^{\mu}\tilde{k}^{\nu}(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{75}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{76}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{76}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= -d_{76}^{(n')}\,\tilde{P}^{\mu}\tilde{k}^{\nu}\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n)}
\end{aligned}
$$

$$
\begin{aligned}
&= d_{76}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{77}^{(n)}\,\sigma^{\mu\nu}P_\mu\Delta_\nu &= d_{77}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0 \\
&= d_{77}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)] = d_{77}^{(n')}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu \\
d_{78}^{(n)}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\Lambda^{(n)} &= d_{78}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0\,\Lambda^{(n')} \\
&= -d_{78}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\,\Lambda^{(n)} \\
&= -d_{78}^{(n')}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\Lambda^{(n)} \\
d_{83}^{(n)}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{83}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{83}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_{83}^{(n')}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{83}^{(n)}\,\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{83}^{(n')}\,\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n')} \\
&= -d_{83}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\,\Lambda^{(n)} \\
&= d_{83}^{(n')}\,\sigma^{\mu\nu}P_\mu\Delta_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{100}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu &= d_{100}^{(n')}\,\gamma_0\gamma_5\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 \\
&= -d_{100}^{(n')}\,\gamma_5\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0 \\
&= -d_{100}^{(n')}\,\tilde{P}^\mu\tilde{k}^\nu\,\gamma_5(i/2) \\
&\quad [(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0)-(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)] \\
&= -d_{100}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu \\
d_{101}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu\,\Lambda^{(n)} &= d_{101}^{(n')}\,\gamma_0\gamma_5\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0\,\Lambda^{(n')} \\
&= d_{101}^{(n')}\,\gamma_5\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{k}_\nu\right)\gamma_0\,\Lambda^{(n)} \\
d_{105}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\Delta_\nu &= d_{105}^{(n')}\,\gamma_0\gamma_5\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0 = -d_{105}^{(n')}\,\gamma_5\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0 \\
&= -d_{105}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,\gamma_5(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0) \\
&\quad -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)] = -d_{105}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,\Delta_\nu \\
d_{106}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\Lambda^{(n)} &= d_{106}^{(n')}\,\gamma_0\gamma_5\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0\,\Lambda^{(n')} \\
&= d_{106}^{(n')}\,\gamma_5\gamma_0\left(\sigma^{\mu\nu}\tilde{P}_\mu\tilde{\Delta}_\nu\right)\gamma_0\,\Lambda^{(n)} \\
&= d_{106}^{(n')}\,\tilde{P}^\mu\tilde{\Delta}^\nu\,\gamma_5(i/2)[(\gamma_0\gamma_\mu\gamma_0)(\gamma_0\gamma_\nu\gamma_0)
\end{aligned}
$$

$$
\begin{aligned}
& -(\gamma_0\gamma_\nu\gamma_0)(\gamma_0\gamma_\mu\gamma_0)]\,\Lambda^{(n)} \\
&= d_{106}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,\Delta_\nu\,\Lambda^{(n)} \\
d_{121}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma &= d_{121}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0 \\
&= d_{121}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\gamma^\mu\gamma_0\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma \\
&= -d_{121}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma \\
d_{122}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{122}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{122}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\gamma^\mu\gamma_0\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\,\Lambda^{(n)} \\
&= d_{122}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{125}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma &= d_{125}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0 \\
&= d_{125}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\gamma^\mu\gamma_0\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma \\
&= -d_{125}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma \\
d_{126}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{126}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{126}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n)} \\
&= d_{126}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{130}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma &= d_{130}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0 \\
&= d_{130}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\,\gamma^5\,\gamma^\mu\gamma_0\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma \\
&= d_{130}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma \\
d_{131}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{131}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{131}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\,\gamma^5\,\gamma^\mu\gamma_0\tilde{\Delta}^\nu\tilde{k}^\rho\tilde{P}^\sigma\,\Lambda^{(n)} \\
&= -d_{131}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{134}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma &= d_{134}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0 \\
&= d_{134}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\,\gamma^5\,\gamma^\mu\gamma_0\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma \\
&= d_{134}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma \\
d_{135}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{135}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{135}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu\tilde{v}^\nu\tilde{k}^\rho\tilde{P}^\sigma\gamma_0\,\Lambda^{(n)} \\
&= -d_{135}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)}
\end{aligned}
\tag{C.287}
$$

C.2.2 Constraint from hermiticity invariance

The hermiticity constraint (4.40) implemented for the helicity correlators reads

$$
\tilde{\Phi}_{\Lambda' i,\Lambda j}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}^\dagger_{\Lambda i,\Lambda' j}(\bar{k},\bar{P},-\Delta)\,\gamma_0 \tag{C.288}
$$

or

$$\tilde{\Phi}^{(1)}_{ij}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}^{(1)\,\dagger}_{ij}(\bar{k},\bar{P},-\Delta)\,\gamma_0 \qquad \tilde{\Phi}^{(2)}_{ij}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}^{(3)\,\dagger}_{ij}(\bar{k},\bar{P},-\Delta)\,\gamma_0$$
$$\tilde{\Phi}^{(3)}_{ij}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}^{(2)\,\dagger}_{ij}(\bar{k},\bar{P},-\Delta)\,\gamma_0 \qquad \tilde{\Phi}^{(4)}_{ij}(\bar{k},\bar{P},\Delta) = \gamma_0\,\tilde{\Phi}^{(4)\,\dagger}_{ij}(\bar{k},\bar{P},-\Delta)\,\gamma_0 \tag{C.289}$$

from which follows

$$\begin{aligned} d^{(\kappa)}_m = d^{(\kappa)*}_m\,, \quad \text{for} \quad m \;=\; & 1,3,6,8,9,11,14,16,17,19,22,24,26,28,29,32,33,\\ & 36,38,39,41,43,45,48,50,51,53,56,58,60,62,63,65,\\ & 67,69,71,74,76,78,80,81,83,86,88,89,91,93,95,98\\ & 100,102,103,105,107,110,111,113,116,118,119,122,\\ & 124,125,128,130,132,135,137 \end{aligned} \tag{C.290}$$

where $\kappa = 1,4$ and

$$\begin{aligned} d^{(2)}_m = d^{(3)*}_m\,, \quad \text{for} \quad m \;=\; & 1,3,6,8,9,11,14,16,17,19,22,24,26,28,29,32,33,\\ & 36,38,39,41,43,45,48,50,51,53,56,58,60,62,63,65,\\ & 67,69,71,74,76,78,80,81,83,86,88,89,91,93,95,98\\ & 100,102,103,105,107,110,111,113,116,118,119,122,\\ & 124,125,128,130,132,135,137 \end{aligned} \tag{C.291}$$

$$\begin{aligned} d^{(1)}_m = -d^{(1)*}_m\,, \quad \text{for} \quad m \;=\; & 2,4,5,7,10,12,13,15,18,20,21,23,25,27,30,31,34,35,\\ & 37,40,42,44,46,47,49,52,54,55,57,59,61,64,66,68,70,\\ & 72,73,75,77,79,82,84,85,87,90,92,94,96,97,99,101,\\ & 104,106,108,109,112,114,115,117,120,121,123,126,127,\\ & 129,131,133,134,136,138 \end{aligned} \tag{C.292}$$

and

$$\begin{aligned} d^{(2)}_m = -d^{(3)*}_m\,, \quad \text{for} \quad m \;=\; & 2,4,5,7,10,12,13,15,18,20,21,23,25,27,30,31,34,35,\\ & 37,40,42,44,46,47,49,52,54,55,57,59,61,64,66,68,70,\\ & 72,73,75,77,79,82,84,85,87,90,92,94,96,97,99,101,\\ & 104,106,108,109,112,114,115,117,120,121,123,126,127,\\ & 129,131,133,134,136,138 \end{aligned} \tag{C.293}$$

since (with $\gamma_5^\dagger = \gamma_5$ and $\gamma_0\gamma^{\mu\dagger}\gamma_0 = \gamma^\mu$ and $\Lambda^{(n')} = \Lambda^{(n)}$ for $n = 1,4$ and $\Lambda^{(n')} = -\Lambda^{(n)}$ for $n = 2,3$)

$$d^{(n)}_1 \quad = \quad d^{(n')*}_1\,\gamma_0\gamma_0$$

$$
\begin{aligned}
d_2^{(n)}\,\Lambda^{(n)} &= d_2^{(n')*}\,\gamma_0\gamma_0\,\Lambda^{(n')} = d_2^{(n)*}\,\Lambda^{(n)} \\
d_7^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_7^{(n')*}\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\gamma_0 \\
&= -d_7^{(n')*}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_8^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_8^{(n')*}\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_8^{(n')*}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_9^{(n)}\,\not{P} &= d_9^{(n')*}\,\gamma_0\,\not{P}^\dagger\gamma_0 = d_9^{(n')*}\,\not{P} \\
d_{10}^{(n)}\,\not{P}\Lambda^{(n)} &= d_{10}^{(n')*}\,\gamma_0\,\not{P}^\dagger\gamma_0\,\Lambda^{(n')} = d_{10}^{(n')*}\,\not{P}\Lambda^{(n)} \\
d_{15}^{(n)}\,\not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{15}^{(n')*}\,\gamma_0\,\not{P}^\dagger\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma \\
&= -d_{15}^{(n')*}\,\not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{16}^{(n)}\,\not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{16}^{(n')*}\,\gamma_0\,\not{P}^\dagger\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\,\Lambda^{(n')} \\
&= -d_{16}^{(n')*}\,\not{P}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{25}^{(n)}\,\not{\Delta} &= d_{25}^{(n')*}\,\gamma_0(-\not{\Delta}^\dagger)\gamma_0 = -d_{25}^{(n')*}\,\not{\Delta} \\
d_{26}^{(n)}\,\not{\Delta}\Lambda^{(n)} &= d_{26}^{(n')*}\,\gamma_0(-\not{\Delta}^\dagger)\gamma_0\,\Lambda^{(n')} = -d_{26}^{(n)*}\,\not{\Delta}\Lambda^{(n)} \\
d_{31}^{(n)}\,\not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{31}^{(n')*}\,\gamma_0(-\not{\Delta}^\dagger)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma \\
&= d_{31}^{(n')*}\,\not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{32}^{(n)}\,\not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{32}^{(n')*}\,\gamma_0(-\not{\Delta}^\dagger)\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\,\Lambda^{(n')} \\
&= d_{32}^{(n')*}\,\not{\Delta}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{35}^{(n)}\,\gamma_5 &= d_{35}^{(n')*}\,\gamma_0\gamma_5^\dagger\gamma_0 = -d_{35}^{(n')*}\,\gamma_5 \\
d_{36}^{(n)}\,\gamma_5\Lambda^{(n)} &= d_{36}^{(n')*}\,\gamma_0\gamma_5^\dagger\gamma_0\,\Lambda^{(n')} = -d_{36}^{(n)*}\,\gamma_5\,\Lambda^{(n)} \\
d_{41}^{(n)}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{41}^{(n')*}\,\gamma_0\gamma_5^\dagger\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma \\
&= d_{41}^{(n')*}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_{42}^{(n)}\,\gamma_5\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{42}^{(n')*}\,\gamma_0\gamma_5^\dagger\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\,\Lambda^{(n')} \\
&= d_{42}^{(n')*}\,\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} \\
d_{43}^{(n)}\,\not{P}\gamma_5 &= d_{43}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0 P_\mu = d_{43}^{(n')*}\,\gamma_0\gamma_5\,\gamma^{\mu\dagger}\gamma_0 \\
&= -d_{43}^{(n')*}\,\gamma_5\gamma_0\gamma^{\mu\dagger}\gamma_0 P_\mu = -d_{43}^{(n')*}\,\gamma_5\gamma^\mu P_\mu \\
&= d_{43}^{(n')*}\,\not{P}\gamma_5 \\
d_{44}^{(n)}\,\not{P}\gamma_5\,\Lambda^{(n)} &= d_{44}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0\,\Lambda^{(n')} = d_{44}^{(n')*}\,\gamma_0\gamma_5\gamma^{\mu\dagger}\gamma_0\,\Lambda^{(n')} \\
&= d_{44}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0\gamma_5 P_\mu\,\Lambda^{(n')} = d_{44}^{(n')*}\,\not{P}\gamma_5\,\Lambda^{(n)} \\
d_{49}^{(n)}\,\not{P}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{49}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,P_\mu\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma \\
&= d_{49}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_5\gamma_0 P_\mu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
&= -d_{49}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0\gamma_5 P_\mu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
&= -d_{49}^{(n')*}\,\not{P}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma
\end{aligned}
$$

$$
\begin{aligned}
d_{50}^{(n)}\ \not{P}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\Lambda^{(n)} &= d_{50}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0\,\Lambda^{(n')}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\\
&= d_{50}^{(n')*}\ \not{P}\gamma_5\,\Lambda^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
d_{59}^{(n)}\ \not{\Delta}\gamma_5 &= d_{59}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0(-\Delta)_\mu = -d_{59}^{(n')*}\,\gamma_0\gamma_5\,\gamma^{\mu\dagger}\gamma_0\Delta_\mu\\
&= d_{59}^{(n')*}\,\gamma_5\gamma_0\gamma^{\mu\dagger}\gamma_0\Delta_\mu\\
&= d_{59}^{(n')*}\,\gamma_5\gamma^\mu\Delta_\mu = -d_{59}^{(n')*}\ \not{\Delta}\gamma_5\\
d_{60}^{(n)}\ \not{\Delta}\gamma_5\,\Lambda^{(n)} &= d_{60}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0\,(-\Delta_\mu)\Lambda^{(n')}\\
&= -d_{60}^{(n')*}\,\gamma_0\gamma_5\gamma^{\mu\dagger}\gamma_0\,\Delta_\mu\Lambda^{(n)}\\
&= -d_{60}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0\gamma_5\Delta_\mu\,\Lambda^{(n)}\\
&= -d_{60}^{(n')*}\ \not{\Delta}\gamma_5\,\Lambda^{(n)}\\
d_{65}^{(n)}\ \not{\Delta}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{65}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,(-\Delta)_\mu\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\\
&= -d_{65}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_5\gamma_0\Delta_\mu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
&= d_{65}^{(n')*}\,\gamma_0\gamma^{\mu\dagger}\gamma_0\gamma_5\Delta_\mu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
&= d_{65}^{(n')*}\ \not{\Delta}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
d_{66}^{(n)}\ \not{\Delta}\gamma_5\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\Lambda^{(n)} &= d_{66}^{(n')*}\,\gamma_0\,(\gamma^\mu\gamma_5)^\dagger\,\gamma_0\,(-\Delta)_\mu\Lambda^{(n')}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\\
&= d_{66}^{(n')*}\ \not{\Delta}\gamma_5\,\Lambda^{(n')}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
d_{69}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu &= d_{69}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\\
&= d_{69}^{(n')*}\,(-i/2)[(\gamma_0\,\not{k}^\dagger\gamma_0)(\gamma_0\,\not{P}^\dagger\gamma_0)-(\gamma_0\,\not{P}^\dagger\gamma_0)(\gamma_0\,\not{k}^\dagger\gamma_0)]\\
&= d_{69}^{(n')*}\,(-i/2)[\,\not{k}\,\not{P}-\ \not{P}\not{k}] = d_{69}^{(n')*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\\
d_{70}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu\Lambda^{(n)} &= d_{70}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\Lambda^{(n')}\\
&= d_{70}^{(n')*}\,(-i/2)[(\gamma_0\,\not{k}^\dagger\gamma_0)(\gamma_0\,\not{P}^\dagger\gamma_0)-(\gamma_0\,\not{P}^\dagger\gamma_0)(\gamma_0\,\not{k}^\dagger\gamma_0)]\Lambda^{(n')}\\
&= d_{70}^{(n')*}\,(-i/2)[\,\not{k}\,\not{P}-\ \not{P}\not{k}]\Lambda^{(n')}\\
&= d_{70}^{(n')*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\Lambda^{(n)}\\
d_{75}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma &= d_{75}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\\
&= -d_{75}^{(n')*}\,(-i/2)[(\gamma_0\,\not{k}^\dagger\gamma_0)(\gamma_0\,\not{P}^\dagger\gamma_0)-(\gamma_0\,\not{P}^\dagger\gamma_0)(\gamma_0\,\not{k}^\dagger\gamma_0)]\\
&\quad\ \epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
&= -d_{75}^{(n')*}\,(-i/2)[\,\not{k}\,\not{P}-\ \not{P}\not{k}]\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
&= -\ d_{75}^{(n')*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
d_{76}^{(n)}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n)} &= d_{76}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha(-\Delta^\beta)P^\rho k^\sigma\,\Lambda^{(n')}\\
&= -d_{76}^{(n')*}\,(-i/2)[(\gamma_0\,\not{k}^\dagger\gamma_0)(\gamma_0\,\not{P}^\dagger\gamma_0)-(\gamma_0\,\not{P}^\dagger\gamma_0)(\gamma_0\,\not{k}^\dagger\gamma_0)]\\
&\quad\ \Lambda^{(n')}\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\\
&= -d_{76}^{(n')*}\,(-i/2)[\,\not{k}\,\not{P}-\ \not{P}\not{k}]\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\,\Lambda^{(n')}\\
&= d_{76}^{(n')*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\,\epsilon_{\alpha\beta\rho\sigma}
\end{aligned}
$$

$$
\begin{aligned}
& \qquad v^{\alpha}\Delta^{\beta}P^{\rho}k^{\sigma}\,\Lambda^{(n')} \\
d_{99}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu &= d_{99}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_5^\dagger\gamma_0 \\
&= -d_{99}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\gamma_5 \\
&= -d_{99}^{(n')*}\,\sigma^{\mu\nu}P_\mu k_\nu\gamma_5 = -d_{99}^{(n')*}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu \\
d_{100}^{(n)}\,\gamma_5\sigma^{\mu\nu}P_\mu k_\nu\Lambda^{(n)} &= d_{100}^{(n')*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_5^\dagger\gamma_0\Lambda^{(n')} \\
&= -d_{100}^{(n)*}\,\gamma_0\,(\sigma^{\mu\nu}P_\mu k_\nu)^\dagger\,\gamma_0\gamma_5\Lambda^{(n)} \\
&= -d_{100}^{(n)*}\,\sigma^{\mu\nu}P_\mu\,k_\nu\gamma_5\Lambda^{(n)} = -d_{100}^{(n)*}\,\gamma_5\sigma^{\mu\nu}P_\mu\,k_\nu\Lambda^{(n)} \\
d_{122}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma &= d_{122}^{(n')*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu\dagger}(-\Delta^\nu)k^\rho P^\sigma\gamma_0 \\
&= -d_{122}^{(n')*}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma \\
d_{123}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{123}^{(n)*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu\dagger}(-\Delta^\nu)k^\rho P^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{123}^{(n)*}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{126}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma &= d_{126}^{(n')*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu\dagger}v^\nu k^\rho P^\sigma\gamma_0 = d_{126}^{(n')*}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma \\
d_{127}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{127}^{(n)*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^{\mu\dagger}v^\nu k^\rho P^\sigma\gamma_0\,\Lambda^{(n')} \\
&= d_{127}^{(n)*}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu v^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{131}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma &= d_{131}^{(n')*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^{\mu\dagger}(-\Delta^\nu)k^\rho P^\sigma\gamma_0 \\
&= d_{131}^{(n')*}\,\epsilon_{\mu\nu\rho a}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma \\
d_{132}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{132}^{(n')*}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}(\gamma^5\,\gamma^\mu)^\dagger\Delta^\nu k^\rho P^\sigma\gamma_0\,\Lambda^{(n')} \\
&= d_{132}^{(n)*}\,\epsilon_{\mu\nu\rho\sigma}\gamma_0\,\gamma^5\,\gamma^\mu\gamma_0\Delta^\nu k^\rho P^\sigma\,\Lambda^{(n)} \\
d_{135}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma &= d_{135}^{(n')}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\,(\gamma^5\,\gamma^\mu)^\dagger v^\nu k^\rho P^\sigma\gamma_0 \\
&= -d_{135}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma \\
d_{136}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu k^\rho P^\sigma\,\Lambda^{(n)} &= d_{136}^{(n)}\,\gamma_0\,\epsilon_{\mu\nu\rho\sigma}\,(\gamma^5\,\gamma^\mu)^\dagger v^\nu k^\rho P^\sigma\gamma_0\,\Lambda^{(n')} \\
&= -d_{136}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\,\gamma^5\,\gamma^\mu v^\nu\tilde{k}^\rho\tilde{P}^\sigma\,\Lambda^{(n)}
\end{aligned}
\tag{C.294}
$$

C.2.3 Constraint from time reversal invariance

The time reversal constraint for helicity off-forward correlators reads

$$
\tilde{\Phi}_{\Lambda i,\,\Lambda^{(n')},j}(\bar{k},\bar{P},\Delta) = (-i\gamma_5 C)\,\tilde{\Phi}_{\Lambda i,\,\Lambda^{(n')},j}(\tilde{\bar{k}},\tilde{\bar{P}},\tilde{\Delta})^*(-i\gamma_5 C)\;. \tag{C.295}
$$

Making use of the following properties of the operator $C = (\,i\gamma^2\gamma^0\,)$

$$
\begin{aligned}
C^* &= \left(i\gamma^2\gamma^0\right)^* = (-i)(\,-\gamma^2\,)\gamma^0 = i\gamma^2\gamma^0 = C \\
C^\dagger &= (C^T)^* = \left(i\gamma^{0T}\gamma^{2T}\right)^* = \left(i\gamma^0\gamma^2\right)^* = (-C)^* = -C \\
C\,C &= \left(i\gamma^2\gamma^0\right)\left(i\gamma^2\gamma^0\right) = -\gamma^2\gamma^0\gamma^2\gamma^0 = -\mathbb{1}
\end{aligned}
$$

from which it follows that $C\,C^\dagger = C^\dagger\,C\mathbb{1}$ the time reversal constraint for a given Dirac structure X can be reformulated as

$$(-i\gamma_5 C)\,X^*\,(-i\gamma_5 C) = -\gamma_5^* C^* X^* \gamma_5^* C^* = -(\gamma_5\,C\,X\,\gamma_5\,C)^* = \left(\,\gamma_5\,C\,X\,C^\dagger\gamma_5\,\right)^* \tag{C.296}$$

Thus we implement the time reversal constraint in the form

$$\tilde{\Phi}_{\Lambda i,\,\Lambda^{(n')},j}(\bar{k},\bar{P},\Delta) = \left[\,\gamma_5 C\,\tilde{\Phi}_{\Lambda i,\,\Lambda^{(n')},j}(\tilde{\bar{k}},\tilde{\bar{P}},\Delta)C^\dagger\gamma_5\,\right]^* . \tag{C.297}$$

obtaining for $k = 1,2,3,4$

$$\begin{array}{lcl}
d^(k)_m = d^(k)_m{}^* & m = & 1,2,3,4,5,6,9,10,11,12,13,14 \\
 & & 17,18,19,20,21,22,25,26,27,28,29,30,33,34, \\
 & & 41,42,43,44,45,46,47,48,51,52,53,54,55,56 \\
 & & 59,60,61,62,63,64,67,68,75,76,83,84,91,92 \\
d^(k)_m = -d_m^{(k)*} & m = & 7,8,15,16,23,24,31,32,35,36, \\
 & & 37,38,39,40,49,50,57,58,65,66,69,70,71,72 \\
 & & 73,74,77,78,79,80,81,82,85,86,87,88,89,90, \\
 & & 93,94,95,96,97,98,99,100,101,102,103,104, \\
 & & 105,106,107,108,109,110,111,112,113,114, \\
 & & 115,116,117,118,119,120,121,122,123,124, \\
 & & 125,126,127,128,129,130,131,132,133,134, \\
 & & 135,136,137,138
\end{array} \tag{C.298}$$

since ($C\,\gamma^\mu C^\dagger = -\gamma^{\mu T}$)

$$\begin{aligned}
d_1^{(n)} &= (\,\gamma_5 C\,C^\dagger\gamma_5\,)^* d_1^{(n')*} = d_1^{(n)*} \\
d_2^{(n)}\Lambda &= (\,\gamma_5 C\,C^\dagger\gamma_5\,)^* d_1^{(n)*}\Lambda = d_2^{(n)*}\Lambda \\
d_7^{(n)}\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma &= (\,\gamma_5 C\,C^\dagger\gamma_5\,)^* = d_7^{(n)*}\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma \\
&= -d_7^{(n)*}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma \\
d_8^{(n)*}\,\epsilon_{\alpha\beta\rho\sigma}\tilde{v}^\alpha\tilde{\Delta}^\beta\tilde{P}^\rho\tilde{k}^\sigma\Lambda &= -d_8^{(n)*}\,\epsilon_{\alpha\beta\rho\sigma}v^\alpha\Delta^\beta P^\rho k^\sigma\Lambda \\
d_9^{(n')}\;\not{P} &= d_2^{(n)*}\,(\gamma_5 C\,\gamma^\mu\,C^\dagger\gamma_5)^*\tilde{P}_\mu = -d_9^{(n)*}\,(C\gamma^\mu C^\dagger)^*\tilde{P}_\mu \\
&= d_9^{(n)*}\,(\gamma^{\mu T})^*\tilde{P}_\mu = d_9^{(n)*}\,\gamma^\mu P_\mu \\
d_{17}^{(n')}\;\not{k} &= d_3^{(n)*}\,(\gamma_5 C\,\gamma^\mu\,C^\dagger\gamma_5)^*\tilde{k}_\mu = -d_{17}^{(n)*}\,(C\gamma^\mu C^\dagger)^*\tilde{k}_\mu \\
&= d_{17}^{(n)*}(\gamma^{\mu T})^*\tilde{k}_\mu = d_{17}^{(n)*}\gamma^\mu k_\mu \\
d_{35}^{(n')}\,\gamma_5 &= d_{35}^{(n)*}\,(\gamma_5 C\,\gamma_5\,C^\dagger\gamma_5)^* = d_{35}^{(n)*}\,\gamma_5
\end{aligned}$$

$$
\begin{aligned}
d_{43}^{(n')}\,\gamma_5\,\not{P} &= d_{43}^{(n)*}\,(\gamma_5 C\,\gamma^\mu\gamma_5\,C^\dagger\gamma_5)^*\,\tilde{P}_\mu \\
&= -d_{43}^{(n)*}\,\gamma_5(C\gamma^\mu C^\dagger)^*\,\tilde{P}_\mu \\
&= d_{43}^{(n)*}\gamma_5\gamma^\mu P_\mu \\
d_{69}^{(n')}\,\sigma^{\mu\nu}P_\mu\,k_\nu &= d_{69}^{(n)*}(\gamma_5 C\,\sigma^{\mu\nu}\,C^\dagger\gamma_5)^*\,\tilde{P}_\mu\tilde{k}_\nu \\
&= d_{69}^{(n)*}(C\,\sigma^{\mu\nu}\,C^\dagger)^*\,\tilde{P}_\mu\tilde{k}_\nu \\
&= d_{69}^{(n)*}[\frac{i}{2}(C\gamma^\mu C^\dagger\,C\gamma^\nu C^\dagger - C\gamma^\nu C^\dagger\,C\gamma^\mu C^\dagger)]^*\,\tilde{P}_\mu\tilde{k}_\nu \\
&= -d_{69}^{(n)*}\frac{i}{2}(\gamma^{\mu\dagger}\,\gamma^{\nu\dagger} - \gamma^{\nu\dagger}\,\gamma^{\mu\dagger})P_\mu\tilde{k}_\nu = -d_{69}^{(n)*}\,\sigma^{\mu\nu}P_\mu k_\nu \\
d_{99}^{(n')}\,\sigma^{\mu\nu}\gamma_5 P_\mu\,k_\nu &= d_{99}^{(n)*}(\gamma_5 C\,\sigma^{\mu\nu}\gamma_5\,C^\dagger\gamma_5)^*\,\tilde{P}_\mu\tilde{k}_\nu \\
&= d_{99}^{(n)*}\,\gamma_5\,(C\,\sigma^{\mu\nu}\,C^\dagger)^*\,\tilde{P}_\mu\tilde{k}_\nu \\
&= -d_{99}^{(n)*}\,\gamma_5\tilde{\sigma^{\mu\nu}}\,\tilde{P}_\mu\tilde{k}_\nu \\
&= -d_{99}^{(n)*}\,\sigma^{\mu\nu}\gamma_5 P_\mu k_\nu \\
d_{125}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho v'^\sigma &= (\gamma_5 C d_{125}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu C^\dagger\gamma_5)^*\,\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= -{d_{125}^{(n)}}^*\,\epsilon_{\mu\nu\rho\sigma}(C\gamma^\mu C^\dagger)^*\,\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= {d_{125}^{(n)}}^*\,\epsilon_{\mu\nu\rho\sigma}\tilde{\gamma}^\mu\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= -{d_{125}^{(n)}}^*\,\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho v'^\sigma \\
d_{134}^{(n')}\,\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu P^\nu k^\rho v'^\sigma &= (\gamma_5 C d_{134}^{(n)}\,\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu C^\dagger\gamma_5)^*\,\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= -{d_{134}^{(n)}}^*\,\gamma_5\epsilon_{\mu\nu\rho\sigma}(\,C\gamma^\mu C^\dagger\,)^*\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= {d_{134}^{(n)}}^*\,\gamma_5\epsilon_{\mu\nu\rho\sigma}\tilde{\gamma}^\mu\tilde{P}^\nu\tilde{k}^\rho\tilde{v}'^\sigma \\
&= -{d_{134}^{(n)}}^*\,\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu P^\nu k^\rho v'^\sigma
\end{aligned}
$$

D Dirac matrices in Weyl representation

$$
\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{D.299}
$$

$$
\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{D.300}
$$

and

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{D.301}$$

with

$$(\gamma^0)^2 = \mathbb{1}\,, \qquad (\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -\mathbb{1}\,, \qquad (\gamma_5)^2 = \mathbb{1} \tag{D.302}$$

and

$$(\gamma^\mu) \;= \begin{cases} \gamma^\mu & \text{for } \mu = 0,1,3 \\ -\gamma^\mu & \text{for } \mu = 2 \end{cases} \qquad\qquad (\gamma^\mu)^T = \begin{cases} \gamma^\mu & \text{for } \mu = 0,2 \\ -\gamma^\mu & \text{for } \mu = 1,3 \end{cases} \tag{D.303}$$

$$\gamma^{0\dagger} = \gamma^0\,, \qquad \vec{\gamma}^\dagger = -\vec{\gamma}\,, \qquad {\gamma_5}^\dagger = \gamma_5 \tag{D.304}$$

$$\bar{\gamma}^\mu = \gamma^0 \gamma^{\mu\dagger} \gamma^0 = \gamma^\mu\,, \qquad \bar{\gamma}_5 = -\gamma_5 \tag{D.305}$$

The σ matrices, defined by

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)\,, \tag{D.306}$$

are explicitly given as

$$\sigma^{00} = \sigma^{11} = \sigma^{22} = \sigma^{33} = 0 \tag{D.307}$$

and

$$\sigma^{01} = -\sigma^{10} = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix} \qquad \sigma^{02} = -\sigma^{20} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{D.308}$$

$$\sigma^{03} = -\sigma^{30} = \begin{pmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \end{pmatrix} \qquad \sigma^{12} = -\sigma^{21} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{D.309}$$

$$\sigma^{13} = -\sigma^{31} = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix} \qquad \sigma^{23} = -\sigma^{32} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{D.310}$$

or expressed in light-cone coordinates

$$\frac{\sigma^{+1}}{\sqrt{2}} = \frac{-\sigma^{1+}}{\sqrt{2}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \frac{\sigma^{+2}}{\sqrt{2}} = \frac{-\sigma^{2+}}{\sqrt{2}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{D.311}$$

$$\sigma^{+-} = -\sigma^{-+} = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix} \qquad \sigma^{12} = -\sigma^{21} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{D.312}$$

$$\sigma^{-1} = -\sigma^{1-} = \begin{pmatrix} 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \end{pmatrix} \qquad \sigma^{-2} = -\sigma^{2-} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{D.313}$$