

# Supplement material for the study “The Thurstonian linked block model: Improving Thurstonian modeling for paired comparison and ranking data”

Markus Thomas Jansen & Ralf Schulze

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# 1 Overview

## 1.1 Simulation

This file gives an overview and the results of the simulation studies from the article “The Thurstonian linked block model: Improving Thurstonian modeling for paired comparison and ranking data”.

All model estimations were done with Mplus. To invoke and perform the simulations, preliminary versions of ThurMod (Jansen, 2023) was used. Versions of software are:

1. R Version 4.0.0
2. RStudio Version RStudio v1.3.1056-1
3. Mplus Version 8.1
4. MIRTJ (ThurMod) version 0.2.021
5. ThurMod version 0.4.3.

Be careful: The Mplus analyses alone would take about 28 years on a single core computer. All analyses were done parallel on a 64 core computer.

All syntaxes for simulation have the following structure:

1. Working directory and packages are loaded.
2. A global seed and sub seeds for every data generation process are defined
3. Initialization of simulation conditions
4. Definition of blocks
5. For each of the 1000 repetitions
  - (a) Create and save data file
  - (b) Create syntax and run the analysis for the Thurstonian factor and IRT model for the different blocks (full model, Blocks 1-5, linked/partially linked/unlinked)

All files to invoke the simulation can be found in the folder 00\_sim\_syntaxes. The definition of a number to each condition is

1. CI\_01 = uncor\_1\_12 = uncorrelated, 1 trait, 12 items total (12 per trait)
2. CI\_02 = uncor\_1\_18 = uncorrelated, 1 trait, 18 items total (18 per trait)
3. CI\_03 = uncor\_3\_18 = uncorrelated, 3 traits, 18 items total (6 per trait)
4. CI\_04 = uncor\_3\_36 = uncorrelated, 3 traits, 36 items total (12 per trait)
5. CI\_05 = uncor\_3\_54 = uncorrelated, 3 traits, 54 items total (18 per trait)
6. CI\_06 = cor\_3\_18 = correlated, 3 traits, 18 items total (6 per trait)
7. CI\_07 = cor\_3\_36 = correlated, 3 traits, 36 items total (12 per trait)

8. CI\_08 = cor\_3\_54 = correlated, 3 traits, 54 items total (18 per trait)
9. CI\_09 = uncor\_5\_30 = uncorrelated, 5 traits, 30 items total (6 per trait)
10. CI\_10 = uncor\_5\_60 = uncorrelated, 5 traits, 60 items total (12 per trait)
11. CI\_11 = uncor\_5\_90 = uncorrelated, 5 traits, 90 items total (18 per trait)
12. CI\_12 = cor\_5\_30 = correlated, 5 traits, 30 items total (6 per trait)
13. CI\_13 = cor\_5\_60 = correlated, 5 traits, 60 items total (12 per trait)
14. CI\_14 = cor\_5\_90 = correlated, 5 traits, 90 items total (18 per trait)

All simulations used triplets as blocks. All simulations used a ranking design (transitive responding).

## 1.2 Results

After analysis of all simulation conditions, results are gathered. All simulated data and Mplus files combined are too large to share. Raw data of the results are shared in excel files which can be found in the folder 01\_raw. Steps for Thurstonian factor model:

1. Test for each repetition if results are plausible. Even for models where estimation converges, some results are not usable. Data is not usable if one of the following conditions apply
  - (a) The SD of the loading estimates is 0.
  - (b) The residual variance of a trait is negative.
  - (c) The SD of the utility estimates is 0.
  - (d) The correlation between traits is larger  $|1|$ .
  - (e) The relative bias of loadings is larger than 15
2. The number of converged data is counted (after exclusion of implausible data).
3. Results are gathered for
  - (a) Fit statistics
  - (b) Bias, recovery and bias of standard errors for loadings
  - (c) Bias, recovery and bias of standard errors for utilities
  - (d) Bias, recovery and bias of standard errors for thresholds
  - (e) Bias/Mean of trait correlations
  - (f) Bias of standard errors or SD of trait correlations
  - (g) Latent trait recovery
  - (h) empirical rejection rates

The relative bias for each estimation is calculated by

$$Bias = \frac{\hat{\theta} - \theta}{\theta}$$

where  $\theta$  is the true parameter and  $\hat{\theta}$  the estimation. Due to identification constraints, true and estimated parameters need to be defined on the same scale. For loadings and utilities the unstandardized estimations are used. This is mainly the case, as for standardized results, some parameters that are defined to be equal, are not anymore. For thresholds no rescaling need to be done, when standardized results are used. Loading estimates that are reversed keyed (they are directly compared to one of the negative items) are rekeyed.

For unlinked and partially linked blocks, the true parameters were regressed on the estimations and rescaled for each (unconnected) block. For linked blocks, the estimations were regressed on the true score and rescaled once for the whole data set. This procedure potentially favors the unlinked block design, therefore, also the recovery (correlation between true and estimated parameters) is reported.

Blocks are

1. Blocks 1: Items are ordered by their utilities. Items with similar utilities (e.g., items with the least three utilities) are combined into blocks.
2. Blocks 2: Items are ordered by their loadings. Items with similar loadings (e.g., items with the least three loadings) are combined into blocks.
3. Blocks 3: All blocks are created randomly.
4. Blocks 4: Initial blocks are purely uni-dimensional.
5. Blocks 5: All blocks are purely multidimensional.

All results are presented by a figure and a corresponding numeric table for reference. If any cell in a table is empty, there are no results available (no convergence, other reasons for exclusion, see above). For any figure, the x-axis has the number of traits per factor (6. vs. 12 vs. 18). The different lines represent the design (unlinked, partially linked, fully linked). For each result variable, a dashed reference line is included, which references the best value (e.g. 1000 for converged repetitions, 0 for bias), a lower limit (e.g. latent trait score recovery), or the corresponding alpha level for the rejection rates.

The y-axis per result variable was chosen to be equal across conditions. Therefore, it is

1. Convergence rates (Factor and IRT model): 0 to 1000
2. Bias (loadings, utilities, thresholds): -.6 to .6
3. Item parameter recovery (loadings, utilities): 0 to 1
4. Bias for SE (loadings, utilities): -1 to 3

5. Item parameter recovery (thresholds): .60 to 1
6. Bias for SE (thresholds): -.1 to .1
7. Bias or mean of trait correlation: -.6 to .6
8. Bias for SE or SD of trait correlation: 0 to .6
9. Recovery of latent trait scores: 0 to 1
10. empirical rejection rates for  $\alpha = .01$ : 0 to .20
11. empirical rejection rates for  $\alpha = .05$ : 0 to .30
12. empirical rejection rates for  $\alpha = .10$ : 0 to .40
13. empirical rejection rates for  $\alpha = .20$ : 0 to .50

## 2 Simulation Study with one trait

### 2.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: one trait
3. Number of items: 12 vs. 18 items per trait (12 vs. 18 total)

### 2.2 Definition of parameters

#### 2.2.1 Correlation matrix $\Phi$ of traits

	Trait 1
Trait 1	1

#### 2.2.2 Matrix of utilities $U$

	Trait 1
Items 1	-0.48978
Items 2	-0.81764
Items 3	0.92617
Items 4	0.40913
Items 5	-0.28216
Items 6	-0.33886
Items 7	0.82277
Items 8	-0.65770
Items 9	-0.92953
Items 10	-0.67635
Items 11	-0.51847
Items 12	-0.43816
Items 13	0.44724
Items 14	0.36591
Items 15	-0.40899
Items 16	0.08061
Items 17	0.13864
Items 18	-0.75349

### 2.2.3 Matrix of loadings $\Lambda$

	Trait 1
Items 1	0.69138
Items 2	0.62816
Items 3	0.66001
Items 4	0.46287
Items 5	0.66380
Items 6	0.33614
Items 7	0.43488
Items 8	0.59709
Items 9	0.88431
Items 10	0.53421
Items 11	0.50945
Items 12	0.30158
Items 13	0.79880
Items 14	0.77966
Items 15	0.67635
Items 16	0.36463
Items 17	0.76807
Items 18	0.61888

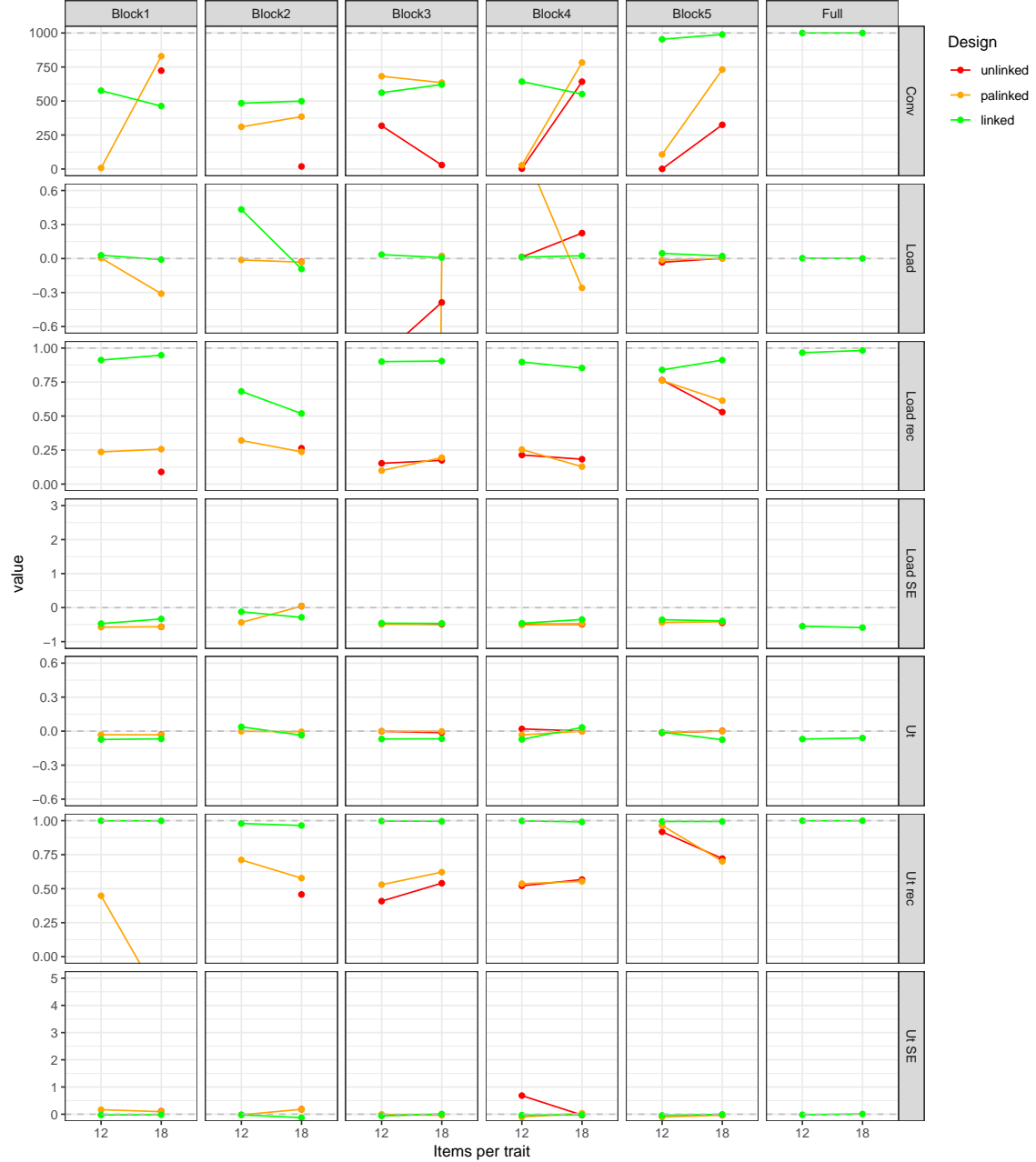


## 2.3 Results

### 2.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 1

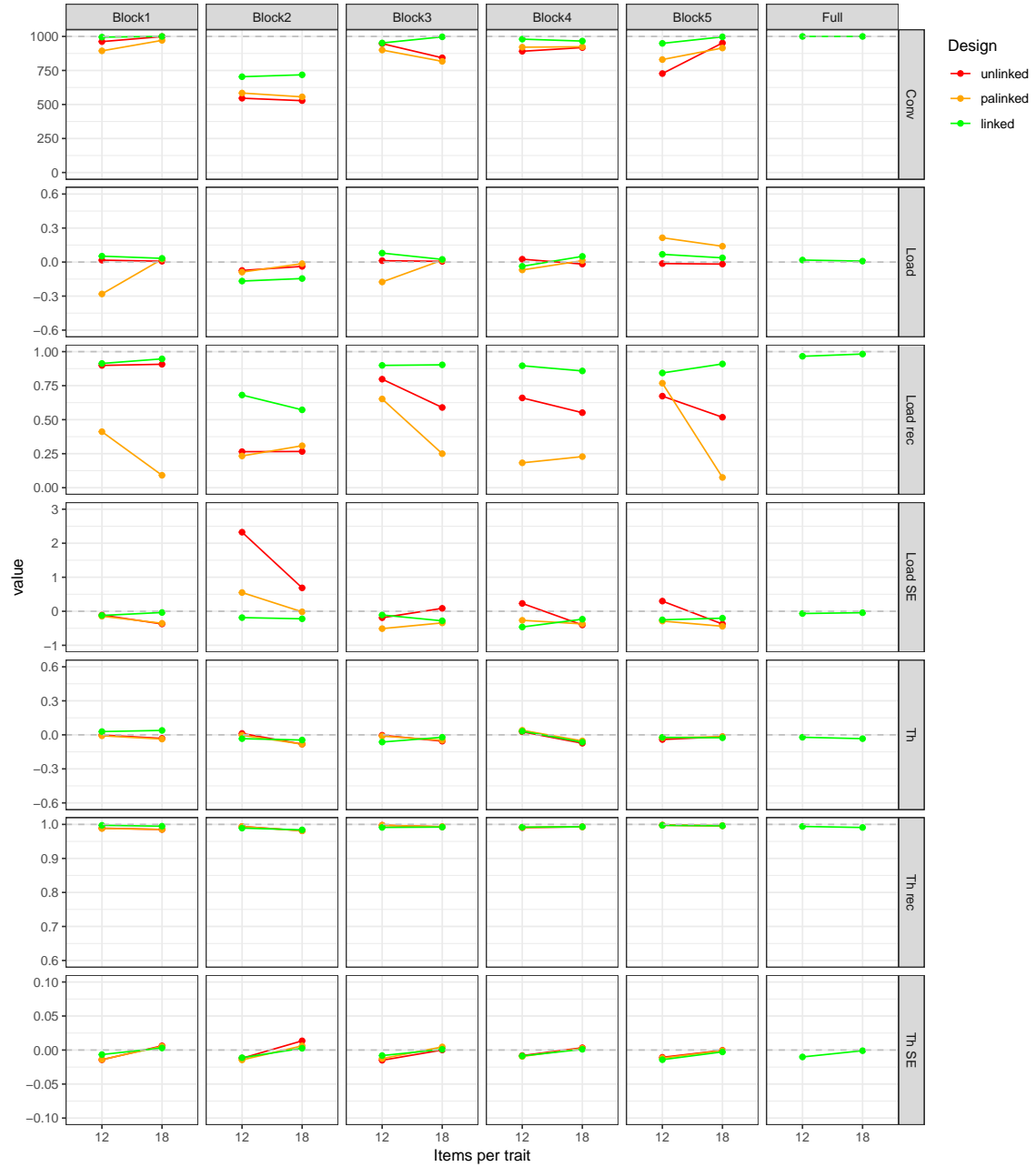
*Bias results for the Thurstonian factor model with one trait.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 2**

*Bias results for the Thurstonian IRT model with one trait.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 1***Bias results for the Thurstonian factor model with one trait.*

	blocks	unlinked		palinked		linked	
		12	18	12	18	12	18
Conv	Block1		723	8	829	576	463
	Block2		19	310	385	484	499
	Block3	318	29	682	635	561	621
	Block4	3	642	28	783	643	551
	Block5	1	325	107	731	954	989
	Full					1000	1000
Load	Block1		−0.92	0.00	−0.31	0.03	−0.01
	Block2		−0.03	−0.01	−0.03	0.43	−0.09
	Block3	−0.86	−0.39	−53.31	0.02	0.03	0.01
	Block4	0.01	0.22	1.00	−0.26	0.01	0.02
	Block5	−0.03	0.00	−0.01	0.00	0.05	0.02
	Full					0.00	0.00
Load rec	Block1		0.09	0.24	0.26	0.91	0.95
	Block2		0.26	0.32	0.24	0.68	0.52
	Block3	0.15	0.17	0.10	0.19	0.90	0.90
	Block4	0.21	0.18	0.25	0.13	0.90	0.85
	Block5	0.76	0.53	0.76	0.61	0.84	0.91
	Full					0.97	0.98
Load SE	Block1		−0.57	−0.58	−0.56	−0.47	−0.33
	Block2		0.04	−0.44	0.04	−0.12	−0.29
	Block3	−0.48	−0.49	−0.49	−0.47	−0.46	−0.47
	Block4	−0.49	−0.49	−0.51	−0.47	−0.46	−0.35
	Block5		−0.45	−0.44	−0.42	−0.36	−0.39
	Full					−0.55	−0.59
Ut	Block1		−0.03	−0.03	−0.03	−0.07	−0.07
	Block2		−0.03	0.00	−0.01	0.04	−0.04
	Block3	0.00	−0.01	0.00	0.00	−0.07	−0.07
	Block4	0.02	0.00	−0.03	0.00	−0.07	0.03
	Block5	−0.01	0.00	−0.01	0.00	−0.01	−0.08
	Full					−0.07	−0.06
Ut rec	Block1		−0.18	0.45	−0.32	1.00	1.00
	Block2		0.46	0.71	0.58	0.98	0.96
	Block3	0.41	0.54	0.53	0.62	1.00	1.00
	Block4	0.52	0.57	0.54	0.55	1.00	0.99
	Block5	0.92	0.72	0.96	0.70	0.99	0.99
	Full					1.00	1.00
Ut SE	Block1		0.11	0.17	0.09	−0.03	−0.02
	Block2		0.18	−0.02	0.18	−0.02	−0.13
	Block3	−0.04	−0.01	−0.01	−0.04	−0.06	0.01
	Block4	0.68	−0.03	−0.10	0.03	−0.03	−0.03
	Block5		−0.03	−0.11	−0.03	−0.04	−0.01
	Full					−0.02	0.01

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 2***Bias results for the Thurstonian IRT model with one trait.*

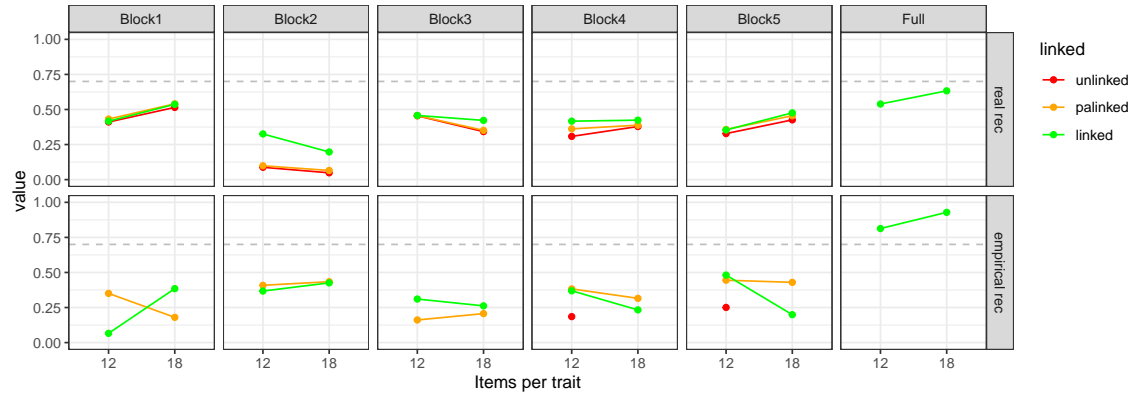
		unlinked		palinked		linked	
	blocks	12	18	12	18	12	18
Conv	Block1	962	999	894	971	995	1000
	Block2	546	528	584	556	704	718
	Block3	947	843	900	817	952	997
	Block4	891	918	921	924	980	966
	Block5	727	952	830	915	949	997
	Full					1000	1000
Load	Block1	0.02	0.01	-0.28	0.02	0.05	0.03
	Block2	-0.07	-0.04	-0.09	-0.01	-0.17	-0.15
	Block3	0.01	0.01	-0.18	0.02	0.08	0.02
	Block4	0.03	-0.02	-0.07	0.01	-0.04	0.05
	Block5	-0.01	-0.02	0.21	0.14	0.07	0.04
	Full					0.02	0.01
Load rec	Block1	0.90	0.91	0.41	0.09	0.91	0.95
	Block2	0.26	0.27	0.23	0.31	0.68	0.57
	Block3	0.80	0.59	0.65	0.25	0.90	0.90
	Block4	0.66	0.55	0.18	0.23	0.90	0.86
	Block5	0.67	0.52	0.77	0.08	0.84	0.91
	Full					0.97	0.98
Load SE	Block1	-0.11	-0.37	-0.15	-0.35	-0.12	-0.04
	Block2	2.32	0.69	0.55	-0.02	-0.19	-0.22
	Block3	-0.19	0.09	-0.51	-0.34	-0.11	-0.28
	Block4	0.23	-0.40	-0.27	-0.37	-0.46	-0.23
	Block5	0.30	-0.37	-0.29	-0.44	-0.25	-0.20
	Full					-0.07	-0.04
Th	Block1	0.00	-0.03	-0.01	-0.04	0.03	0.04
	Block2	0.01	-0.08	0.00	-0.08	-0.03	-0.05
	Block3	0.00	-0.06	-0.01	-0.05	-0.06	-0.02
	Block4	0.03	-0.07	0.04	-0.05	0.03	-0.06
	Block5	-0.04	-0.02	-0.03	-0.02	-0.02	-0.03
	Full					-0.02	-0.03
Th rec	Block1	0.99	0.99	0.99	0.99	1.00	0.99
	Block2	0.99	0.98	0.99	0.98	0.99	0.98
	Block3	1.00	0.99	1.00	0.99	0.99	0.99
	Block4	0.99	0.99	0.99	0.99	0.99	0.99
	Block5	1.00	1.00	1.00	1.00	1.00	1.00
	Full					0.99	0.99
Th SE	Block1	-0.01	0.01	-0.01	0.01	-0.01	0.00
	Block2	-0.01	0.01	-0.01	0.01	-0.01	0.00
	Block3	-0.02	0.00	-0.01	0.00	-0.01	0.00
	Block4	-0.01	0.00	-0.01	0.00	-0.01	0.00
	Block5	-0.01	0.00	-0.01	0.00	-0.01	0.00
	Full					-0.01	0.00

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 2.3.2 Results for latent trait recovery

**Figure 3**

*Latent trait recovery results with one trait.*



*Note.* Abbreviations: rec: recovery.

**Table 3**

*Latent trait recovery results with one trait.*

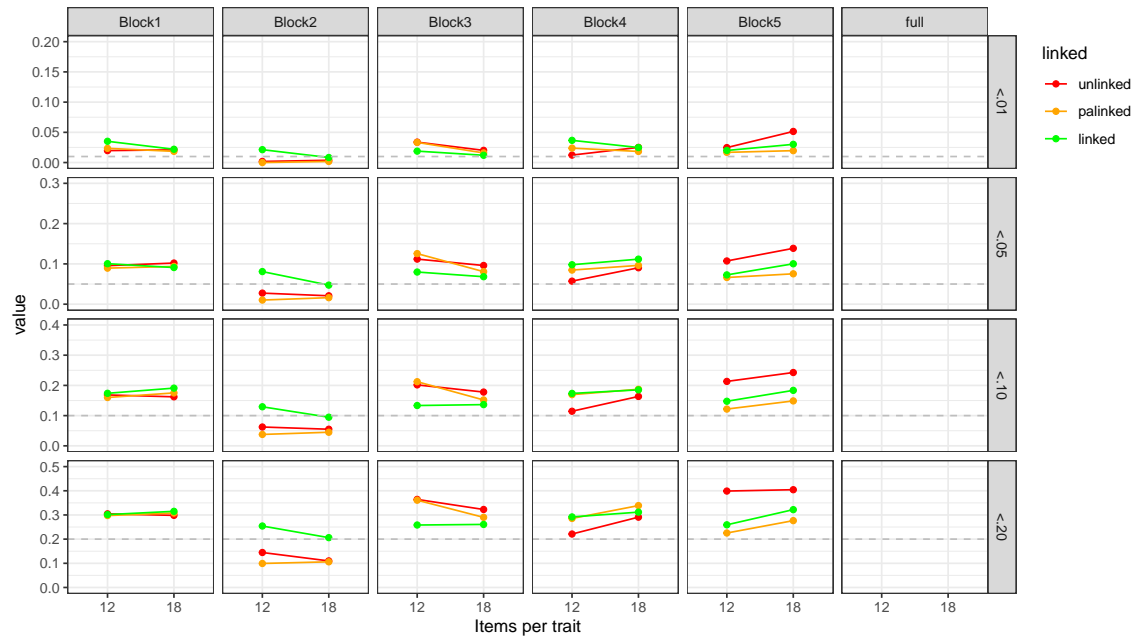
		unlinked		palinked		linked	
	blocks	12	18	12	18	12	18
real rec	Block1	0.411	0.515	0.432	0.540	0.417	0.537
	Block2	0.088	0.048	0.099	0.065	0.326	0.197
	Block3	0.455	0.342	0.456	0.351	0.457	0.422
	Block4	0.308	0.379	0.362	0.388	0.417	0.424
	Block5	0.328	0.426	0.358	0.455	0.354	0.476
	Full					0.539	0.633
empirical rec	Block1			0.351	0.180	0.066	0.385
	Block2			0.408	0.434	0.368	0.426
	Block3			0.161	0.207	0.311	0.262
	Block4	0.185		0.383	0.316	0.369	0.233
	Block5	0.251		0.445	0.430	0.482	0.199
	Full					0.813	0.929

*Note.* Abbreviations: rec: recovery.

### 2.3.3 Results for empirical rejection rates

Figure 4

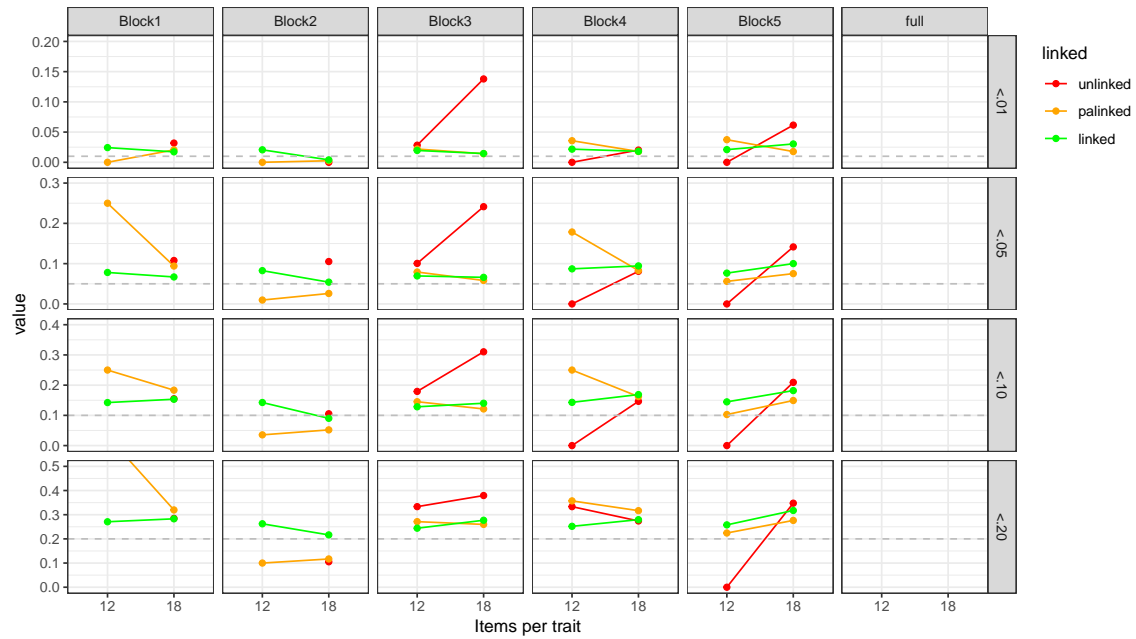
*Empirical rejection rates for the Thurstonian factor model with one trait.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

Figure 5

*Empirical rejection rates for the Thurstonian IRT model with one trait.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 4***Empirical rejection rates for the Thurstonian factor model with one trait.*

		unlinked		palinked		linked	
	blocks	12	18	12	18	12	18
<.01	Block1	0.020	0.021	0.023	0.019	0.035	0.022
	Block2	0.002	0.004	0.000	0.002	0.021	0.008
	Block3	0.034	0.020	0.033	0.016	0.019	0.012
	Block4	0.012	0.025	0.024	0.018	0.037	0.025
	Block5	0.025	0.051	0.017	0.020	0.020	0.030
	full					0.929	1.000
<.05	Block1	0.096	0.102	0.089	0.094	0.101	0.091
	Block2	0.027	0.021	0.010	0.016	0.081	0.047
	Block3	0.112	0.096	0.126	0.081	0.080	0.068
	Block4	0.057	0.090	0.085	0.096	0.098	0.112
	Block5	0.107	0.139	0.066	0.075	0.073	0.100
	full					0.995	1.000
<.10	Block1	0.168	0.162	0.160	0.175	0.174	0.191
	Block2	0.062	0.055	0.038	0.045	0.129	0.095
	Block3	0.202	0.178	0.212	0.152	0.133	0.136
	Block4	0.114	0.163	0.169	0.187	0.173	0.185
	Block5	0.213	0.243	0.122	0.149	0.148	0.184
	full					0.999	1.000
<.20	Block1	0.305	0.298	0.298	0.308	0.302	0.315
	Block2	0.145	0.110	0.099	0.106	0.254	0.206
	Block3	0.364	0.323	0.361	0.290	0.258	0.261
	Block4	0.221	0.291	0.286	0.339	0.292	0.312
	Block5	0.399	0.404	0.225	0.277	0.259	0.322
	full					1.000	1.000

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 5**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with one trait.*

	blocks	unlinked		palinked		linked	
		12	18	12	18	12	18
$\chi^2$	Block1	49	129	101	250	130	317
	Block2	44	120	95	194	129	311
	Block3	49	126	86	224	131	317
	Block4	48	129	101	206	130	292
	Block5	48	129	100	227	130	319
	full					2130	11618
$\chi^2$ SD	Block1	10	15	13	21	16	24
	Block2	8	13	11	17	15	22
	Block3	9	15	12	19	15	24
	Block4	9	15	13	18	16	22
	Block5	9	16	13	20	15	24
	full					57	113
df	Block1	50	129	102	250	131	317
	Block2	50	129	102	205	131	317
	Block3	50	129	87	227	131	317
	Block4	50	129	102	205	131	291
	Block5	50	129	102	227	131	317
	full					2123	11594
Corrected df	Block1	46	123	96	241	124	307
	Block2	45	122	98	199	125	308
	Block3	44	120	79	217	127	311
	Block4	46	123	96	196	124	281
	Block5	42	119	98	221	125	308
	full					1903	10778



**Table 6***Empirical rejection rates for the Thurstonian IRT model with one trait.*

		unlinked		palinked		linked	
	blocks	12	18	12	18	12	18
<.01	Block1		0.032	0.000	0.021	0.024	0.017
	Block2		0.000	0.000	0.003	0.021	0.004
	Block3	0.028	0.138	0.022	0.014	0.020	0.014
	Block4	0.000	0.020	0.036	0.018	0.022	0.018
	Block5	0.000	0.062	0.037	0.018	0.021	0.030
	full					0.925	1.000
<.05	Block1		0.108	0.250	0.094	0.078	0.067
	Block2		0.105	0.010	0.026	0.083	0.054
	Block3	0.101	0.241	0.079	0.058	0.070	0.066
	Block4	0.000	0.081	0.179	0.083	0.087	0.094
	Block5	0.000	0.142	0.056	0.075	0.077	0.100
	full					0.993	1.000
<.10	Block1		0.155	0.250	0.183	0.142	0.153
	Block2		0.105	0.035	0.052	0.143	0.090
	Block3	0.179	0.310	0.145	0.121	0.128	0.140
	Block4	0.000	0.146	0.250	0.162	0.143	0.169
	Block5	0.000	0.209	0.103	0.149	0.145	0.182
	full					0.999	1.000
<.20	Block1		0.285	0.625	0.320	0.271	0.283
	Block2		0.105	0.100	0.117	0.262	0.216
	Block3	0.333	0.379	0.271	0.260	0.244	0.277
	Block4	0.333	0.274	0.357	0.317	0.252	0.279
	Block5	0.000	0.348	0.224	0.276	0.258	0.317
	full					1.000	1.000

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 7**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with one trait.*

	blocks	unlinked		palinked		linked	
		12	18	12	18	12	18
$\chi^2$	Block1		135	118	262	138	326
	Block2		128	101	204	137	322
	Block3	53	137	92	234	138	328
	Block4	54	135	113	214	138	301
	Block5	54	135	109	237	137	329
	full					2185	11754
$\chi^2$ SD	Block1		18	13	26	16	24
	Block2		12	12	17	15	21
	Block3	10	21	13	19	15	24
	Block4	6	15	13	18	16	23
	Block5		16	13	21	16	25
	full					57	114
df	Block1		135	109	261	138	327
	Block2		135	109	214	138	327
	Block3	54	135	93	237	138	327
	Block4	54	135	109	214	138	300
	Block5	54	135	109	237	138	327
	full					2178	11730
Corrected df	Block1		129	103	252	133	319
	Block2		125	105	208	132	318
	Block3	48	126	88	229	134	321
	Block4	50	129	103	205	133	293
	Block5	47	125	105	231	132	318
	full					1958	10914

### 3 Simulation Study with three uncorrelated traits

#### 3.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: three traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (18 vs. 36 vs. 54 total)

#### 3.2 Definition of parameters

##### 3.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3
Trait 1	1	0	0
Trait 2	0	1	0
Trait 3	0	0	1

##### 3.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.60214	0.55420	0.70129
Items 4/5/6	0.83584	-0.82974	-0.33638
Items 7/8/9	0.26087	-0.31988	-0.87237
Items 10/11/12	0.65395	0.60920	0.52512
Items 13/14/15	0.56438	-0.52383	0.48212
Items 16/17/18	-0.40190	-0.01101	-0.04458
Items 19/20/21	-0.46988	0.26912	0.80706
Items 22/23/24	0.99636	0.57226	0.74355
Items 25/26/27	-0.58206	-0.00674	-0.45567
Items 28/29/30	-0.22311	0.44484	-0.64701
Items 31/32/33	0.16485	0.02783	-0.12827
Items 34/35/36	0.19929	-0.02725	-0.94064
Items 37/38/39	-0.79683	0.38148	-0.20727
Items 40/41/42	-0.23520	0.67593	-0.38561
Items 43/44/45	-0.14346	0.52521	-0.79865
Items 46/47/48	-0.89717	0.23819	-0.60556
Items 49/50/51	0.75287	-0.92923	-0.13654
Items 52/53/54	0.36866	-0.44479	0.09615

### 3.2.3 Matrix of loadings $\Lambda$

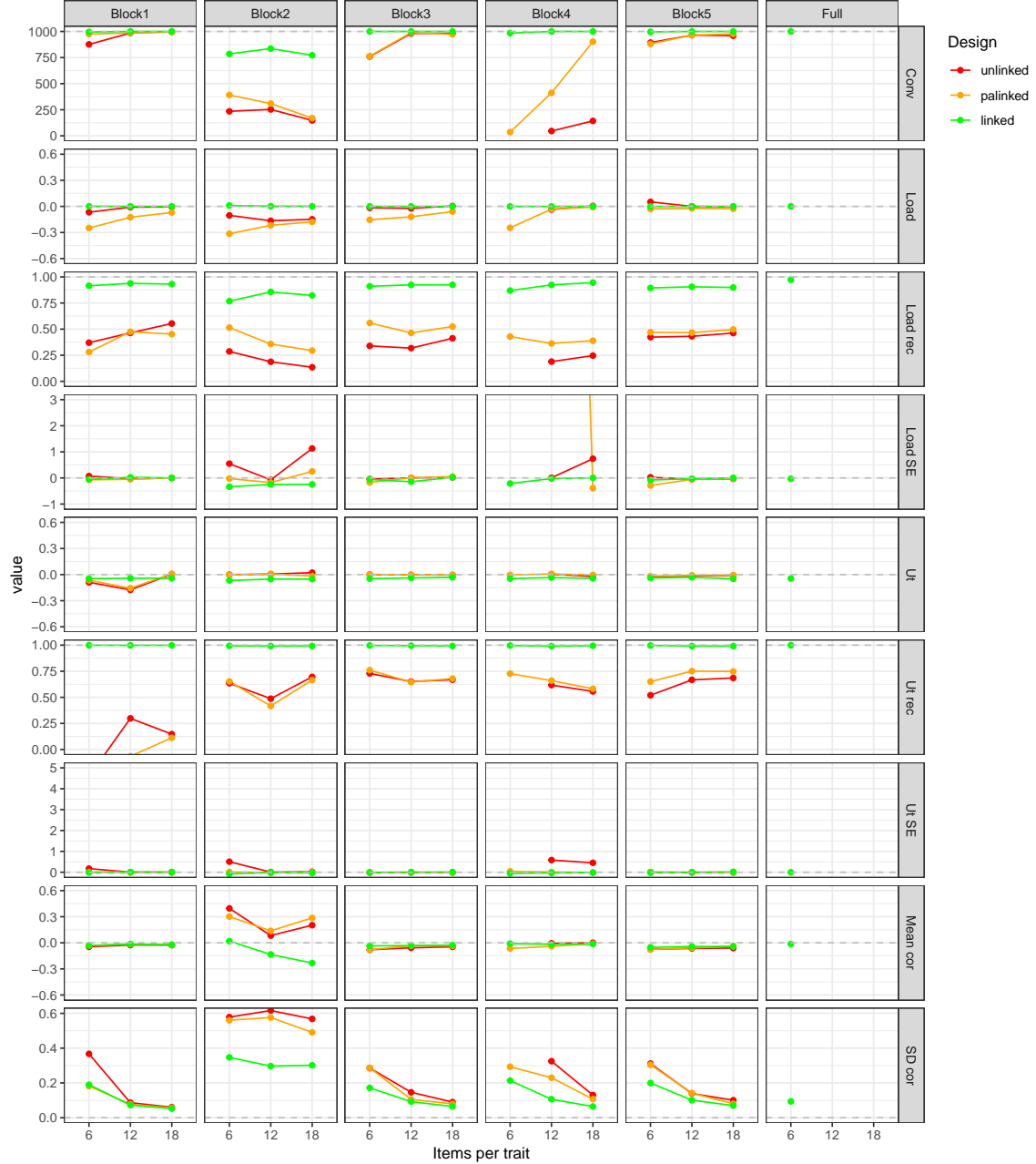
	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.66560	0.30552	0.45042
Items 4/5/6	0.54022	0.30174	0.39008
Items 7/8/9	0.78315	0.58585	0.80319
Items 10/11/12	0.45948	0.67285	0.57439
Items 13/14/15	0.75821	0.73864	0.51631
Items 16/17/18	0.31710	0.74134	0.53943
Items 19/20/21	0.69617	0.71690	0.73394
Items 22/23/24	0.59674	0.65789	0.75229
Items 25/26/27	0.81445	0.46456	0.84991
Items 28/29/30	0.38511	0.56578	0.30548
Items 31/32/33	0.66549	0.82177	0.79742
Items 34/35/36	0.64335	0.70405	0.38461
Items 37/38/39	0.74185	0.31318	0.54122
Items 40/41/42	0.67769	0.86301	0.52383
Items 43/44/45	0.81144	0.82512	0.87679
Items 46/47/48	0.76865	0.78362	0.83513
Items 49/50/51	0.50213	0.82578	0.37268
Items 52/53/54	0.43420	0.44544	0.66294

### 3.3 Results

#### 3.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 6

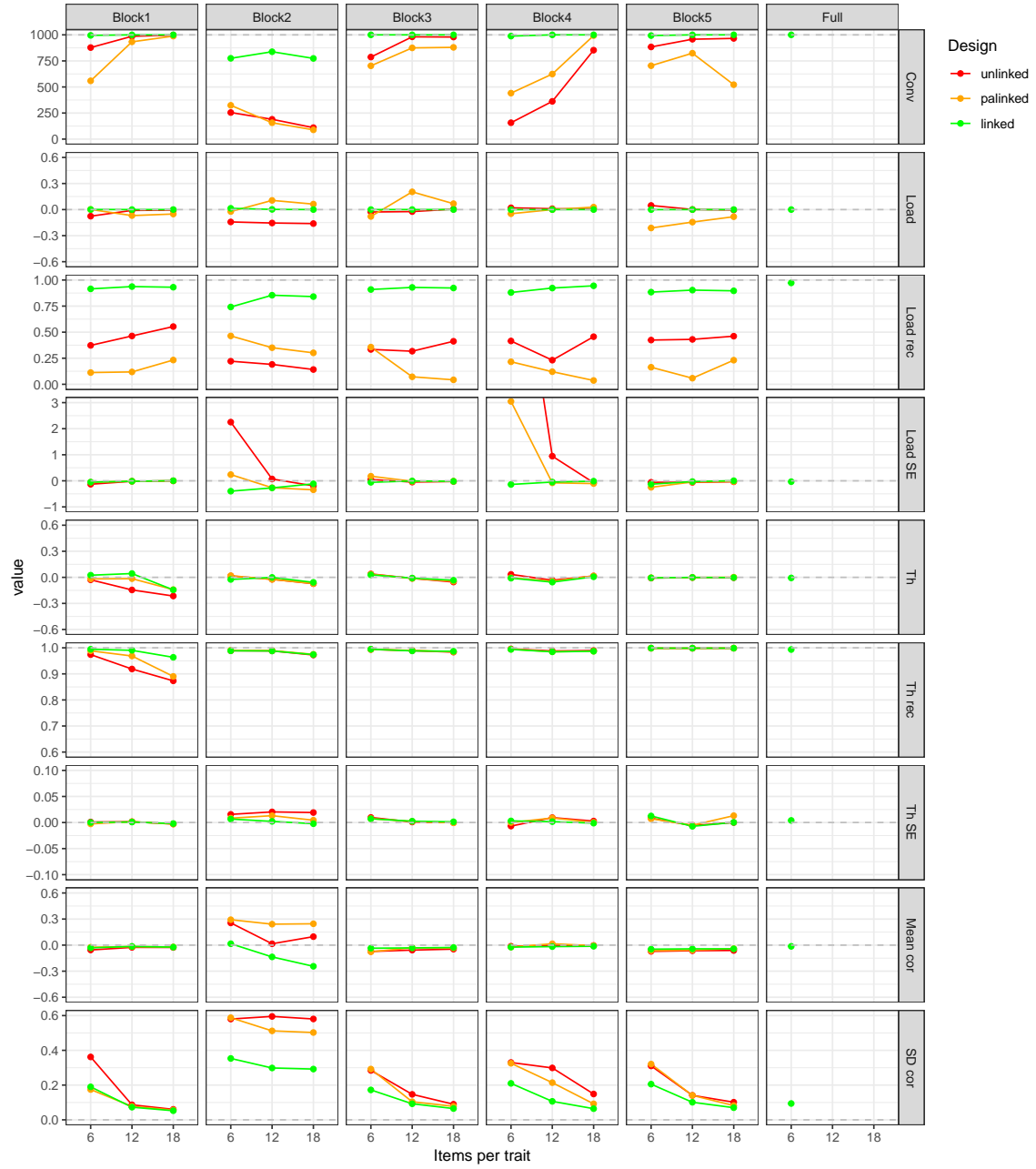
*Bias results for the Thurstonian factor model with three uncorrelated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 7**

*Bias results for the Thurstonian IRT model with three uncorrelated traits*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 8**

*Bias results for the Thurstonian factor model with three uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	876	985	998	975	986	994	995	1000	1000
	Block2	235	253	148	391	309	170	785	836	772
	Block3	760	981	982	763	989	972	1000	1000	1000
	Block4		46	143	36	412	902	984	1000	1000
	Block5	893	965	959	881	965	977	996	1000	1000
	Full							1000		
Load	Block1	-0.07	-0.01	-0.01	-0.25	-0.13	-0.07	0.00	0.00	0.00
	Block2	-0.10	-0.17	-0.15	-0.31	-0.22	-0.18	0.01	0.00	0.00
	Block3	-0.02	-0.02	0.00	-0.15	-0.12	-0.06	0.00	0.00	0.00
	Block4		-0.04	0.01	-0.25	-0.03	-0.01	0.00	0.00	0.00
	Block5	0.05	0.00	0.00	-0.03	-0.02	-0.03	0.00	0.00	0.00
	Full							0.00		
Load rec	Block1	0.37	0.46	0.55	0.28	0.47	0.45	0.92	0.94	0.93
	Block2	0.29	0.19	0.13	0.51	0.36	0.29	0.77	0.86	0.82
	Block3	0.34	0.32	0.41	0.56	0.46	0.52	0.91	0.92	0.93
	Block4		0.19	0.25	0.43	0.36	0.39	0.87	0.92	0.95
	Block5	0.42	0.43	0.46	0.47	0.47	0.50	0.89	0.91	0.90
	Full							0.97		
Load SE	Block1	0.07	-0.03	0.00	-0.07	-0.05	0.00	-0.05	0.02	0.00
	Block2	0.55	-0.07	1.13	-0.02	-0.18	0.25	-0.34	-0.25	-0.24
	Block3	-0.04	0.00	0.02	-0.17	0.01	0.05	-0.06	-0.15	0.03
	Block4		0.01	0.74	1154.84	46.95	-0.39	-0.21	-0.02	0.01
	Block5	0.02	-0.05	-0.03	-0.29	-0.05	-0.02	-0.08	-0.03	0.00
	Full							-0.03		
Ut	Block1	-0.09	-0.18	0.00	-0.06	-0.16	0.01	-0.05	-0.04	-0.04
	Block2	0.00	0.01	0.02	0.00	0.01	-0.02	-0.07	-0.05	-0.05
	Block3	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	-0.04	-0.03
	Block4		0.00	-0.02	0.00	0.01	0.00	-0.05	-0.03	-0.04
	Block5	-0.03	-0.01	-0.01	-0.02	-0.01	-0.01	-0.04	-0.03	-0.05
	Full							-0.04		
Ut rec	Block1	-0.26	0.30	0.15	-0.51	-0.07	0.11	1.00	1.00	1.00
	Block2	0.64	0.49	0.70	0.65	0.42	0.67	0.99	0.99	0.99
	Block3	0.73	0.65	0.67	0.76	0.65	0.68	1.00	0.99	0.99
	Block4		0.62	0.56	0.73	0.66	0.58	1.00	0.99	0.99
	Block5	0.52	0.67	0.69	0.65	0.75	0.75	1.00	0.99	0.99
	Full							1.00		
Ut SE	Block1	0.17	0.00	0.00	0.01	0.00	0.00	-0.01	0.01	0.01
	Block2	0.51	0.01	0.02	0.02	-0.02	0.00	-0.08	-0.01	-0.03
	Block3	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	0.01	0.01
	Block4		0.58	0.45	0.04	0.01	-0.01	-0.05	-0.03	-0.02
	Block5	0.00	0.00	0.00	0.01	0.00	-0.01	-0.02	0.00	0.02
	Full							0.00		
Mean cor	Block1	-0.05	-0.03	-0.03	-0.03	-0.02	-0.02	-0.03	-0.02	-0.02
	Block2	0.40	0.08	0.20	0.30	0.14	0.29	0.02	-0.13	-0.23
	Block3	-0.08	-0.06	-0.05	-0.08	-0.03	-0.04	-0.04	-0.03	-0.03
	Block4		-0.01	0.00	-0.07	-0.04	-0.01	-0.01	-0.02	-0.01
	Block5	-0.07	-0.06	-0.06	-0.08	-0.06	-0.04	-0.05	-0.04	-0.04
	Full							-0.01		
SD cor	Block1	0.37	0.09	0.06	0.18	0.08	0.06	0.19	0.07	0.05
	Block2	0.58	0.62	0.57	0.56	0.58	0.49	0.35	0.30	0.30
	Block3	0.29	0.15	0.09	0.29	0.11	0.08	0.17	0.09	0.06
	Block4		0.33	0.13	0.29	0.23	0.11	0.21	0.11	0.06
	Block5	0.31	0.14	0.10	0.30	0.14	0.08	0.20	0.10	0.07
	Full							0.09		

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 9***Bias results for the Thurstonian IRT model with three uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	878	986	998	559	933	988	995	1000	1000
	Block2	255	190	110	324	157	89	775	838	774
	Block3	787	981	979	703	875	880	1000	1000	1000
	Block4	157	362	853	441	624	993	988	1000	1000
	Block5	884	958	966	704	824	522	993	1000	1000
	Full							1000		
Load	Block1	-0.08	-0.01	-0.01	0.00	-0.07	-0.05	0.00	0.00	0.00
	Block2	-0.14	-0.15	-0.16	-0.02	0.11	0.06	0.01	0.00	0.00
	Block3	-0.03	-0.02	0.00	-0.08	0.20	0.07	0.00	0.00	0.00
	Block4	0.02	0.01	0.01	-0.05	0.00	0.03	0.00	0.00	0.00
	Block5	0.05	0.00	-0.01	-0.21	-0.14	-0.08	0.00	0.00	0.00
	Full							0.00		
Load rec	Block1	0.37	0.46	0.55	0.11	0.12	0.23	0.92	0.94	0.93
	Block2	0.22	0.19	0.14	0.46	0.35	0.30	0.74	0.85	0.84
	Block3	0.33	0.32	0.41	0.36	0.07	0.04	0.91	0.93	0.92
	Block4	0.42	0.23	0.46	0.22	0.12	0.04	0.88	0.92	0.94
	Block5	0.42	0.43	0.46	0.16	0.06	0.23	0.88	0.90	0.90
	Full							0.97		
Load SE	Block1	-0.13	-0.02	0.00	-0.05	-0.02	0.00	-0.05	-0.02	0.00
	Block2	2.25	0.07	-0.19	0.24	-0.26	-0.34	-0.40	-0.27	-0.12
	Block3	0.05	-0.05	-0.03	0.18	-0.02	-0.02	-0.06	-0.01	-0.02
	Block4	11.36	0.95	-0.07	3.05	-0.08	-0.11	-0.14	-0.05	-0.01
	Block5	-0.06	-0.06	-0.04	-0.25	-0.04	-0.03	-0.13	-0.03	0.00
	Full							-0.03		
Th	Block1	-0.03	-0.14	-0.21	-0.02	-0.02	-0.14	0.02	0.04	-0.14
	Block2	0.02	-0.02	-0.07	0.02	-0.02	-0.07	-0.02	0.00	-0.06
	Block3	0.04	-0.01	-0.05	0.04	-0.01	-0.04	0.03	-0.01	-0.03
	Block4	0.03	-0.03	0.02	0.00	-0.04	0.02	-0.01	-0.06	0.01
	Block5	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00
	Full							-0.01		
Th rec	Block1	0.97	0.92	0.87	0.99	0.97	0.89	0.99	0.99	0.96
	Block2	0.99	0.99	0.97	0.99	0.99	0.97	0.99	0.99	0.97
	Block3	0.99	0.99	0.98	0.99	0.99	0.98	0.99	0.99	0.99
	Block4	1.00	0.99	0.99	0.99	0.98	0.99	0.99	0.98	0.99
	Block5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Full							0.99		
Th SE	Block1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Block2	0.02	0.02	0.02	0.01	0.01	0.00	0.01	0.00	0.00
	Block3	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
	Block4	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
	Block5	0.01	-0.01	0.00	0.01	-0.01	0.01	0.01	-0.01	0.00
	Full							0.00		
Mean cor	Block1	-0.06	-0.03	-0.03	-0.02	-0.02	-0.02	-0.03	-0.02	-0.02
	Block2	0.26	0.02	0.10	0.29	0.24	0.25	0.02	-0.14	-0.24
	Block3	-0.07	-0.06	-0.05	-0.08	-0.03	-0.03	-0.04	-0.03	-0.03
	Block4	-0.01	0.00	0.00	-0.02	0.02	-0.01	-0.02	-0.02	-0.01
	Block5	-0.07	-0.06	-0.06	-0.06	-0.06	-0.04	-0.05	-0.04	-0.04
	Full							-0.01		
SD cor	Block1	0.36	0.09	0.06	0.17	0.08	0.06	0.19	0.07	0.05
	Block2	0.58	0.60	0.58	0.59	0.51	0.50	0.35	0.30	0.29
	Block3	0.28	0.15	0.09	0.29	0.10	0.08	0.17	0.09	0.06
	Block4	0.33	0.30	0.15	0.33	0.21	0.09	0.21	0.11	0.06
	Block5	0.31	0.14	0.10	0.32	0.14	0.08	0.21	0.10	0.07
	Full							0.09		

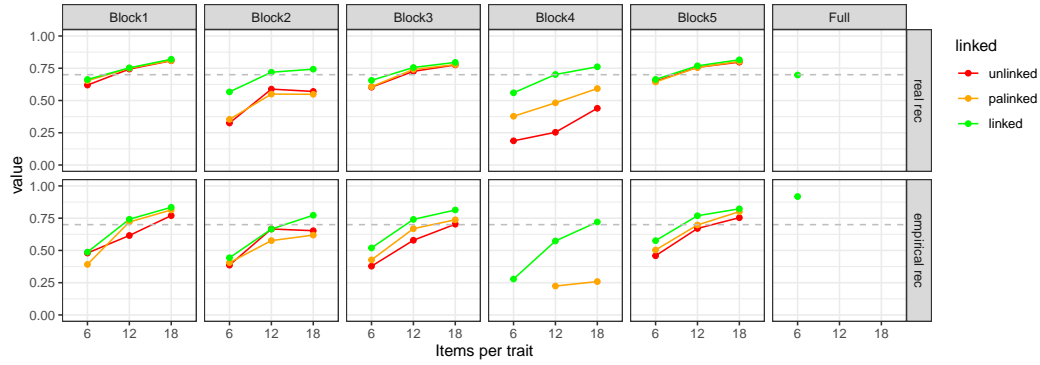
*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.



### 3.3.2 Results for latent trait recovery

Figure 8

*Latent trait recovery results with three uncorrelated traits.*



*Note.* Abbreviations: rec: recovery.

Table 10

*Latent trait recovery results with three uncorrelated traits.*

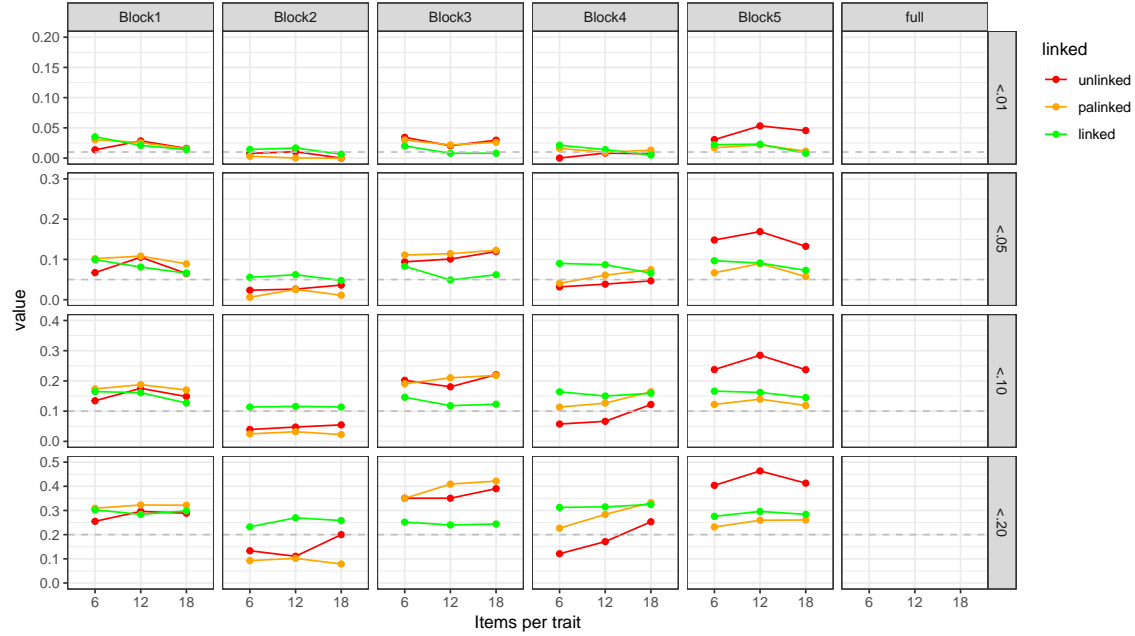
		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
real rec	Block1	0.620	0.743	0.808	0.653	0.751	0.810	0.663	0.753	0.820
	Block2	0.325	0.588	0.570	0.354	0.550	0.548	0.567	0.719	0.743
	Block3	0.603	0.726	0.776	0.607	0.741	0.778	0.656	0.756	0.795
	Block4	0.187	0.254	0.440	0.378	0.482	0.593	0.560	0.701	0.761
	Block5	0.650	0.757	0.797	0.644	0.756	0.803	0.663	0.769	0.815
	Full							0.698		
empirical rec	Block1	0.480	0.616	0.770	0.392	0.720	0.814	0.487	0.742	0.834
	Block2	0.386	0.666	0.653	0.405	0.576	0.619	0.443	0.666	0.773
	Block3	0.378	0.579	0.703	0.427	0.669	0.737	0.520	0.741	0.813
	Block4					0.224	0.259	0.278	0.573	0.720
	Block5	0.459	0.670	0.754	0.504	0.697	0.801	0.576	0.769	0.823
	Full							0.918		

*Note.* Abbreviations: rec: recovery.

### 3.3.3 Results for empirical rejection rates

**Figure 9**

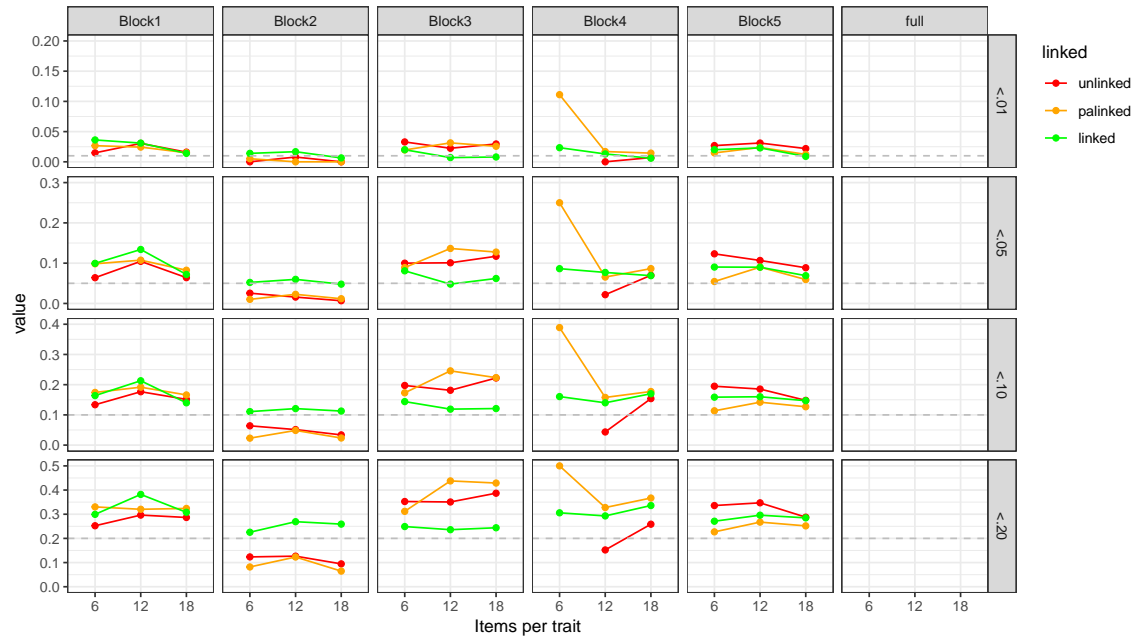
*Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Figure 10**

*Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 11**

*Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
<.01	Block1	0.014	0.028	0.016	0.030	0.026	0.015	0.035	0.021	0.014
	Block2	0.008	0.011	0.000	0.003	0.000	0.000	0.014	0.017	0.006
	Block3	0.034	0.020	0.030	0.030	0.022	0.026	0.020	0.008	0.008
	Block4	0.000	0.008	0.007	0.016	0.010	0.013	0.021	0.014	0.005
	Block5	0.031	0.053	0.046	0.017	0.022	0.011	0.022	0.023	0.008
	full							1.000		
<.05	Block1	0.067	0.105	0.065	0.102	0.108	0.089	0.099	0.081	0.066
	Block2	0.024	0.026	0.036	0.006	0.025	0.011	0.055	0.062	0.048
	Block3	0.094	0.101	0.120	0.111	0.114	0.123	0.083	0.049	0.062
	Block4	0.032	0.039	0.047	0.041	0.061	0.075	0.090	0.087	0.066
	Block5	0.148	0.169	0.133	0.067	0.090	0.057	0.097	0.091	0.073
	full							1.000		
<.10	Block1	0.134	0.175	0.148	0.174	0.188	0.170	0.165	0.161	0.127
	Block2	0.039	0.047	0.055	0.025	0.032	0.022	0.114	0.116	0.114
	Block3	0.202	0.180	0.221	0.191	0.210	0.218	0.146	0.118	0.123
	Block4	0.057	0.066	0.122	0.113	0.127	0.164	0.164	0.150	0.159
	Block5	0.238	0.285	0.237	0.122	0.140	0.119	0.166	0.162	0.145
	full							1.000		
<.20	Block1	0.255	0.296	0.289	0.309	0.323	0.322	0.303	0.284	0.298
	Block2	0.133	0.111	0.200	0.093	0.102	0.079	0.232	0.270	0.258
	Block3	0.351	0.351	0.390	0.350	0.409	0.422	0.252	0.240	0.244
	Block4	0.121	0.171	0.253	0.227	0.284	0.332	0.313	0.315	0.325
	Block5	0.404	0.463	0.413	0.232	0.260	0.261	0.276	0.296	0.284
	full							1.000		

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 12**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	118	570	1338	242	885	1797	286	1359	3131
	Block2	111	548	1315	208	901	1833	309	1353	3130
	Block3	119	568	1345	241	884	1866	313	1358	3137
	Block4	117	570	1355	217	887	1879	313	1363	3138
	Block5	118	568	1338	217	798	2186	312	1360	3133
	full	11598								
$\chi^2$ SD	Block1	14	32	45	22	39	52	24	46	63
	Block2	12	27	38	17	36	44	23	46	60
	Block3	14	29	45	21	35	51	24	44	65
	Block4	13	28	42	19	35	48	23	43	58
	Block5	14	30	45	20	36	55	24	46	65
	full	128								
df	Block1	120	569	1338	242	884	1797	287	1357	3130
	Block2	121	567	1340	220	926	1861	313	1356	3130
	Block3	120	568	1340	242	883	1861	313	1357	3130
	Block4	126	579	1356	223	891	1875	313	1357	3130
	Block5	120	567	1338	219	797	2183	313	1357	3130
	full	11590								
Corrected df	Block1	114	557	1320	233	866	1770	277	1339	3097
	Block2	111	551	1304	214	914	1843	304	1338	3103
	Block3	111	550	1313	230	857	1825	307	1345	3112
	Block4	120	567	1338	214	873	1848	303	1339	3097
	Block5	108	541	1302	213	785	2165	304	1339	3103
	full	10774								

**Table 13***Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.015	0.030	0.016	0.027	0.024	0.015	0.036	0.031	0.014
	Block2	0.000	0.008	0.000	0.005	0.000	0.000	0.014	0.017	0.006
	Block3	0.033	0.022	0.030	0.020	0.031	0.026	0.020	0.007	0.008
	Block4		0.000	0.007	0.111	0.017	0.014	0.023	0.013	0.006
	Block5	0.027	0.031	0.022	0.015	0.024	0.012	0.020	0.023	0.009
	full							1.000		
<.05	Block1	0.064	0.105	0.064	0.098	0.108	0.082	0.099	0.134	0.072
	Block2	0.026	0.016	0.007	0.010	0.023	0.012	0.052	0.060	0.048
	Block3	0.100	0.101	0.117	0.089	0.137	0.128	0.081	0.048	0.062
	Block4		0.022	0.070	0.250	0.066	0.086	0.086	0.077	0.069
	Block5	0.123	0.107	0.089	0.054	0.090	0.059	0.090	0.090	0.069
	full							1.000		
<.10	Block1	0.134	0.177	0.151	0.174	0.192	0.166	0.164	0.213	0.140
	Block2	0.064	0.051	0.034	0.023	0.049	0.024	0.111	0.121	0.113
	Block3	0.197	0.181	0.222	0.173	0.246	0.223	0.144	0.119	0.121
	Block4		0.043	0.154	0.389	0.158	0.177	0.161	0.140	0.170
	Block5	0.195	0.185	0.148	0.114	0.142	0.127	0.159	0.160	0.147
	full							1.000		
<.20	Block1	0.252	0.296	0.287	0.330	0.320	0.324	0.299	0.382	0.308
	Block2	0.123	0.126	0.095	0.082	0.123	0.065	0.225	0.269	0.259
	Block3	0.353	0.351	0.387	0.312	0.438	0.429	0.249	0.236	0.244
	Block4		0.152	0.259	0.500	0.328	0.367	0.306	0.293	0.336
	Block5	0.336	0.347	0.288	0.227	0.267	0.252	0.271	0.296	0.285
	full							1.000		

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 14**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with three uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	124	582	1356	254	905	1822	295	1378	3159
	Block2	116	559	1334	216	925	1855	319	1373	3158
	Block3	125	580	1363	252	903	1893	323	1377	3165
	Block4		577	1375	247	913	1909	323	1382	3166
	Block5	124	580	1356	226	816	2215	321	1380	3161
	full							11734		
$\chi^2$ SD	Block1	14	33	46	22	39	52	24	47	64
	Block2	13	27	35	17	36	43	23	46	60
	Block3	15	29	46	21	36	51	24	44	65
	Block4		29	44	25	36	49	24	43	58
	Block5	14	31	46	20	36	56	25	47	66
	full							128		
df	Block1	126	581	1356	253	903	1822	296	1376	3158
	Block2	127	579	1358	230	946	1887	323	1376	3158
	Block3	126	580	1358	253	902	1887	323	1376	3158
	Block4		591	1374	234	912	1902	323	1376	3158
	Block5	126	579	1356	229	814	2213	323	1376	3158
	full							11726		
Corrected df	Block1	120	569	1338	244	885	1795	286	1346	3122
	Block2	117	563	1338	224	934	1869	314	1358	3131
	Block3	117	562	1331	243	872	1851	317	1364	3140
	Block4		579	1356	225	894	1875	313	1360	3122
	Block5	116	563	1336	223	802	2195	314	1358	3131
	full							10910		

## 4 Simulation Study with three correlated traits

### 4.1 Conditions

1. Correlation of traits: correlated traits
2. Number of traits: three traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (18 vs. 36 vs. 54 total)

### 4.2 Definition of parameters

#### 4.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3
Trait 1	1.0	0.2	0.3
Trait 2	0.2	1.0	0.5
Trait 3	0.3	0.5	1.0

#### 4.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.66313	0.68495	0.23673
Items 4/5/6	-0.01962	0.96208	-0.45111
Items 7/8/9	-0.78583	0.46908	0.24392
Items 10/11/12	0.22657	-0.88302	-0.83380
Items 13/14/15	-0.10953	-0.52574	0.63187
Items 16/17/18	0.95749	0.62862	0.92821
Items 19/20/21	0.91897	-0.69029	-0.23159
Items 22/23/24	-0.78072	0.25508	-0.24633
Items 25/26/27	-0.00629	-0.62912	-0.68637
Items 28/29/30	-0.25695	0.43070	0.11076
Items 31/32/33	0.00552	-0.34708	0.16990
Items 34/35/36	0.24643	0.58285	-0.80722
Items 37/38/39	-0.41388	0.57632	-0.57120
Items 40/41/42	-0.87580	-0.33450	0.13569
Items 43/44/45	-0.74707	0.30484	0.54187
Items 46/47/48	-0.47004	0.55212	0.71244
Items 49/50/51	-0.12585	-0.70236	0.82168
Items 52/53/54	-0.19953	0.46597	-0.92932

### 4.2.3 Matrix of loadings $\Lambda$

	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.69346	0.41061	0.53898
Items 4/5/6	0.31649	0.44403	0.52048
Items 7/8/9	0.36916	0.36613	0.31971
Items 10/11/12	0.66118	0.35091	0.49624
Items 13/14/15	0.83274	0.68795	0.67882
Items 16/17/18	0.43069	0.77565	0.47052
Items 19/20/21	0.46902	0.85293	0.60289
Items 22/23/24	0.46630	0.70542	0.40274
Items 25/26/27	0.82667	0.54489	0.71654
Items 28/29/30	0.71396	0.62572	0.34277
Items 31/32/33	0.38473	0.63116	0.84787
Items 34/35/36	0.66461	0.38114	0.57857
Items 37/38/39	0.37252	0.51616	0.79192
Items 40/41/42	0.81273	0.40619	0.34437
Items 43/44/45	0.61965	0.42866	0.63735
Items 46/47/48	0.87876	0.85378	0.76889
Items 49/50/51	0.51305	0.44540	0.41166
Items 52/53/54	0.41010	0.70313	0.51560

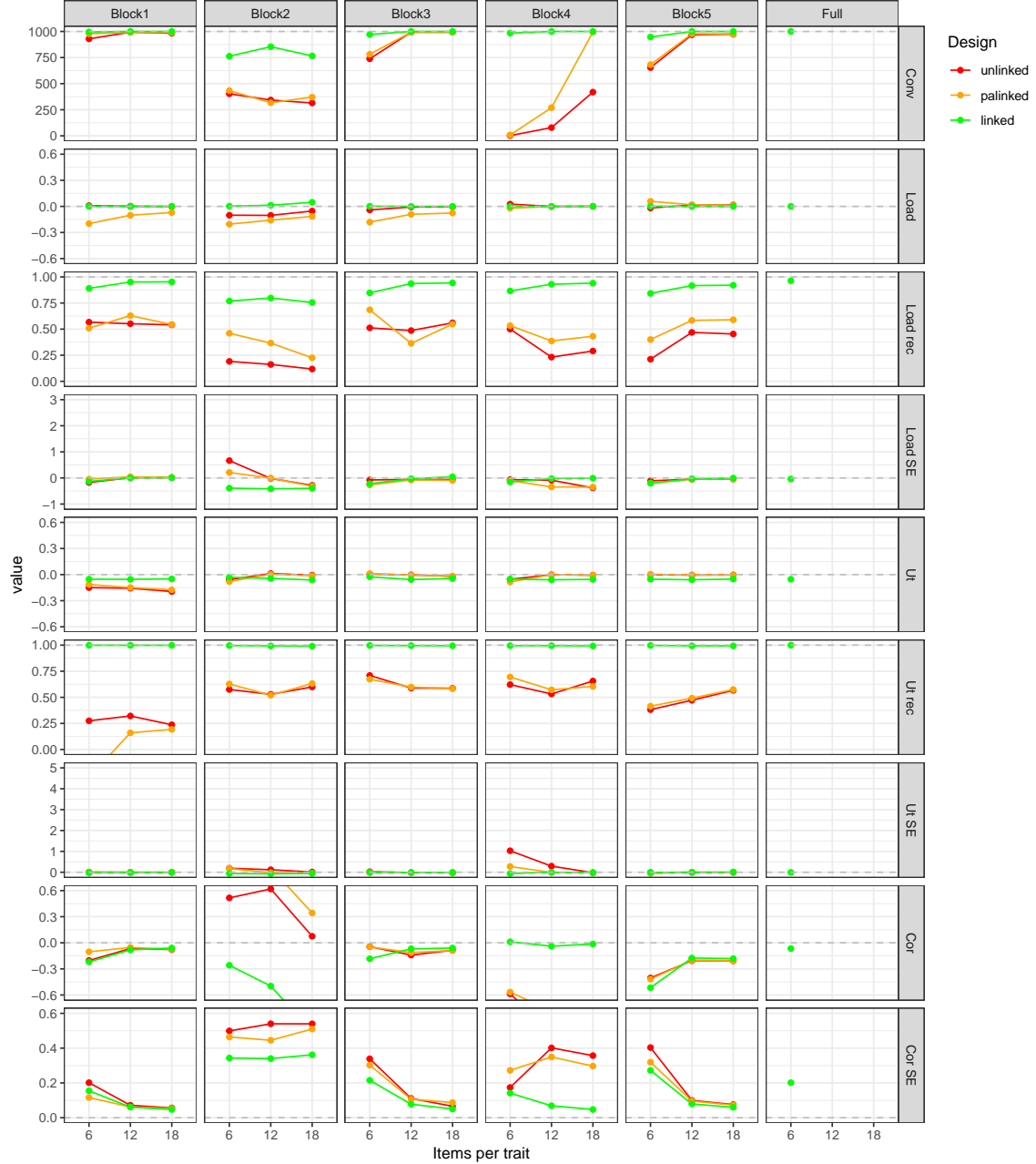


## 4.3 Results

### 4.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 11

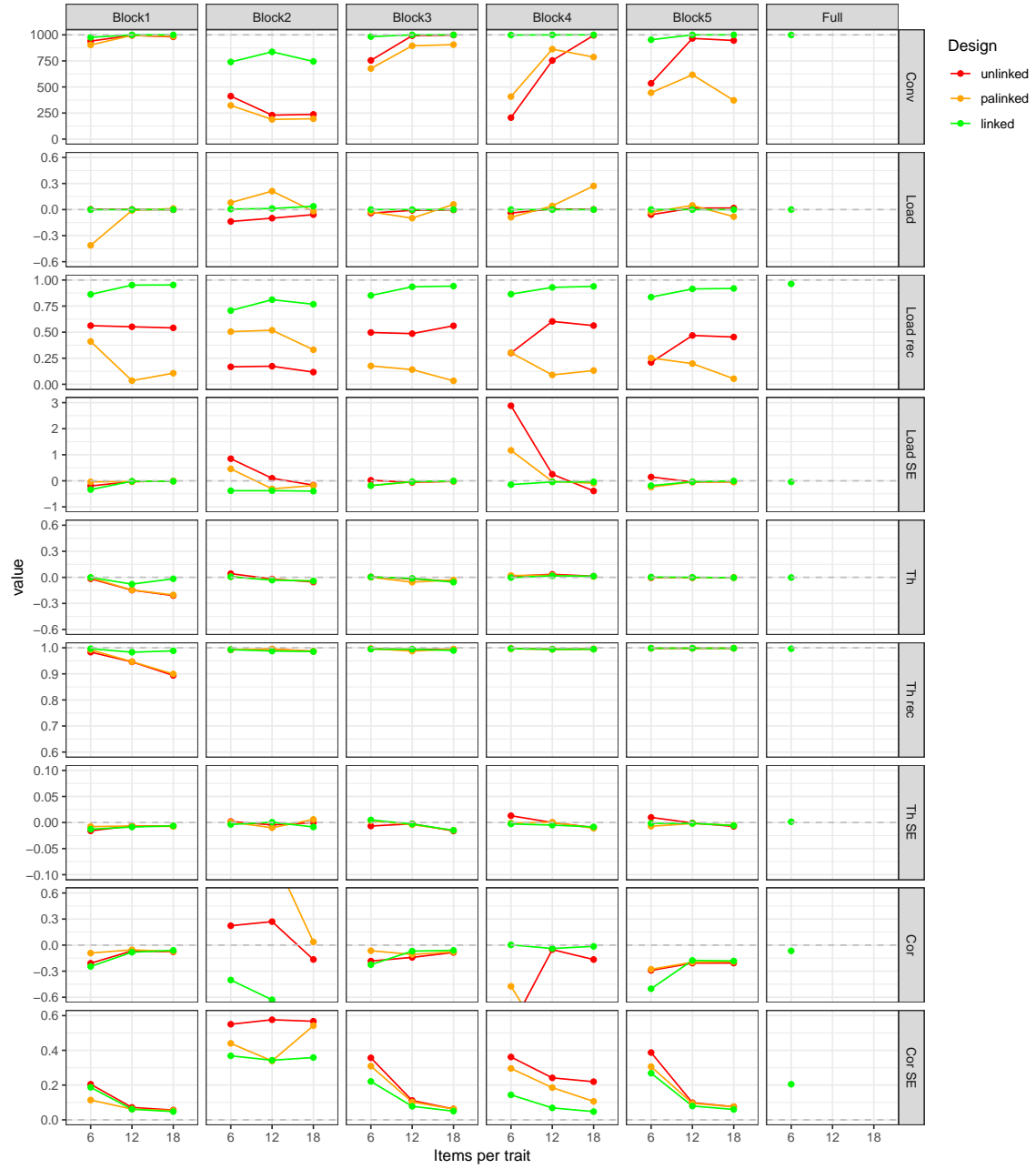
*Bias results for the Thurstonian factor model with three correlated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 12**

*Bias results for the Thurstonian IRT model with three correlated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 15**

*Bias results for the Thurstonian factor model with three correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	929	993	983	978	991	990	995	1000	1000
	Block2	403	343	315	433	317	371	763	854	766
	Block3	738	993	993	782	992	992	971	1000	1000
	Block4	2	79	419	8	270	994	984	1000	1000
	Block5	654	968	974	682	978	976	947	1000	1000
	Full							1000		
Load	Block1	0.01	0.00	0.00	-0.20	-0.10	-0.07	0.00	0.00	0.00
	Block2	-0.10	-0.10	-0.05	-0.20	-0.16	-0.11	0.00	0.01	0.05
	Block3	-0.04	-0.01	0.00	-0.18	-0.09	-0.08	0.00	0.00	0.00
	Block4	0.03	0.00	0.00	-0.02	0.00	0.00	0.00	0.00	0.00
	Block5	-0.02	0.01	0.02	0.06	0.02	0.01	0.00	0.00	0.00
	Full							0.00		
Load rec	Block1	0.57	0.55	0.54	0.51	0.63	0.54	0.89	0.95	0.95
	Block2	0.19	0.16	0.12	0.46	0.37	0.23	0.77	0.80	0.76
	Block3	0.51	0.49	0.56	0.69	0.36	0.55	0.85	0.94	0.94
	Block4	0.50	0.23	0.29	0.53	0.39	0.43	0.87	0.93	0.94
	Block5	0.21	0.47	0.45	0.40	0.58	0.59	0.84	0.92	0.92
	Full							0.96		
Load SE	Block1	-0.17	0.01	0.01	-0.05	0.04	0.02	-0.13	0.00	0.01
	Block2	0.66	-0.02	-0.28	0.21	-0.01	-0.32	-0.39	-0.42	-0.40
	Block3	-0.08	-0.06	-0.05	-0.27	-0.08	-0.09	-0.22	-0.03	0.05
	Block4	-0.07	-0.09	-0.38	-0.10	-0.34	-0.35	-0.15	-0.03	-0.01
	Block5	-0.11	-0.05	-0.04	-0.20	-0.05	-0.04	-0.20	-0.04	-0.01
	Full							-0.04		
Ut	Block1	-0.15	-0.16	-0.19	-0.11	-0.15	-0.17	-0.05	-0.05	-0.05
	Block2	-0.06	0.01	-0.01	-0.08	0.00	-0.01	-0.03	-0.04	-0.06
	Block3	0.01	0.00	-0.02	0.01	0.00	-0.02	-0.03	-0.06	-0.05
	Block4	-0.05	0.00	-0.01	-0.09	0.00	-0.01	-0.05	-0.06	-0.05
	Block5	0.00	-0.01	0.00	0.00	0.00	0.00	-0.05	-0.06	-0.05
	Full							-0.05		
Ut rec	Block1	0.28	0.32	0.24	-0.27	0.16	0.19	1.00	1.00	1.00
	Block2	0.58	0.53	0.60	0.63	0.52	0.63	1.00	0.99	0.99
	Block3	0.71	0.59	0.59	0.67	0.60	0.58	1.00	1.00	0.99
	Block4	0.62	0.53	0.66	0.70	0.57	0.60	0.99	0.99	0.99
	Block5	0.38	0.47	0.57	0.42	0.49	0.57	1.00	0.99	0.99
	Full							1.00		
Ut SE	Block1	0.01	0.00	0.00	0.00	0.00	0.00	-0.03	-0.02	-0.01
	Block2	0.19	0.12	0.01	0.18	-0.02	-0.04	-0.06	-0.07	-0.06
	Block3	0.02	-0.01	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.02
	Block4	1.03	0.29	-0.02	0.27	-0.01	-0.01	-0.06	0.00	-0.02
	Block5	-0.01	0.00	-0.01	-0.06	0.00	-0.01	-0.01	-0.01	0.01
	Full							-0.01		
Cor	Block1	-0.20	-0.07	-0.08	-0.10	-0.05	-0.07	-0.22	-0.08	-0.06
	Block2	0.52	0.62	0.08	0.72	0.86	0.34	-0.26	-0.50	-1.02
	Block3	-0.04	-0.14	-0.09	-0.05	-0.11	-0.09	-0.18	-0.07	-0.06
	Block4	-0.59	-1.02	-1.02	-0.57	-0.78	-0.76	0.01	-0.04	-0.01
	Block5	-0.40	-0.21	-0.21	-0.42	-0.20	-0.21	-0.52	-0.17	-0.18
	Full							-0.07		
Cor SE	Block1	0.20	0.07	0.06	0.12	0.06	0.05	0.15	0.06	0.05
	Block2	0.50	0.54	0.54	0.46	0.45	0.51	0.34	0.34	0.36
	Block3	0.34	0.11	0.07	0.30	0.11	0.09	0.22	0.08	0.05
	Block4	0.17	0.40	0.36	0.27	0.35	0.30	0.14	0.07	0.05
	Block5	0.40	0.10	0.08	0.32	0.10	0.07	0.27	0.08	0.06
	Full							0.20		

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 16**

*Bias results for the Thurstonian IRT model with three correlated traits.*

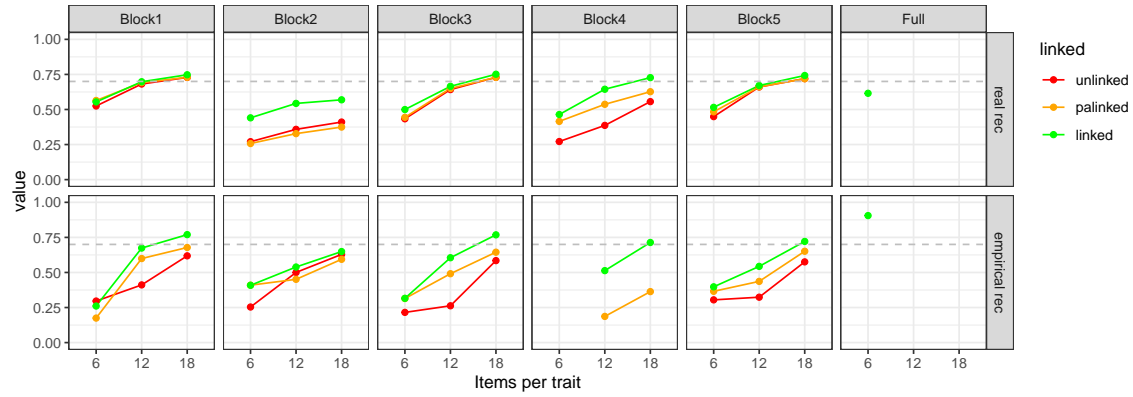
	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	940	995	981	903	995	986	976	1000	1000
	Block2	412	230	237	323	189	195	740	837	745
	Block3	755	992	998	677	895	906	983	1000	1000
	Block4	206	754	995	408	863	787	998	1000	1000
	Block5	535	966	946	445	617	372	953	1000	1000
	Full							999		
Load	Block1	0.00	0.00	0.00	-0.41	-0.01	0.01	0.00	0.00	0.00
	Block2	-0.14	-0.10	-0.06	0.08	0.21	-0.02	0.01	0.01	0.04
	Block3	-0.04	-0.01	0.00	-0.03	-0.10	0.06	0.00	0.00	0.00
	Block4	-0.04	0.01	0.00	-0.09	0.04	0.27	0.00	0.00	0.00
	Block5	-0.06	0.02	0.02	-0.03	0.05	-0.08	0.00	0.00	0.00
	Full							0.00		
Load rec	Block1	0.56	0.55	0.54	0.41	0.03	0.11	0.86	0.95	0.95
	Block2	0.17	0.17	0.12	0.51	0.52	0.33	0.71	0.81	0.77
	Block3	0.50	0.49	0.56	0.18	0.14	0.03	0.85	0.94	0.94
	Block4	0.30	0.60	0.56	0.30	0.09	0.13	0.86	0.93	0.94
	Block5	0.21	0.47	0.45	0.25	0.20	0.05	0.84	0.91	0.92
	Full							0.96		
Load SE	Block1	-0.20	-0.03	-0.01	-0.04	-0.03	-0.01	-0.34	-0.02	-0.02
	Block2	0.85	0.10	-0.17	0.46	-0.31	-0.18	-0.38	-0.38	-0.40
	Block3	0.02	-0.06	-0.02	-0.18	-0.04	-0.02	-0.18	-0.03	0.00
	Block4	2.88	0.25	-0.39	1.17	-0.02	-0.09	-0.15	-0.04	-0.04
	Block5	0.15	-0.05	-0.04	-0.24	-0.04	-0.04	-0.19	-0.04	-0.01
	Full							-0.04		
Th	Block1	-0.02	-0.15	-0.21	-0.01	-0.15	-0.20	0.00	-0.08	-0.02
	Block2	0.04	-0.02	-0.05	0.01	-0.03	-0.04	0.01	-0.03	-0.04
	Block3	0.00	-0.02	-0.05	0.00	-0.06	-0.03	0.00	-0.01	-0.06
	Block4	0.01	0.03	0.01	0.02	0.02	0.01	0.00	0.02	0.01
	Block5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Full							0.00		
Th rec	Block1	0.98	0.95	0.89	0.99	0.95	0.90	1.00	0.98	0.99
	Block2	0.99	1.00	0.99	0.99	1.00	0.99	0.99	0.99	0.99
	Block3	1.00	0.99	0.99	1.00	0.99	0.99	0.99	0.99	0.99
	Block4	1.00	0.99	1.00	1.00	0.99	0.99	1.00	0.99	0.99
	Block5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Full							1.00		
Th SE	Block1	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	Block2	0.00	0.00	0.00	0.00	-0.01	0.01	0.00	0.00	-0.01
	Block3	-0.01	0.00	-0.02	0.00	0.00	-0.01	0.00	0.00	-0.01
	Block4	0.01	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	-0.01
	Block5	0.01	0.00	-0.01	-0.01	0.00	-0.01	0.00	0.00	-0.01
	Full							0.00		
Cor	Block1	-0.21	-0.07	-0.08	-0.09	-0.05	-0.07	-0.25	-0.08	-0.06
	Block2	0.22	0.27	-0.17	0.71	1.06	0.04	-0.40	-0.63	-1.10
	Block3	-0.18	-0.14	-0.08	-0.06	-0.11	-0.08	-0.23	-0.07	-0.06
	Block4	-0.95	-0.05	-0.17	-0.47	-1.24	-1.34	0.00	-0.04	-0.01
	Block5	-0.29	-0.21	-0.21	-0.28	-0.20	-0.19	-0.50	-0.18	-0.18
	Full							-0.07		
Cor SE	Block1	0.20	0.07	0.06	0.11	0.06	0.05	0.19	0.06	0.05
	Block2	0.55	0.58	0.57	0.44	0.34	0.54	0.37	0.34	0.36
	Block3	0.36	0.11	0.06	0.31	0.10	0.06	0.22	0.08	0.05
	Block4	0.36	0.24	0.22	0.30	0.19	0.11	0.14	0.07	0.05
	Block5	0.39	0.10	0.08	0.31	0.10	0.08	0.27	0.08	0.06
	Full							0.20		

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 4.3.2 Results for latent trait recovery

**Figure 13**

*Latent trait recovery results with three correlated traits.*



*Note.* Abbreviations: rec: recovery.

**Table 17**

*Latent trait recovery results with three correlated traits.*

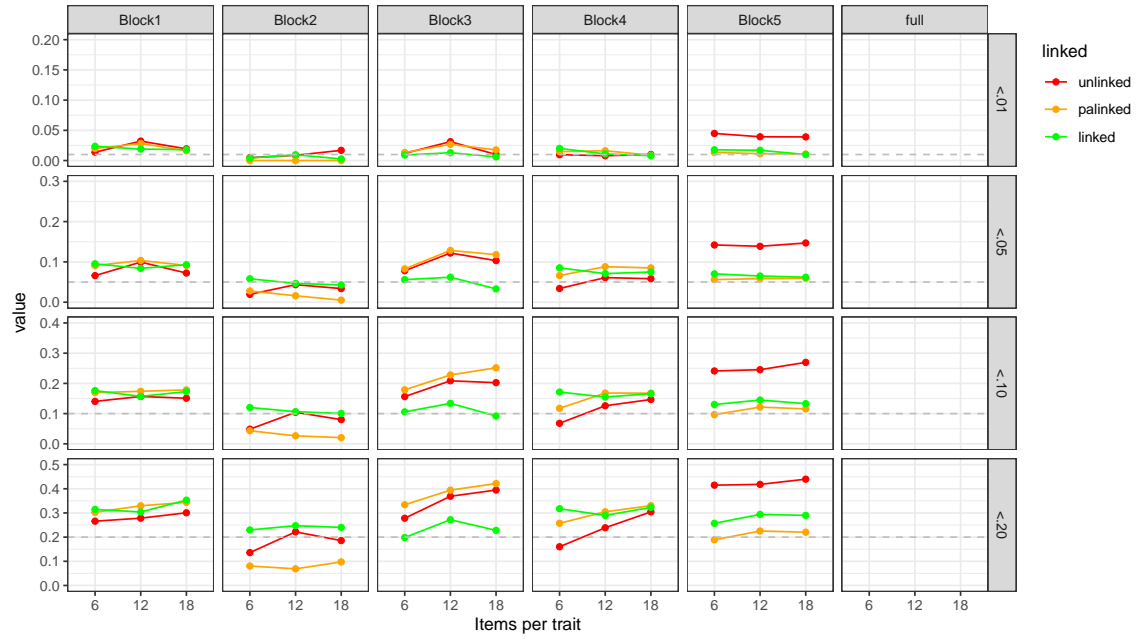
		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
real rec	Block1	0.525	0.682	0.729	0.564	0.692	0.729	0.554	0.698	0.748
	Block2	0.270	0.358	0.410	0.257	0.328	0.375	0.440	0.543	0.569
	Block3	0.434	0.642	0.729	0.444	0.650	0.729	0.500	0.665	0.751
	Block4	0.272	0.387	0.556	0.415	0.537	0.627	0.464	0.644	0.727
	Block5	0.449	0.660	0.720	0.483	0.661	0.719	0.515	0.671	0.742
	Full							0.615		
empirical rec	Block1	0.296	0.411	0.618	0.175	0.599	0.678	0.261	0.674	0.770
	Block2	0.254	0.500	0.629	0.409	0.451	0.594	0.409	0.539	0.649
	Block3	0.215	0.262	0.585	0.314	0.491	0.645	0.316	0.605	0.768
	Block4					0.187	0.363		0.513	0.714
	Block5	0.305	0.324	0.576	0.366	0.436	0.651	0.398	0.543	0.721
	Full							0.905		

*Note.* Abbreviations: rec: recovery.

### 4.3.3 Results for empirical rejection rates

Figure 14

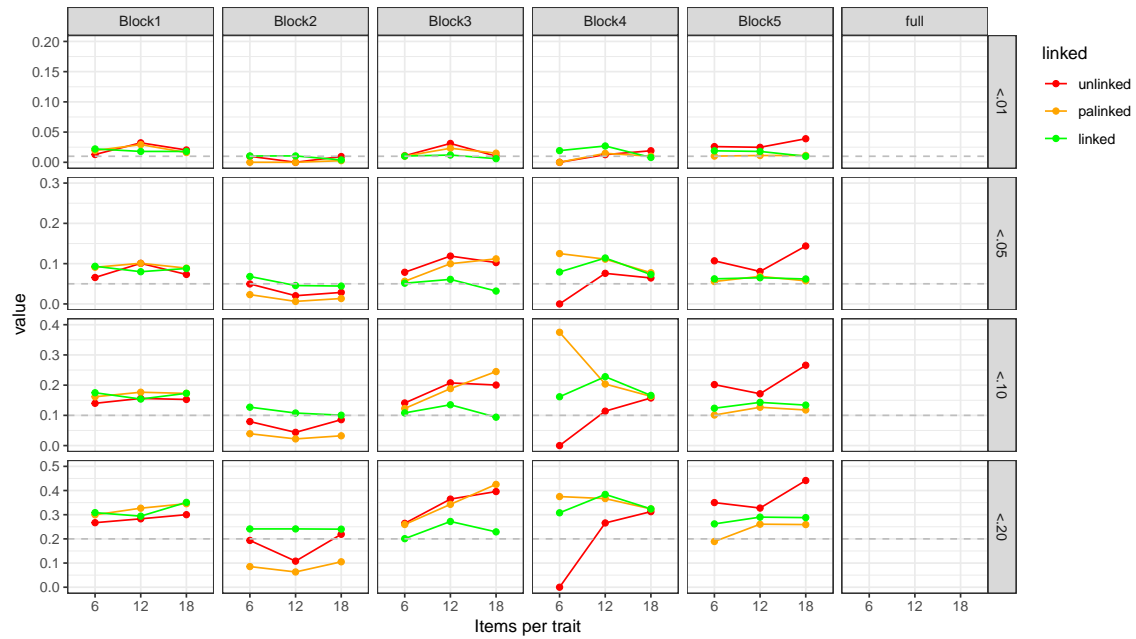
*Empirical rejection rates for the Thurstonian factor model with three correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

Figure 15

*Empirical rejection rates for the Thurstonian IRT model with three correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 18***Empirical rejection rates for the Thurstonian factor model with three correlated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.014	0.032	0.019	0.020	0.028	0.016	0.024	0.019	0.018
	Block2	0.005	0.009	0.017	0.000	0.000	0.000	0.004	0.010	0.003
	Block3	0.012	0.031	0.010	0.013	0.027	0.018	0.009	0.013	0.006
	Block4	0.010	0.008	0.010	0.015	0.016	0.009	0.020	0.011	0.008
	Block5	0.045	0.039	0.039	0.013	0.011	0.011	0.018	0.017	0.010
	full							1.000		
<.05	Block1	0.066	0.099	0.072	0.091	0.104	0.091	0.095	0.084	0.093
	Block2	0.019	0.043	0.034	0.028	0.016	0.005	0.058	0.047	0.043
	Block3	0.078	0.122	0.103	0.083	0.128	0.118	0.056	0.062	0.033
	Block4	0.034	0.061	0.058	0.066	0.088	0.085	0.085	0.071	0.075
	Block5	0.142	0.139	0.147	0.056	0.058	0.059	0.070	0.065	0.062
	full							1.000		
<.10	Block1	0.140	0.157	0.151	0.169	0.174	0.178	0.176	0.157	0.173
	Block2	0.049	0.104	0.080	0.043	0.026	0.021	0.120	0.106	0.101
	Block3	0.156	0.209	0.202	0.179	0.228	0.252	0.106	0.134	0.092
	Block4	0.068	0.126	0.147	0.118	0.168	0.168	0.171	0.155	0.166
	Block5	0.241	0.245	0.270	0.097	0.122	0.116	0.130	0.145	0.133
	full							1.000		
<.20	Block1	0.266	0.278	0.301	0.302	0.330	0.344	0.315	0.304	0.353
	Block2	0.136	0.222	0.186	0.080	0.069	0.097	0.230	0.247	0.240
	Block3	0.278	0.369	0.395	0.334	0.394	0.422	0.198	0.272	0.228
	Block4	0.160	0.239	0.305	0.257	0.305	0.330	0.318	0.290	0.323
	Block5	0.415	0.418	0.440	0.189	0.225	0.220	0.257	0.294	0.290
	full							1.000		

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 19**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	118	569	1344	242	885	1866	313	1359	3137
	Block2	110	546	1310	186	812	1831	307	1354	3130
	Block3	116	569	1344	238	930	1866	310	1361	3135
	Block4	119	575	1360	243	848	1878	313	1360	3137
	Block5	116	566	1341	214	795	2324	311	1306	3137
	full	11603								
$\chi^2$ SD	Block1	14	33	45	20	40	52	23	47	64
	Block2	12	28	41	16	35	46	23	43	57
	Block3	13	30	42	18	38	49	22	45	58
	Block4	14	30	41	20	36	48	23	44	58
	Block5	15	30	43	19	35	55	22	44	61
	full	124								
df	Block1	120	569	1341	242	884	1862	313	1357	3130
	Block2	121	568	1338	198	840	1859	313	1357	3130
	Block3	120	567	1340	242	926	1860	313	1357	3129
	Block4	126	579	1356	246	848	1874	313	1357	3130
	Block5	120	567	1338	219	797	2320	313	1304	3130
	full	11590								
Corrected df	Block1	114	557	1323	233	866	1835	303	1339	3094
	Block2	112	538	1302	192	828	1841	304	1339	3103
	Block3	111	549	1313	228	903	1824	307	1345	3111
	Block4	120	567	1338	237	830	1847	303	1339	3094
	Block5	106	544	1302	213	785	2302	304	1286	3103
	full	10774								



**Table 20***Empirical rejection rates for the Thurstonian IRT model with three correlated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.013	0.032	0.020	0.019	0.029	0.016	0.022	0.018	0.018
	Block2	0.010	0.000	0.010	0.000	0.000	0.003	0.010	0.011	0.004
	Block3	0.011	0.031	0.010	0.010	0.023	0.015	0.010	0.012	0.006
	Block4	0.000	0.013	0.019	0.000	0.015	0.010	0.019	0.027	0.008
	Block5	0.026	0.025	0.039	0.010	0.011	0.011	0.019	0.018	0.010
	full							1.000		
<.05	Block1	0.066	0.101	0.073	0.091	0.101	0.089	0.093	0.080	0.088
	Block2	0.050	0.020	0.029	0.023	0.006	0.013	0.068	0.046	0.044
	Block3	0.079	0.119	0.103	0.056	0.100	0.112	0.051	0.061	0.032
	Block4	0.000	0.076	0.064	0.125	0.111	0.077	0.079	0.114	0.073
	Block5	0.107	0.081	0.144	0.056	0.069	0.057	0.062	0.065	0.062
	full							1.000		
<.10	Block1	0.140	0.156	0.153	0.162	0.177	0.173	0.175	0.154	0.173
	Block2	0.079	0.044	0.086	0.039	0.022	0.032	0.127	0.108	0.101
	Block3	0.141	0.207	0.200	0.123	0.189	0.245	0.108	0.135	0.094
	Block4	0.000	0.114	0.158	0.375	0.204	0.163	0.162	0.228	0.166
	Block5	0.202	0.171	0.266	0.101	0.127	0.118	0.124	0.143	0.134
	full							1.000		
<.20	Block1	0.267	0.283	0.300	0.300	0.327	0.345	0.309	0.294	0.351
	Block2	0.194	0.108	0.219	0.085	0.063	0.105	0.241	0.241	0.240
	Block3	0.264	0.365	0.396	0.260	0.343	0.425	0.201	0.272	0.229
	Block4	0.000	0.266	0.313	0.375	0.367	0.324	0.308	0.384	0.324
	Block5	0.350	0.327	0.441	0.189	0.261	0.259	0.262	0.290	0.288
	full							1.000		

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 21**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with three correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	125	581	1362	253	904	1892	323	1378	3165
	Block2	116	557	1332	194	829	1856	318	1373	3158
	Block3	122	581	1362	249	950	1891	321	1380	3164
	Block4	124	586	1379	263	872	1906	323	1379	3165
	Block5	122	578	1359	224	813	2358	321	1324	3165
	full	11739								
$\chi^2$ SD	Block1	14	33	45	20	40	52	24	47	65
	Block2	13	28	40	17	35	47	24	43	58
	Block3	14	30	42	20	39	49	22	46	58
	Block4	16	32	44	24	37	49	24	45	58
	Block5	14	30	43	20	35	54	23	44	61
	full	125								
df	Block1	126	581	1359	253	903	1888	323	1376	3158
	Block2	127	580	1356	207	858	1885	323	1376	3158
	Block3	126	579	1358	253	946	1886	323	1376	3158
	Block4	132	591	1374	258	868	1902	323	1376	3158
	Block5	126	579	1356	229	814	2352	323	1322	3158
	full	11726								
Corrected df	Block1	120	569	1341	244	885	1861	313	1359	3122
	Block2	115	564	1320	201	846	1867	314	1358	3131
	Block3	117	561	1331	243	929	1850	317	1364	3140
	Block4	126	579	1356	249	850	1875	314	1346	3122
	Block5	114	563	1320	223	802	2334	314	1304	3131
	full	10910								

## 5 Simulation Study with five uncorrelated traits

### 5.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: five traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (30 vs. 60 vs. 90 total)

### 5.2 Definition of parameters

#### 5.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Trait 1	1	0	0	0	0
Trait 2	0	1	0	0	0
Trait 3	0	0	1	0	0
Trait 4	0	0	0	1	0
Trait 5	0	0	0	0	1

#### 5.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	0.24518	0.66086	0.48827	0.98181	0.41005
Items 6/7/8/9/10	0.53322	0.59655	0.68037	0.88188	0.64907
Items 11/12/13/14/15	0.01248	-0.97261	0.35896	-0.44649	-0.87150
Items 16/17/18/19/20	-0.73687	0.80658	0.76670	-0.52175	0.61697
Items 21/22/23/24/25	0.23831	0.09530	-0.56324	-0.03489	0.33794
Items 26/27/28/29/30	0.90647	0.70024	0.05044	0.26504	0.74103
Items 31/32/33/34/35	0.46643	-0.60082	-0.59229	-0.68363	0.92553
Items 36/37/38/39/40	0.43739	0.03060	0.62571	0.40679	0.80148
Items 41/42/43/44/45	0.08417	-0.06825	0.33246	-0.36060	0.09666
Items 46/47/48/49/50	-0.99867	-0.37256	0.53908	-0.67790	-0.96947
Items 51/52/53/54/55	0.77804	0.98856	0.96448	-0.80124	-0.03354
Items 56/57/58/59/60	0.56907	0.33700	0.74427	-0.19280	-0.99186
Items 61/62/63/64/65	-0.82825	-0.94413	-0.62604	-0.54077	-0.84006
Items 66/67/68/69/70	-0.28553	0.45733	-0.03167	0.48286	0.70799
Items 71/72/73/74/75	-0.94310	-0.87433	-0.35558	-0.35707	-0.47810
Items 76/77/78/79/80	0.30574	-0.68826	-0.57978	-0.09968	0.70322
Items 81/82/83/84/85	-0.90868	-0.26185	-0.38711	0.14885	-0.48436
Items 86/87/88/89/90	0.42411	0.14044	0.10763	0.54818	0.52393

### 5.2.3 Matrix of loadings $\Lambda$

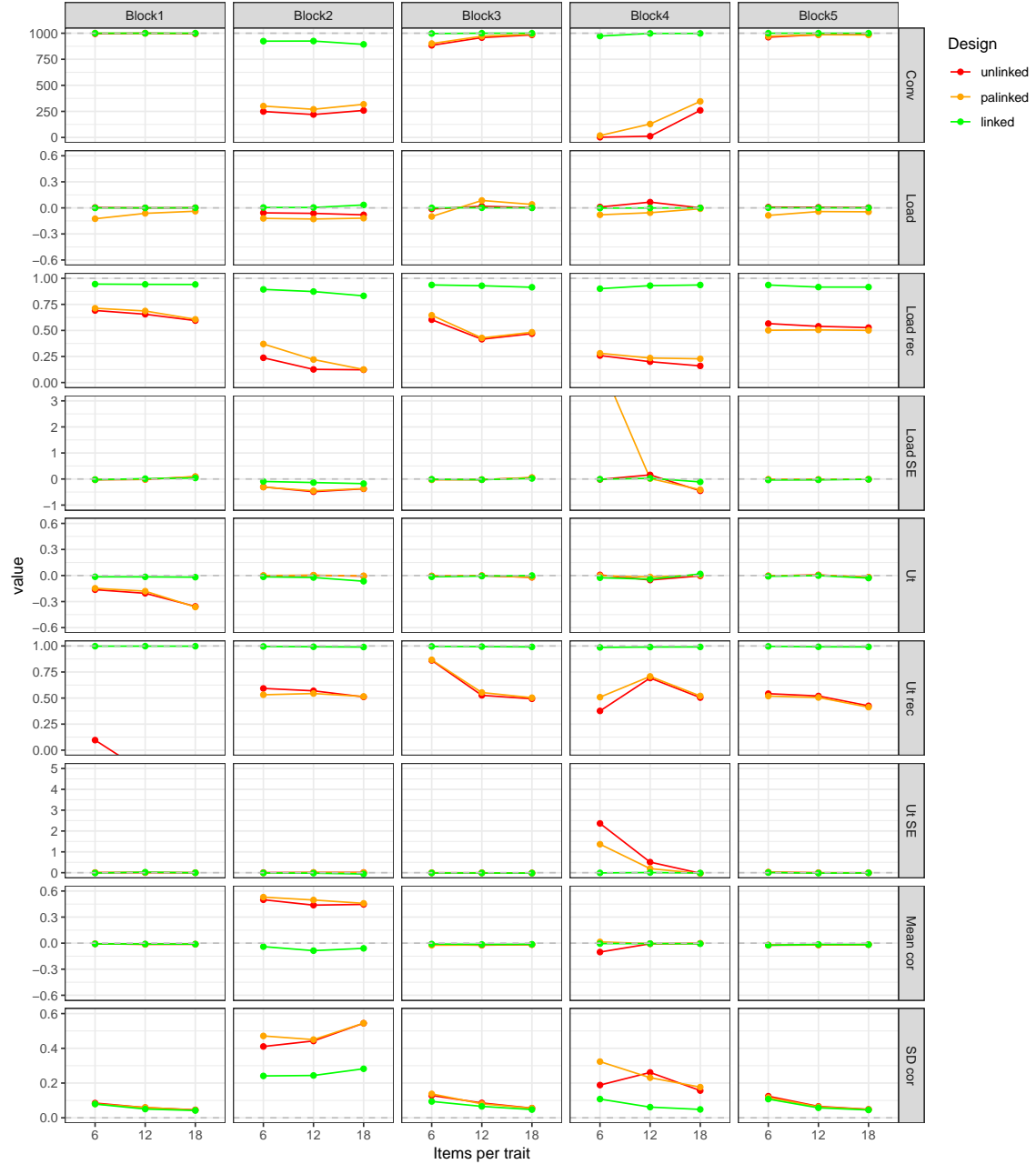
		Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items	1/2/3/4/5	0.46748	0.86478	0.54644	0.72531	0.83987
Items	6/7/8/9/10	0.59732	0.59741	0.35959	0.58580	0.69257
Items	11/12/13/14/15	0.55521	0.34879	0.55931	0.54952	0.51125
Items	16/17/18/19/20	0.88763	0.42499	0.62199	0.37931	0.86241
Items	21/22/23/24/25	0.40280	0.78846	0.66230	0.82522	0.32848
Items	26/27/28/29/30	0.79658	0.55206	0.40157	0.83745	0.83381
Items	31/32/33/34/35	0.74689	0.67933	0.83296	0.52845	0.34842
Items	36/37/38/39/40	0.60969	0.83238	0.58650	0.54992	0.74615
Items	41/42/43/44/45	0.73127	0.50223	0.70054	0.85724	0.63864
Items	46/47/48/49/50	0.71052	0.31451	0.54640	0.34536	0.73091
Items	51/52/53/54/55	0.63810	0.50938	0.76545	0.54699	0.84756
Items	56/57/58/59/60	0.31990	0.52766	0.59275	0.75889	0.53042
Items	61/62/63/64/65	0.38567	0.83131	0.39300	0.88053	0.81445
Items	66/67/68/69/70	0.34524	0.37958	0.42946	0.49213	0.36015
Items	71/72/73/74/75	0.77532	0.36426	0.55718	0.46028	0.81884
Items	76/77/78/79/80	0.86569	0.67936	0.67434	0.42420	0.84114
Items	81/82/83/84/85	0.46344	0.54230	0.78847	0.42550	0.58489
Items	86/87/88/89/90	0.60683	0.83880	0.66717	0.56154	0.49534

## 5.3 Results

### 5.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 16

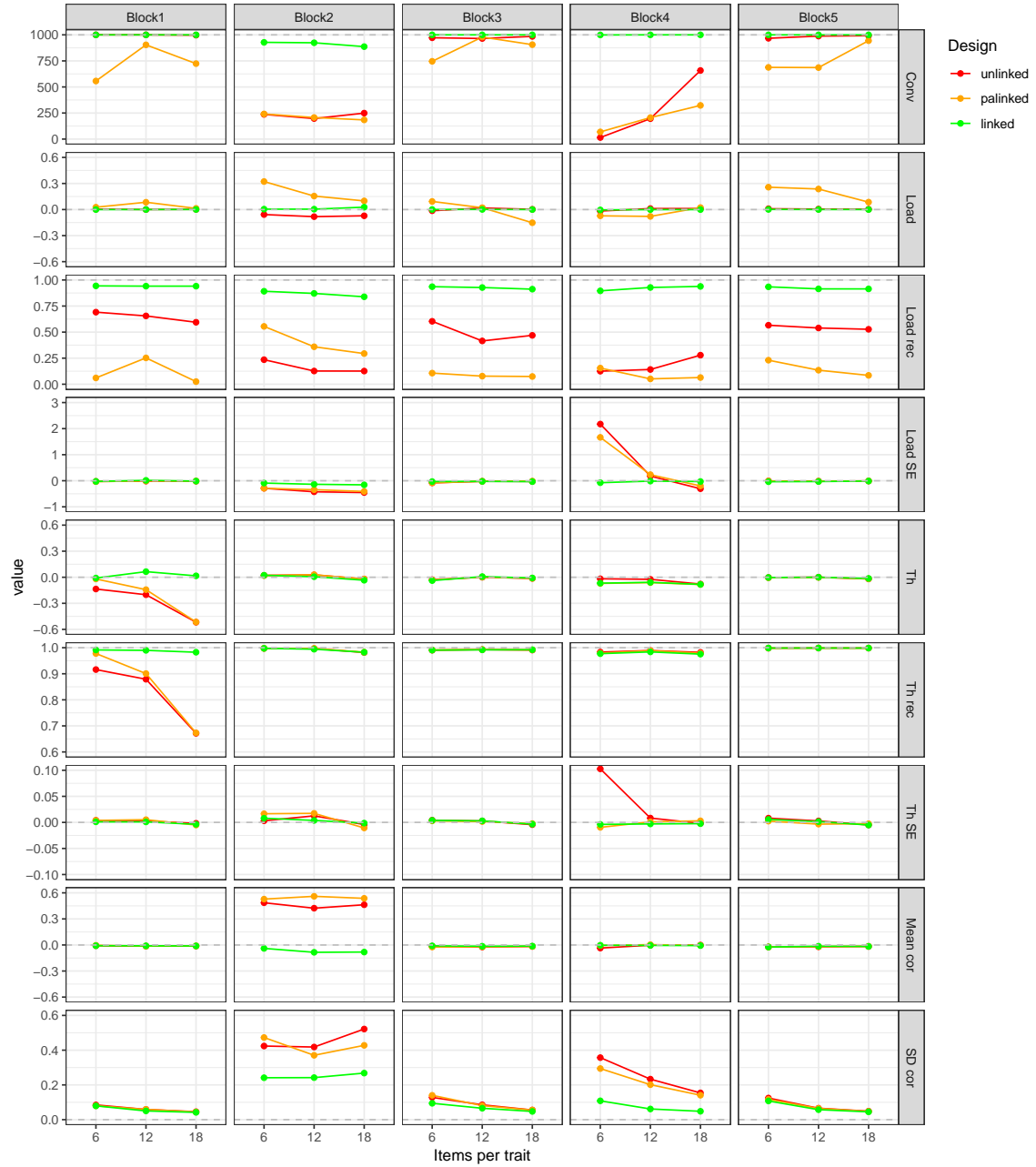
*Bias results for the Thurstonian factor model with five uncorrelated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 17**

*Bias results for the Thurstonian IRT model with five uncorrelated traits*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 22***Bias results for the Thurstonian factor model with five uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	995	1000	995	1000	999	999	1000	1000	1000
	Block2	249	220	259	301	270	318	924	925	893
	Block3	884	958	984	901	971	991	997	1000	1000
	Block4	2	12	260	18	129	346	973	998	998
	Block5	962	986	989	971	986	984	1000	1000	1000
Load	Block1	0.00	0.00	0.00	-0.13	-0.06	-0.04	0.00	0.00	0.00
	Block2	-0.06	-0.06	-0.08	-0.12	-0.13	-0.12	0.00	0.00	0.03
	Block3	-0.01	0.02	0.00	-0.10	0.09	0.04	0.00	0.00	0.00
	Block4	0.01	0.07	0.00	-0.08	-0.06	-0.01	0.00	0.00	0.00
	Block5	0.01	0.01	0.00	-0.09	-0.04	-0.05	0.00	0.00	0.00
Load rec	Block1	0.69	0.66	0.59	0.71	0.69	0.61	0.94	0.94	0.94
	Block2	0.24	0.13	0.12	0.37	0.22	0.13	0.89	0.87	0.83
	Block3	0.60	0.42	0.47	0.65	0.43	0.48	0.94	0.93	0.91
	Block4	0.26	0.20	0.16	0.28	0.24	0.23	0.90	0.93	0.94
	Block5	0.57	0.54	0.53	0.50	0.50	0.50	0.94	0.92	0.92
Load SE	Block1	-0.03	-0.02	0.09	-0.03	-0.01	0.10	-0.02	0.01	0.04
	Block2	-0.30	-0.49	-0.38	-0.31	-0.45	-0.37	-0.09	-0.14	-0.18
	Block3	-0.02	-0.02	0.05	-0.03	-0.02	0.06	-0.01	-0.02	0.02
	Block4	-0.02	0.16	-0.45	4.48	0.02	-0.41	-0.01	0.03	-0.11
	Block5	-0.02	-0.02	-0.01	-0.01	-0.02	-0.01	-0.04	-0.03	-0.01
Ut	Block1	-0.16	-0.20	-0.36	-0.15	-0.18	-0.36	-0.01	-0.02	-0.02
	Block2	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	-0.02	-0.07
	Block3	-0.01	0.00	-0.02	-0.01	0.00	-0.02	-0.01	-0.01	0.00
	Block4	0.01	-0.05	0.00	0.00	-0.02	0.01	-0.03	-0.04	0.02
	Block5	0.00	0.01	-0.02	0.00	0.00	-0.02	-0.01	0.00	-0.03
Ut rec	Block1	0.10	-0.21	-0.14	-0.32	-0.24	-0.19	1.00	1.00	1.00
	Block2	0.59	0.57	0.51	0.53	0.54	0.52	0.99	0.99	0.99
	Block3	0.86	0.53	0.49	0.87	0.55	0.50	1.00	0.99	0.99
	Block4	0.38	0.69	0.50	0.51	0.71	0.52	0.99	0.99	0.99
	Block5	0.54	0.52	0.43	0.52	0.51	0.41	1.00	0.99	0.99
Ut SE	Block1	0.01	0.01	0.00	0.01	0.01	0.00	-0.01	0.02	0.00
	Block2	0.01	0.01	0.01	-0.01	0.01	0.00	-0.01	-0.02	-0.06
	Block3	-0.01	-0.01	-0.02	0.00	-0.01	-0.01	-0.01	-0.01	-0.02
	Block4	2.36	0.51	-0.03	1.37	0.20	-0.04	-0.01	0.01	-0.01
	Block5	0.02	0.00	-0.01	0.03	0.00	0.00	0.01	-0.02	-0.01
Mean cor	Block1	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	Block2	0.50	0.44	0.45	0.53	0.50	0.46	-0.04	-0.09	-0.06
	Block3	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01
	Block4	-0.10	-0.01	0.00	0.01	-0.01	0.00	-0.01	-0.01	-0.01
	Block5	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.02
SD cor	Block1	0.08	0.06	0.05	0.08	0.06	0.04	0.08	0.05	0.04
	Block2	0.41	0.44	0.54	0.47	0.45	0.54	0.24	0.24	0.28
	Block3	0.13	0.09	0.06	0.14	0.08	0.06	0.09	0.07	0.05
	Block4	0.19	0.26	0.16	0.32	0.23	0.18	0.11	0.06	0.05
	Block5	0.12	0.07	0.05	0.12	0.06	0.05	0.11	0.06	0.04

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 23***Bias results for the Thurstonian IRT model with five uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	1000	1000	997	557	904	724	1000	1000	1000
	Block2	237	197	249	241	207	184	928	924	887
	Block3	972	965	985	746	979	906	1000	1000	1000
	Block4	15	196	658	69	206	323	999	1000	1000
	Block5	967	988	992	688	686	944	1000	1000	1000
Load	Block1	0.00	0.00	0.00	0.03	0.08	0.01	0.00	0.00	0.00
	Block2	-0.06	-0.08	-0.07	0.32	0.16	0.10	0.00	0.00	0.03
	Block3	-0.01	0.02	0.00	0.09	0.02	-0.15	0.00	0.00	0.00
	Block4	-0.02	0.01	0.01	-0.07	-0.08	0.02	0.00	0.00	0.00
	Block5	0.01	0.01	0.00	0.26	0.24	0.09	0.00	0.00	0.00
Load rec	Block1	0.69	0.66	0.59	0.06	0.25	0.03	0.94	0.94	0.94
	Block2	0.24	0.13	0.13	0.55	0.36	0.29	0.89	0.87	0.84
	Block3	0.60	0.42	0.47	0.11	0.08	0.07	0.94	0.93	0.91
	Block4	0.13	0.14	0.28	0.15	0.05	0.06	0.90	0.93	0.94
	Block5	0.57	0.54	0.53	0.23	0.13	0.08	0.93	0.91	0.91
Load SE	Block1	-0.03	-0.01	-0.02	-0.03	-0.01	-0.01	-0.02	0.01	-0.01
	Block2	-0.29	-0.42	-0.45	-0.29	-0.34	-0.41	-0.09	-0.14	-0.16
	Block3	-0.08	-0.02	-0.03	-0.08	-0.02	-0.02	-0.03	-0.02	-0.03
	Block4	2.17	0.18	-0.31	1.66	0.23	-0.20	-0.08	-0.01	-0.03
	Block5	-0.01	-0.02	-0.01	-0.01	-0.02	-0.01	-0.04	-0.03	-0.01
Th	Block1	-0.14	-0.20	-0.52	-0.02	-0.14	-0.52	-0.01	0.06	0.02
	Block2	0.02	0.03	-0.02	0.02	0.03	-0.02	0.02	0.01	-0.03
	Block3	-0.03	0.00	-0.01	-0.03	0.00	-0.01	-0.04	0.01	-0.01
	Block4	-0.02	-0.02	-0.08	-0.07	-0.06	-0.08	-0.07	-0.06	-0.08
	Block5	0.00	0.00	-0.02	-0.01	0.00	-0.02	0.00	0.00	-0.01
Th rec	Block1	0.92	0.88	0.67	0.98	0.90	0.67	0.99	0.99	0.98
	Block2	1.00	1.00	0.98	1.00	1.00	0.98	1.00	0.99	0.98
	Block3	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	Block4	0.98	0.99	0.98	0.98	0.99	0.98	0.98	0.98	0.98
	Block5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Th SE	Block1	0.00	0.00	0.00	0.00	0.01	-0.01	0.00	0.00	0.00
	Block2	0.00	0.01	0.00	0.02	0.02	-0.01	0.01	0.00	0.00
	Block3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Block4	0.10	0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00
	Block5	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Mean cor	Block1	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01
	Block2	0.49	0.42	0.46	0.53	0.56	0.54	-0.04	-0.08	-0.08
	Block3	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01
	Block4	-0.04	0.00	0.00	-0.01	0.00	0.00	0.00	-0.01	-0.01
	Block5	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.02
SD cor	Block1	0.08	0.06	0.05	0.08	0.06	0.04	0.08	0.05	0.04
	Block2	0.42	0.42	0.52	0.47	0.37	0.43	0.24	0.24	0.27
	Block3	0.13	0.09	0.06	0.14	0.08	0.06	0.09	0.07	0.05
	Block4	0.36	0.23	0.15	0.29	0.20	0.14	0.11	0.06	0.05
	Block5	0.12	0.07	0.05	0.12	0.06	0.05	0.11	0.06	0.04

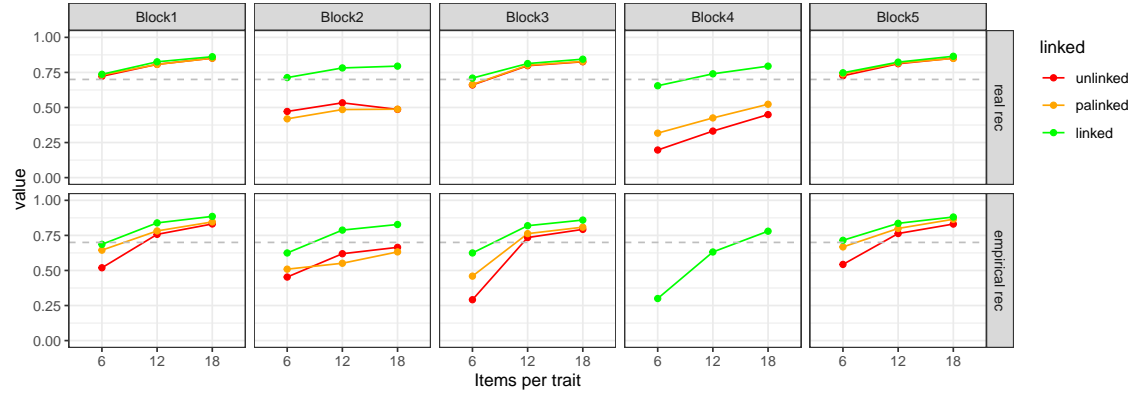
*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.



### 5.3.2 Results for latent trait recovery

Figure 18

*Latent trait recovery results with five uncorrelated traits.*



*Note.* Abbreviations: rec: recovery.

Table 24

*Latent trait recovery results with five uncorrelated traits.*

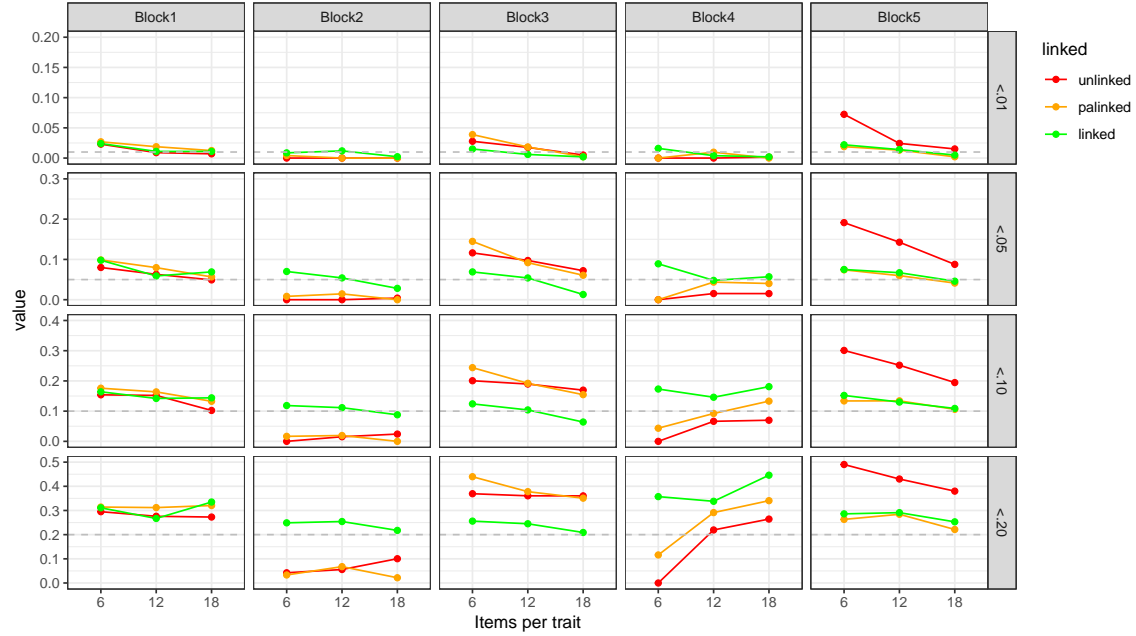
	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
real rec	Block1	0.722	0.807	0.852	0.732	0.807	0.852	0.737	0.826	0.862
	Block2	0.472	0.533	0.486	0.419	0.485	0.488	0.713	0.782	0.795
	Block3	0.660	0.798	0.826	0.664	0.801	0.828	0.710	0.813	0.844
	Block4	0.197	0.332	0.449	0.317	0.426	0.523	0.655	0.740	0.795
	Block5	0.726	0.812	0.851	0.740	0.817	0.850	0.747	0.823	0.865
empirical rec	Block1	0.519	0.757	0.831	0.645	0.782	0.846	0.686	0.839	0.886
	Block2	0.453	0.619	0.665	0.509	0.551	0.632	0.625	0.788	0.828
	Block3	0.291	0.735	0.793	0.460	0.763	0.809	0.625	0.819	0.859
	Block4							0.300	0.632	0.780
	Block5	0.543	0.763	0.831	0.668	0.800	0.865	0.715	0.836	0.881

*Note.* Abbreviations: rec: recovery.

### 5.3.3 Results for empirical rejection rates

**Figure 19**

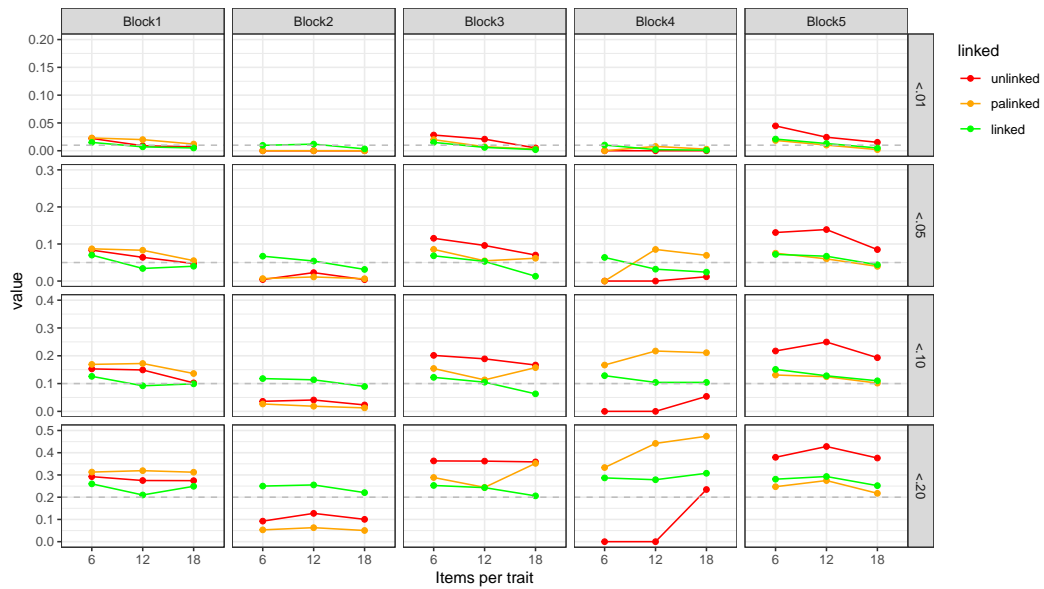
*Empirical rejection rates for the Thurstonian factor model with five uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Figure 20**

*Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 25**

*Empirical rejection rates for the Thurstonian factor model with five uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
<.01	Block1	0.023	0.009	0.007	0.027	0.019	0.012	0.024	0.011	0.011
	Block2	0.000	0.000	0.000	0.004	0.000	0.000	0.009	0.012	0.002
	Block3	0.028	0.018	0.005	0.039	0.018	0.002	0.015	0.006	0.002
	Block4	0.000	0.000	0.002	0.000	0.010	0.000	0.016	0.004	0.002
	Block5	0.072	0.024	0.015	0.019	0.013	0.002	0.022	0.014	0.005
<.05	Block1	0.080	0.063	0.049	0.099	0.080	0.057	0.098	0.059	0.069
	Block2	0.000	0.000	0.004	0.008	0.014	0.000	0.070	0.054	0.028
	Block3	0.116	0.097	0.072	0.145	0.092	0.061	0.069	0.054	0.013
	Block4	0.000	0.015	0.015	0.000	0.044	0.040	0.089	0.048	0.057
	Block5	0.191	0.143	0.088	0.074	0.060	0.041	0.075	0.067	0.046
<.10	Block1	0.154	0.152	0.102	0.176	0.164	0.133	0.164	0.142	0.144
	Block2	0.000	0.015	0.024	0.017	0.019	0.000	0.119	0.111	0.088
	Block3	0.201	0.190	0.170	0.244	0.192	0.155	0.124	0.104	0.064
	Block4	0.000	0.066	0.070	0.043	0.092	0.133	0.173	0.146	0.181
	Block5	0.301	0.252	0.195	0.134	0.134	0.106	0.152	0.130	0.109
<.20	Block1	0.295	0.276	0.273	0.314	0.312	0.320	0.311	0.267	0.335
	Block2	0.042	0.056	0.100	0.033	0.068	0.022	0.249	0.254	0.218
	Block3	0.369	0.361	0.360	0.440	0.378	0.351	0.256	0.245	0.209
	Block4	0.000	0.219	0.264	0.116	0.291	0.341	0.357	0.338	0.446
	Block5	0.490	0.430	0.380	0.263	0.284	0.221	0.286	0.291	0.253

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 26**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	375	1660	3849	606	2234	4694	921	3876	8859
	Block2	349	1622	3793	542	2124	4630	916	3879	8855
	Block3	376	1661	3854	606	2167	4700	922	3880	8869
	Block4	363	1681	3887	634	2254	4734	927	3893	8886
	Block5	374	1662	3850	567	2173	5947	921	3883	8730
$\chi^2$ SD	Block1	26	49	66	33	57	71	39	70	95
	Block2	20	40	55	26	51	60	39	68	88
	Block3	25	49	59	31	54	64	39	72	82
	Block4	15	38	49	23	45	57	34	55	70
	Block5	26	48	67	31	54	81	39	71	91
df	Block1	375	1660	3846	605	2235	4691	921	3876	8856
	Block2	375	1660	3846	569	2167	4691	921	3876	8856
	Block3	376	1660	3846	606	2167	4691	921	3876	8855
	Block4	385	1680	3875	649	2252	4718	921	3876	8855
	Block5	375	1660	3845	569	2167	5943	921	3876	8722
Corrected df	Block1	365	1640	3816	590	2205	4646	903	3842	8783
	Block2	363	1635	3798	559	2147	4661	906	3846	8811
	Block3	361	1630	3801	582	2129	4643	911	3856	8825
	Block4	375	1660	3845	634	2222	4673	903	3842	8782
	Block5	351	1622	3797	559	2147	5913	906	3846	8677

**Table 27***Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.022	0.009	0.007	0.023	0.020	0.012	0.015	0.007	0.005
	Block2	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.012	0.003
	Block3	0.028	0.021	0.005	0.020	0.007	0.003	0.015	0.006	0.002
	Block4	0.000	0.000	0.000	0.000	0.008	0.003	0.010	0.002	0.001
	Block5	0.045	0.024	0.015	0.019	0.010	0.002	0.021	0.013	0.005
<.05	Block1	0.083	0.064	0.047	0.087	0.083	0.055	0.070	0.034	0.040
	Block2	0.004	0.023	0.004	0.007	0.011	0.006	0.067	0.054	0.031
	Block3	0.115	0.096	0.070	0.085	0.055	0.062	0.068	0.053	0.013
	Block4	0.000	0.000	0.012	0.000	0.085	0.069	0.064	0.032	0.024
	Block5	0.131	0.139	0.085	0.075	0.060	0.040	0.072	0.067	0.044
<.10	Block1	0.153	0.149	0.103	0.169	0.172	0.136	0.126	0.092	0.099
	Block2	0.036	0.041	0.023	0.027	0.019	0.013	0.118	0.114	0.090
	Block3	0.201	0.189	0.167	0.154	0.113	0.157	0.122	0.105	0.063
	Block4	0.000	0.000	0.054	0.167	0.217	0.211	0.128	0.104	0.104
	Block5	0.217	0.249	0.193	0.131	0.125	0.102	0.151	0.128	0.110
<.20	Block1	0.292	0.275	0.274	0.313	0.319	0.312	0.260	0.210	0.249
	Block2	0.092	0.127	0.100	0.053	0.063	0.050	0.250	0.255	0.221
	Block3	0.363	0.362	0.359	0.289	0.244	0.352	0.253	0.243	0.206
	Block4	0.000	0.000	0.235	0.333	0.442	0.474	0.287	0.279	0.308
	Block5	0.379	0.428	0.376	0.247	0.275	0.217	0.281	0.293	0.252

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 28**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five uncorrelated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	385	1680	3879	621	2264	4732	937	3907	8905
	Block2	360	1646	3821	556	2154	4672	932	3910	8902
	Block3	386	1681	3884	623	2194	4739	938	3911	8917
	Block4	380	1671	3912	675	2303	4791	943	3924	8933
	Block5	384	1682	3880	582	2197	5995	937	3914	8775
$\chi^2$ SD	Block1	26	50	66	33	57	71	40	70	95
	Block2	22	43	54	28	47	63	40	68	89
	Block3	26	50	60	32	54	65	39	72	82
	Block4	5	27	48	21	47	55	34	55	70
	Block5	26	48	67	32	54	80	39	71	91
df	Block1	385	1680	3876	621	2263	4729	937	3907	8902
	Block2	385	1680	3876	584	2194	4729	937	3907	8902
	Block3	386	1680	3876	622	2194	4729	937	3907	8902
	Block4	395	1700	3905	668	2282	4757	937	3907	8902
	Block5	385	1680	3875	584	2194	5992	937	3907	8767
Corrected df	Block1	375	1660	3846	606	2233	4684	925	3888	8854
	Block2	368	1642	3828	574	2174	4699	922	3877	8857
	Block3	371	1650	3831	610	2175	4681	927	3887	8872
	Block4	385	1680	3875	653	2252	4712	925	3882	8854
	Block5	368	1642	3827	574	2174	5962	922	3877	8722

## 6 Simulation Study with five correlated traits

### 6.1 Conditions

1. Correlation of traits: correlated traits
2. Number of traits: five traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (30 vs. 60 vs. 90 total)

### 6.2 Definition of parameters

#### 6.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Trait 1	1.00	-0.21	0.02	-0.25	-0.53
Trait 2	-0.21	1.00	0.40	0.04	0.27
Trait 3	0.02	0.40	1.00	-0.02	-0.02
Trait 4	-0.25	0.04	-0.02	1.00	0.24
Trait 5	-0.53	0.27	-0.02	0.24	1.00

#### 6.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	0.60214	0.55420	0.70129	0.83584	-0.82974
Items 6/7/8/9/10	-0.33638	0.26087	-0.31988	-0.87237	0.65395
Items 11/12/13/14/15	0.60920	0.52512	0.56438	-0.52383	0.48212
Items 16/17/18/19/20	-0.40190	-0.01101	-0.04458	-0.46988	0.26912
Items 21/22/23/24/25	0.80706	0.99636	0.57226	0.74355	-0.58206
Items 26/27/28/29/30	-0.00674	-0.45567	-0.22311	0.44484	-0.64701
Items 31/32/33/34/35	0.16485	0.02783	-0.12827	0.19929	-0.02725
Items 36/37/38/39/40	-0.94064	-0.79683	0.38148	-0.20727	-0.23520
Items 41/42/43/44/45	0.67593	-0.38561	-0.14346	0.52521	-0.79865
Items 46/47/48/49/50	-0.89717	0.23819	-0.60556	0.75287	-0.92923
Items 51/52/53/54/55	-0.13654	0.36866	-0.44479	0.09615	0.23225
Items 56/57/58/59/60	-0.81727	0.92177	0.95524	0.69880	-0.50336
Items 61/62/63/64/65	0.84490	0.76463	-0.62866	-0.65849	0.77938
Items 66/67/68/69/70	0.59967	0.28728	0.65635	-0.47333	-0.49743
Items 71/72/73/74/75	-0.83619	-0.57984	0.40354	-0.30743	-0.91097
Items 76/77/78/79/80	0.40734	0.77604	-0.84526	-0.36176	0.10683
Items 81/82/83/84/85	-0.46310	-0.16666	0.24651	-0.13813	-0.49169
Items 86/87/88/89/90	-0.50218	-0.37893	0.82843	0.77273	0.73938

### 6.2.3 Matrix of loadings $\Lambda$

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	0.66560	0.30552	0.45042	0.54022	0.30174
Items 6/7/8/9/10	0.39008	0.78315	0.58585	0.80319	0.45948
Items 11/12/13/14/15	0.67285	0.57439	0.75821	0.73864	0.51631
Items 16/17/18/19/20	0.31710	0.74134	0.53943	0.69617	0.71690
Items 21/22/23/24/25	0.73394	0.59674	0.65789	0.75229	0.81445
Items 26/27/28/29/30	0.46456	0.84991	0.38511	0.56578	0.30548
Items 31/32/33/34/35	0.66549	0.82177	0.79742	0.64335	0.70405
Items 36/37/38/39/40	0.38461	0.74185	0.31318	0.54122	0.67769
Items 41/42/43/44/45	0.86301	0.52383	0.81144	0.82512	0.87679
Items 46/47/48/49/50	0.76865	0.78362	0.83513	0.50213	0.82578
Items 51/52/53/54/55	0.37268	0.43420	0.44544	0.66294	0.64984
Items 56/57/58/59/60	0.69808	0.39913	0.89827	0.88314	0.63887
Items 61/62/63/64/65	0.64359	0.42596	0.51341	0.65399	0.77683
Items 66/67/68/69/70	0.85219	0.30972	0.38484	0.61440	0.80709
Items 71/72/73/74/75	0.63604	0.59204	0.55033	0.55909	0.43551
Items 76/77/78/79/80	0.88231	0.32762	0.48370	0.64456	0.44946
Items 81/82/83/84/85	0.43149	0.47511	0.62588	0.89012	0.76663
Items 86/87/88/89/90	0.57287	0.67858	0.41352	0.54581	0.53112

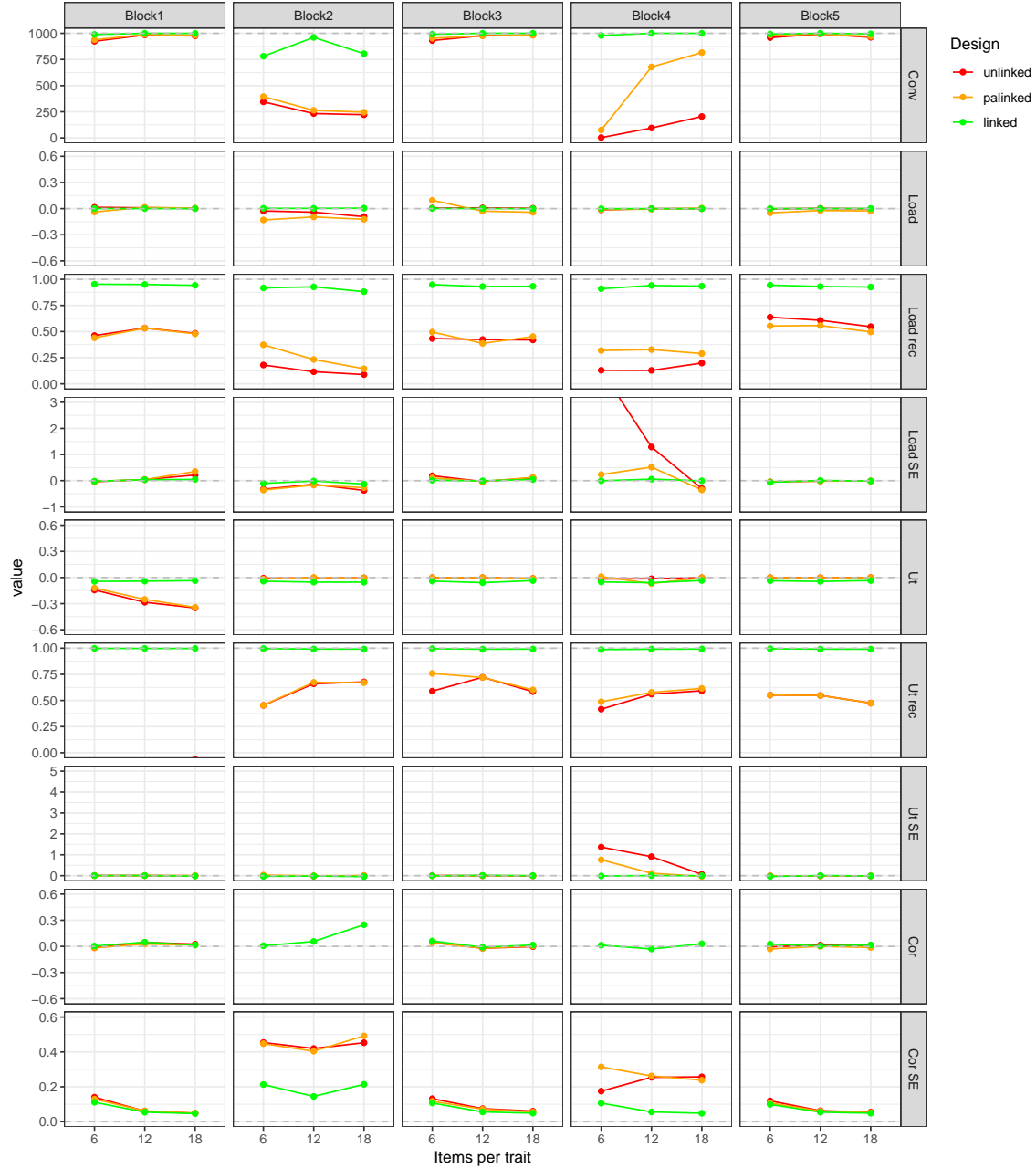


## 6.3 Results

### 6.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 21

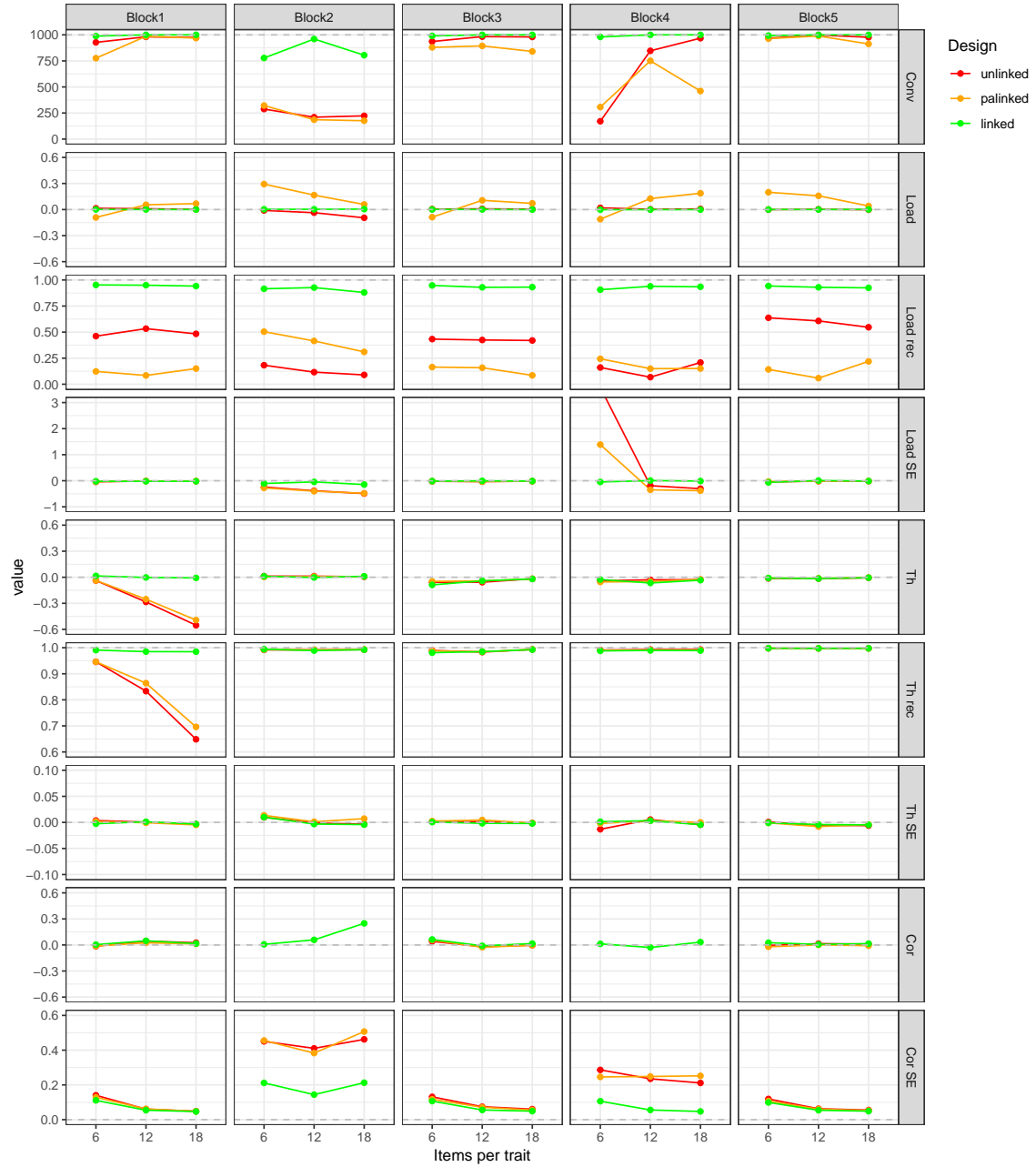
*Bias results for the Thurstonian factor model with five correlated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 22**

*Bias results for the Thurstonian IRT model with five correlated traits.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 29**

*Bias results for the Thurstonian factor model with five correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	924	984	975	938	989	981	989	1000	999
	Block2	345	233	222	395	264	246	781	961	805
	Block3	931	978	983	953	976	982	991	1000	1000
	Block4	3	95	205	75	678	816	979	1000	1000
	Block5	959	993	962	984	993	970	993	1000	995
Load	Block1	0.01	0.01	0.00	−0.04	0.01	0.01	0.00	0.00	0.00
	Block2	−0.03	−0.04	−0.09	−0.13	−0.10	−0.12	0.00	0.00	0.01
	Block3	0.01	0.01	0.00	0.10	−0.03	−0.04	0.00	0.00	0.00
	Block4	−0.01	0.00	0.00	−0.01	−0.01	0.01	0.00	0.00	0.00
	Block5	0.00	0.00	0.00	−0.05	−0.02	−0.03	0.00	0.00	0.00
Load rec	Block1	0.46	0.53	0.48	0.44	0.53	0.48	0.95	0.95	0.94
	Block2	0.18	0.12	0.09	0.37	0.23	0.14	0.92	0.93	0.88
	Block3	0.43	0.42	0.42	0.49	0.39	0.45	0.95	0.93	0.93
	Block4	0.13	0.13	0.20	0.32	0.33	0.29	0.91	0.94	0.93
	Block5	0.64	0.61	0.55	0.55	0.56	0.50	0.94	0.93	0.93
Load SE	Block1	−0.04	0.04	0.21	−0.03	0.04	0.35	−0.02	0.04	0.05
	Block2	−0.33	−0.14	−0.37	−0.36	−0.17	−0.27	−0.11	−0.01	−0.13
	Block3	0.19	−0.03	0.10	0.10	−0.03	0.12	0.01	−0.01	0.05
	Block4	4.23	1.29	−0.30	0.23	0.52	−0.35	0.00	0.06	−0.01
	Block5	−0.05	−0.02	−0.01	−0.04	−0.02	−0.01	−0.06	0.01	−0.01
Ut	Block1	−0.14	−0.28	−0.35	−0.12	−0.25	−0.34	−0.04	−0.04	−0.04
	Block2	−0.01	0.00	0.00	−0.02	0.00	−0.01	−0.04	−0.05	−0.05
	Block3	0.00	0.00	−0.01	0.00	0.00	−0.01	−0.04	−0.06	−0.04
	Block4	−0.02	−0.01	0.00	0.01	−0.07	0.00	−0.05	−0.06	−0.03
	Block5	0.00	0.00	0.00	0.00	0.00	0.00	−0.04	−0.04	−0.03
Ut rec	Block1	−0.17	−0.11	−0.06	−0.33	−0.21	−0.16	1.00	1.00	1.00
	Block2	0.45	0.66	0.68	0.45	0.67	0.67	1.00	0.99	0.99
	Block3	0.59	0.72	0.58	0.76	0.72	0.60	1.00	0.99	0.99
	Block4	0.42	0.56	0.59	0.49	0.58	0.62	0.99	0.99	0.99
	Block5	0.55	0.55	0.47	0.55	0.55	0.47	1.00	0.99	0.99
Ut SE	Block1	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.01	−0.02
	Block2	0.00	−0.02	0.00	0.03	−0.01	0.00	−0.03	−0.02	−0.04
	Block3	0.01	0.00	−0.01	−0.01	0.00	−0.01	−0.01	0.01	0.00
	Block4	1.37	0.91	0.06	0.76	0.11	−0.03	−0.01	0.00	0.00
	Block5	−0.01	−0.01	−0.01	0.00	−0.02	−0.01	−0.04	0.01	−0.01
Cor	Block1	−0.01	0.05	0.03	−0.02	0.03	0.02	0.00	0.05	0.02
	Block2	−1.89	−1.70	−0.93	−1.82	−2.15	−1.95	0.01	0.06	0.25
	Block3	0.05	−0.02	0.00	0.04	−0.02	0.00	0.06	−0.01	0.02
	Block4	−2.13	−0.79	−1.03	−1.09	−0.95	−0.94	0.01	−0.03	0.03
	Block5	0.00	0.02	0.01	−0.03	0.00	−0.01	0.03	0.01	0.02
Cor SE	Block1	0.14	0.06	0.05	0.13	0.06	0.05	0.11	0.05	0.05
	Block2	0.45	0.42	0.45	0.45	0.40	0.49	0.21	0.14	0.21
	Block3	0.13	0.07	0.06	0.12	0.07	0.06	0.11	0.06	0.05
	Block4	0.17	0.25	0.26	0.31	0.26	0.24	0.11	0.06	0.05
	Block5	0.12	0.06	0.06	0.11	0.06	0.05	0.10	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 30***Bias results for the Thurstonian IRT model with five correlated traits.*

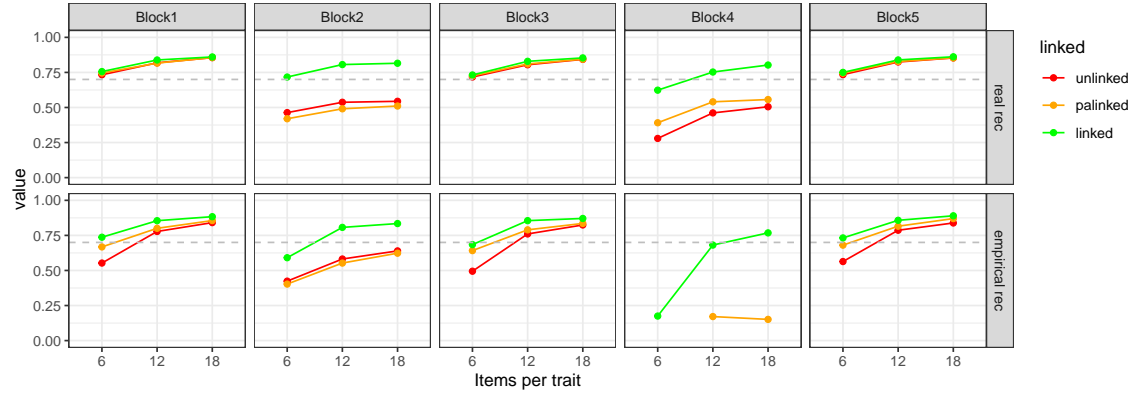
	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
Conv	Block1	928	981	975	776	986	969	988	1000	1000
	Block2	288	210	223	322	186	176	778	960	805
	Block3	936	983	980	880	894	841	990	1000	1000
	Block4	171	847	968	307	751	461	980	1000	1000
	Block5	968	998	977	964	989	913	993	1000	1000
Load	Block1	0.01	0.01	0.00	-0.09	0.05	0.07	0.00	0.00	0.00
	Block2	-0.01	-0.04	-0.10	0.29	0.17	0.06	0.00	0.00	0.01
	Block3	0.00	0.01	0.00	-0.09	0.11	0.07	0.00	0.00	0.00
	Block4	0.02	0.00	0.01	-0.11	0.13	0.19	0.00	0.00	0.00
	Block5	0.00	0.00	0.00	0.20	0.16	0.04	0.00	0.00	0.00
Load rec	Block1	0.46	0.53	0.48	0.12	0.08	0.15	0.95	0.95	0.94
	Block2	0.18	0.12	0.09	0.50	0.42	0.31	0.92	0.93	0.88
	Block3	0.43	0.42	0.42	0.16	0.16	0.09	0.95	0.93	0.93
	Block4	0.16	0.07	0.21	0.24	0.15	0.15	0.91	0.94	0.93
	Block5	0.64	0.61	0.55	0.14	0.06	0.22	0.94	0.93	0.92
Load SE	Block1	-0.04	-0.01	-0.02	-0.03	-0.01	-0.01	-0.02	-0.02	-0.02
	Block2	-0.24	-0.38	-0.49	-0.28	-0.40	-0.48	-0.11	-0.04	-0.15
	Block3	-0.02	-0.03	-0.02	-0.01	-0.03	-0.01	-0.01	0.00	-0.01
	Block4	3.58	-0.19	-0.30	1.39	-0.35	-0.38	-0.05	0.01	-0.01
	Block5	-0.05	-0.02	-0.02	-0.03	-0.02	-0.01	-0.06	0.01	-0.01
Th	Block1	-0.04	-0.28	-0.55	-0.04	-0.25	-0.49	0.02	0.00	-0.01
	Block2	0.01	0.01	0.01	0.00	0.01	0.01	0.01	0.00	0.01
	Block3	-0.06	-0.06	-0.02	-0.05	-0.04	-0.02	-0.09	-0.04	-0.02
	Block4	-0.04	-0.03	-0.03	-0.05	-0.05	-0.02	-0.03	-0.07	-0.03
	Block5	-0.01	-0.02	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01
Th rec	Block1	0.95	0.83	0.65	0.95	0.86	0.70	0.99	0.98	0.98
	Block2	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	Block3	0.99	0.98	0.99	0.99	0.98	0.99	0.98	0.98	0.99
	Block4	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
	Block5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Th SE	Block1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Block2	0.01	0.00	0.00	0.01	0.00	0.01	0.01	0.00	0.00
	Block3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Block4	-0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Block5	0.00	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00
Cor	Block1	-0.01	0.04	0.03	-0.01	0.03	0.01	0.00	0.05	0.02
	Block2	-1.42	-1.87	-0.80	-1.82	-2.31	-2.44	0.01	0.06	0.25
	Block3	0.04	-0.02	0.00	0.06	-0.03	0.00	0.06	-0.01	0.02
	Block4	-1.03	-1.12	-1.09	-1.02	-1.09	-1.06	0.01	-0.03	0.03
	Block5	-0.01	0.01	0.01	-0.02	0.00	-0.01	0.03	0.01	0.02
Cor SE	Block1	0.14	0.06	0.05	0.13	0.06	0.05	0.11	0.05	0.05
	Block2	0.45	0.41	0.46	0.46	0.38	0.51	0.21	0.14	0.21
	Block3	0.13	0.07	0.06	0.12	0.07	0.06	0.11	0.06	0.05
	Block4	0.29	0.24	0.21	0.25	0.25	0.25	0.11	0.06	0.05
	Block5	0.12	0.06	0.06	0.11	0.06	0.05	0.10	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 6.3.2 Results for latent trait recovery

**Figure 23**

*Latent trait recovery results with five correlated traits.*



*Note.* Abbreviations: rec: recovery.

**Table 31**

*Latent trait recovery results with five correlated traits.*

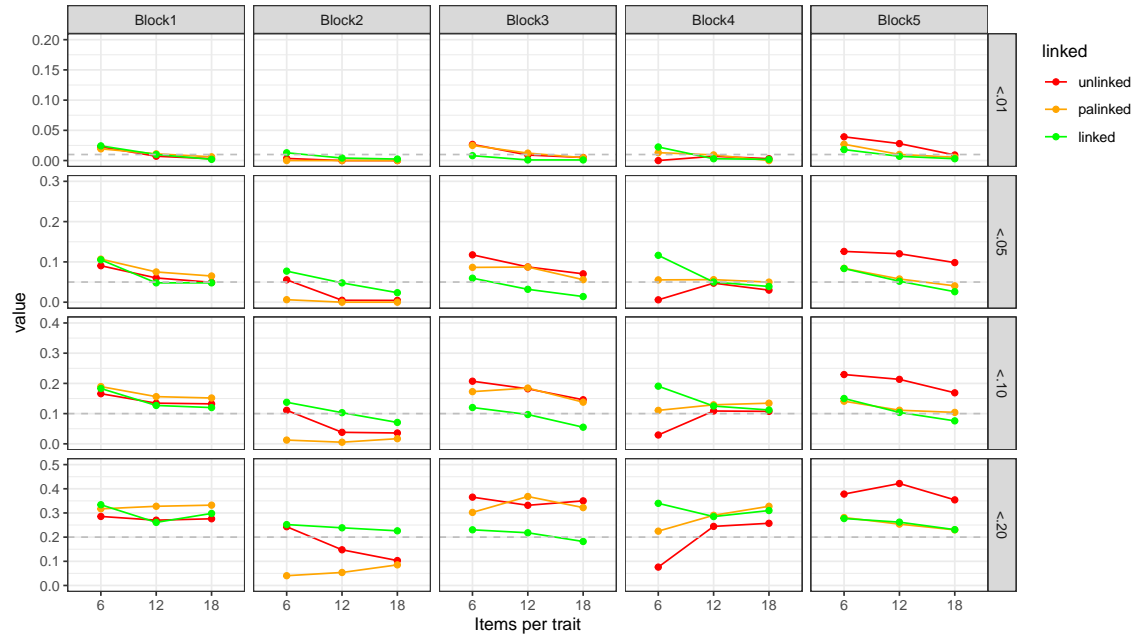
	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
real rec	Block1	0.733	0.818	0.856	0.746	0.816	0.856	0.756	0.839	0.860
	Block2	0.463	0.537	0.544	0.420	0.491	0.510	0.717	0.806	0.815
	Block3	0.717	0.804	0.843	0.726	0.810	0.844	0.732	0.829	0.853
	Block4	0.279	0.461	0.505	0.391	0.540	0.557	0.624	0.753	0.803
	Block5	0.734	0.824	0.853	0.745	0.828	0.853	0.750	0.839	0.861
empirical rec	Block1	0.553	0.778	0.841	0.668	0.801	0.856	0.737	0.855	0.883
	Block2	0.424	0.582	0.639	0.403	0.553	0.623	0.591	0.807	0.834
	Block3	0.495	0.760	0.824	0.642	0.790	0.835	0.684	0.855	0.871
	Block4					0.171	0.151	0.175	0.681	0.768
	Block5	0.564	0.787	0.839	0.681	0.816	0.870	0.732	0.858	0.890

*Note.* Abbreviations: rec: recovery.

### 6.3.3 Results for empirical rejection rates

Figure 24

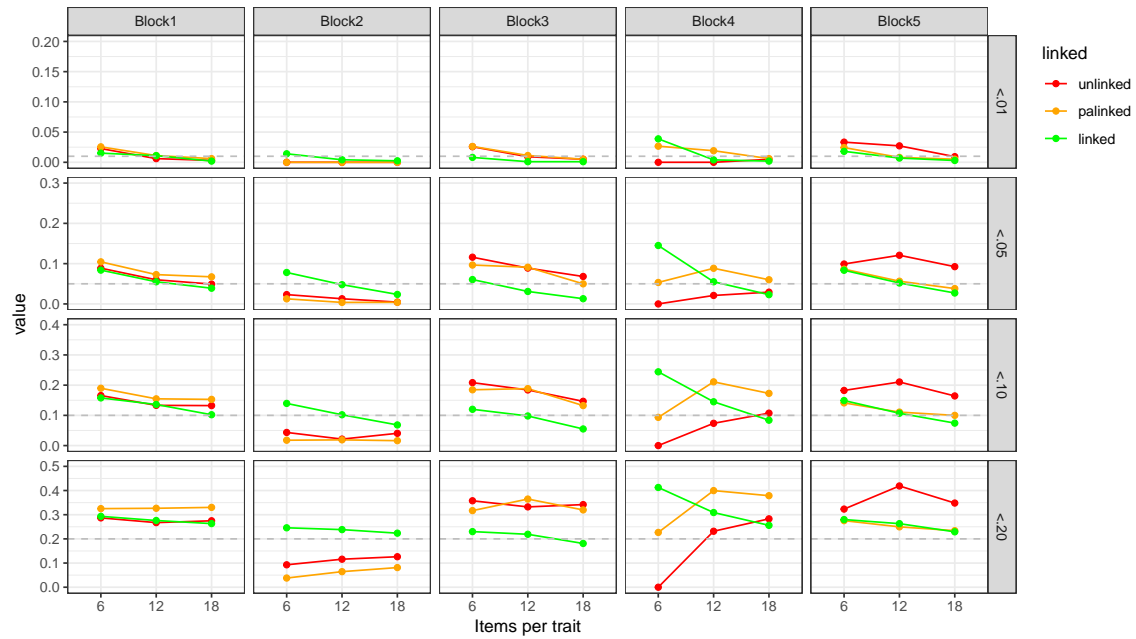
*Empirical rejection rates for the Thurstonian factor model with five correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

Figure 25

*Empirical rejection rates for the Thurstonian IRT model with five correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 32***Empirical rejection rates for the Thurstonian factor model with five correlated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.023	0.007	0.003	0.019	0.011	0.006	0.024	0.010	0.002
	Block2	0.003	0.000	0.000	0.000	0.000	0.000	0.013	0.004	0.002
	Block3	0.027	0.009	0.005	0.025	0.012	0.005	0.008	0.001	0.001
	Block4	0.000	0.007	0.003	0.013	0.009	0.000	0.022	0.003	0.002
	Block5	0.039	0.028	0.009	0.027	0.010	0.005	0.018	0.007	0.003
<.05	Block1	0.091	0.060	0.049	0.107	0.075	0.065	0.105	0.048	0.048
	Block2	0.056	0.005	0.004	0.006	0.000	0.000	0.077	0.048	0.024
	Block3	0.118	0.087	0.070	0.086	0.087	0.056	0.060	0.032	0.014
	Block4	0.006	0.047	0.030	0.055	0.056	0.050	0.116	0.050	0.039
	Block5	0.126	0.120	0.098	0.084	0.058	0.041	0.084	0.052	0.026
<.10	Block1	0.166	0.135	0.132	0.189	0.156	0.152	0.183	0.127	0.120
	Block2	0.111	0.038	0.036	0.012	0.005	0.017	0.138	0.103	0.071
	Block3	0.207	0.182	0.146	0.173	0.185	0.138	0.120	0.097	0.055
	Block4	0.029	0.109	0.107	0.111	0.129	0.134	0.191	0.125	0.112
	Block5	0.229	0.213	0.169	0.141	0.111	0.104	0.150	0.104	0.076
<.20	Block1	0.286	0.270	0.276	0.317	0.328	0.332	0.334	0.261	0.298
	Block2	0.243	0.148	0.103	0.040	0.054	0.085	0.252	0.239	0.226
	Block3	0.365	0.332	0.350	0.302	0.368	0.322	0.230	0.218	0.182
	Block4	0.076	0.244	0.257	0.225	0.290	0.328	0.340	0.285	0.310
	Block5	0.378	0.422	0.354	0.281	0.254	0.230	0.277	0.262	0.231

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 33**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	374	1661	3849	568	2237	4694	921	3874	8859
	Block2	353	1624	3800	544	2192	4647	917	3876	8859
	Block3	376	1661	3850	682	2237	4693	922	3878	8861
	Block4	368	1678	3881	536	2251	4725	922	3881	8869
	Block5	375	1661	3848	605	2168	5620	920	3879	8860
$\chi^2$ SD	Block1	26	47	67	32	54	74	38	68	92
	Block2	23	42	60	26	46	66	39	65	85
	Block3	25	45	62	31	52	69	37	65	83
	Block4	22	45	59	28	51	66	39	62	78
	Block5	27	48	65	33	53	78	40	67	86
df	Block1	375	1661	3846	569	2236	4691	921	3876	8856
	Block2	375	1661	3846	569	2236	4691	921	3876	8856
	Block3	376	1660	3846	681	2235	4691	921	3876	8856
	Block4	385	1680	3875	542	2252	4718	921	3875	8854
	Block5	375	1660	3845	605	2167	5616	921	3876	8856
Corrected df	Block1	365	1641	3816	554	2206	4646	900	3842	8796
	Block2	347	1623	3798	559	2216	4661	906	3846	8811
	Block3	361	1630	3801	665	2197	4643	911	3856	8826
	Block4	375	1660	3845	527	2222	4673	900	3841	8794
	Block5	359	1622	3797	595	2147	5586	906	3846	8811



**Table 34***Empirical rejection rates for the Thurstonian IRT model with five correlated traits.*

		unlinked			palinked			linked		
	blocks	6	12	18	6	12	18	6	12	18
<.01	Block1	0.023	0.006	0.003	0.026	0.011	0.006	0.015	0.011	0.002
	Block2	0.000	0.000	0.000	0.000	0.000	0.000	0.014	0.004	0.002
	Block3	0.026	0.009	0.005	0.026	0.011	0.005	0.008	0.001	0.001
	Block4	0.000	0.000	0.005	0.027	0.019	0.006	0.039	0.004	0.002
	Block5	0.033	0.027	0.009	0.024	0.008	0.005	0.018	0.007	0.003
<.05	Block1	0.089	0.060	0.049	0.104	0.073	0.067	0.084	0.055	0.039
	Block2	0.023	0.013	0.005	0.013	0.004	0.004	0.078	0.048	0.024
	Block3	0.116	0.089	0.068	0.097	0.091	0.050	0.061	0.031	0.013
	Block4	0.000	0.021	0.029	0.053	0.088	0.060	0.145	0.055	0.023
	Block5	0.099	0.121	0.093	0.086	0.056	0.038	0.084	0.052	0.027
<.10	Block1	0.166	0.133	0.132	0.190	0.155	0.153	0.158	0.136	0.102
	Block2	0.043	0.021	0.041	0.018	0.019	0.016	0.140	0.102	0.068
	Block3	0.208	0.184	0.146	0.185	0.189	0.132	0.120	0.098	0.055
	Block4	0.000	0.074	0.107	0.093	0.211	0.173	0.244	0.145	0.084
	Block5	0.182	0.210	0.164	0.141	0.111	0.100	0.149	0.107	0.074
<.20	Block1	0.287	0.267	0.275	0.325	0.327	0.330	0.293	0.276	0.263
	Block2	0.093	0.116	0.126	0.038	0.064	0.081	0.246	0.238	0.224
	Block3	0.358	0.332	0.342	0.317	0.365	0.320	0.230	0.219	0.181
	Block4	0.000	0.232	0.283	0.227	0.400	0.379	0.413	0.309	0.256
	Block5	0.323	0.419	0.348	0.275	0.250	0.234	0.280	0.263	0.229

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 35**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five correlated traits.*

	blocks	unlinked			palinked			linked		
		6	12	18	6	12	18	6	12	18
$\chi^2$	Block1	384	1681	3879	584	2265	4732	937	3905	8905
	Block2	363	1640	3830	558	2222	4687	934	3907	8904
	Block3	386	1681	3880	700	2266	4731	938	3909	8907
	Block4	353	1699	3912	552	2297	4775	938	3913	8917
	Block5	385	1681	3877	622	2195	5666	936	3910	8906
$\chi^2$ SD	Block1	27	47	67	32	54	74	38	68	92
	Block2	25	42	62	28	47	67	39	65	85
	Block3	25	46	62	32	52	68	37	66	83
	Block4	32	41	60	29	53	64	39	62	78
	Block5	27	48	65	34	54	78	41	67	86
df	Block1	385	1681	3876	584	2264	4729	937	3907	8902
	Block2	385	1681	3876	584	2264	4729	937	3907	8902
	Block3	386	1680	3876	699	2263	4729	937	3907	8902
	Block4	395	1700	3905	557	2282	4757	937	3907	8902
	Block5	385	1680	3875	621	2194	5662	937	3907	8902
Corrected df	Block1	375	1661	3846	569	2234	4684	920	3869	8854
	Block2	373	1643	3828	574	2244	4699	922	3877	8857
	Block3	371	1650	3831	682	2225	4681	927	3887	8872
	Block4	385	1680	3875	542	2252	4712	909	3869	8854
	Block5	373	1642	3827	611	2174	5632	922	3877	8857

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## 9 References

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