

# Supplement material for the study “Forced-choice with Thurstonian models: Assessment design and the role of item keying”

Markus Thomas Jansen & Ralf Schulze

2023-03-01

## Contents

<b>1</b>	<b>Overview</b>	<b>3</b>
1.1	Simulation . . . . .	3
1.2	Results . . . . .	4
<b>2</b>	<b>Simulation Study with one trait</b>	<b>7</b>
2.1	Conditions . . . . .	7
2.2	Definition of parameters . . . . .	7
2.2.1	Correlation matrix $\Phi$ of traits . . . . .	7
2.2.2	Matrix of utilities $U$ . . . . .	7
2.2.3	Matrix of loadings $\Lambda$ (one negatively keyed item) . . . . .	8
2.2.4	Matrix of loadings $\Lambda$ (two negatively keyed items) . . . . .	8
2.3	Results . . . . .	9
2.3.1	Results for convergence, mean relative bias and recovery of item parameters . . . . .	9
2.3.2	Results for latent trait recovery . . . . .	17
2.3.3	Results for empirical rejection rates . . . . .	19
<b>3</b>	<b>Simulation Study with three uncorrelated traits</b>	<b>25</b>
3.1	Conditions . . . . .	25
3.2	Definition of parameters . . . . .	25
3.2.1	Correlation matrix $\Phi$ of traits . . . . .	25
3.2.2	Matrix of utilities $U$ . . . . .	25
3.2.3	Matrix of loadings $\Lambda$ (one negatively keyed item) . . . . .	26
3.2.4	Matrix of loadings $\Lambda$ (two negatively keyed items) . . . . .	26
3.3	Results . . . . .	27
3.3.1	Results for convergence, mean relative bias and recovery of item parameters . . . . .	27
3.3.2	Results for latent trait recovery . . . . .	35

3.3.3	Results for empirical rejection rates . . . . .	37
<b>4</b>	<b>Simulation Study with three correlated traits</b>	<b>43</b>
4.1	Conditions . . . . .	43
4.2	Definition of parameters . . . . .	43
4.2.1	Correlation matrix $\Phi$ of traits . . . . .	43
4.2.2	Matrix of utilities $U$ . . . . .	43
4.2.3	Matrix of loadings $\Lambda$ (one negatively keyed item) . . . . .	44
4.2.4	Matrix of loadings $\Lambda$ (two negatively keyed items) . . . . .	44
4.3	Results . . . . .	45
4.3.1	Results for convergence, mean relative bias and recovery of item parameters . . . . .	45
4.3.2	Results for latent trait recovery . . . . .	53
4.3.3	Results for empirical rejection rates . . . . .	55
<b>5</b>	<b>Simulation Study with five uncorrelated traits</b>	<b>61</b>
5.1	Conditions . . . . .	61
5.2	Definition of parameters . . . . .	61
5.2.1	Correlation matrix $\Phi$ of traits . . . . .	61
5.2.2	Matrix of utilities $U$ . . . . .	61
5.2.3	Matrix of loadings $\Lambda$ (one negatively keyed item) . . . . .	62
5.2.4	Matrix of loadings $\Lambda$ (two negatively keyed items) . . . . .	62
5.3	Results . . . . .	63
5.3.1	Results for convergence, mean relative bias and recovery of item parameters . . . . .	63
5.3.2	Results for latent trait recovery . . . . .	71
5.3.3	Results for empirical rejection rates . . . . .	73
<b>6</b>	<b>Simulation Study with five correlated traits</b>	<b>79</b>
6.1	Conditions . . . . .	79
6.2	Definition of parameters . . . . .	79
6.2.1	Correlation matrix $\Phi$ of traits . . . . .	79
6.2.2	Matrix of utilities $U$ . . . . .	79
6.2.3	Matrix of loadings $\Lambda$ (one negatively keyed item) . . . . .	80
6.2.4	Matrix of loadings $\Lambda$ (two negatively keyed items) . . . . .	80
6.3	Results . . . . .	81
6.3.1	Results for convergence, mean relative bias and recovery of item parameters . . . . .	81
6.3.2	Results for latent trait recovery . . . . .	89
6.3.3	Results for empirical rejection rates . . . . .	91
<b>7</b>	<b>List of Figures</b>	<b>97</b>
<b>8</b>	<b>List of Tables</b>	<b>99</b>
<b>9</b>	<b>References</b>	<b>102</b>

# 1 Overview

## 1.1 Simulation

This file gives an overview and the results of the simulation studies from the article “Forced-choice with Thurstonian models: The role of items key direction and current recommendations for test construction”.

All model estimations were done with Mplus. To invoke and perform the simulations, preliminary versions of ThurMod (Jansen, 2023) was used. Versions of software are:

1. R Version 4.0.3
2. RStudio Version 2022.07.2 Build 576
3. Mplus Version 8.4
4. MIRTJ (ThurMod) version 0.3.001 for conditions with one negatively keyed item
5. MIRTJ (ThurMod) version 0.3.004 for conditions with two negatively keyed items
6. ThurMod version 0.4.3.

Be careful: The Mplus analyses alone would take about 53 years on a single core computer. All analyses were done parallel on a 64 core computer.

All syntaxes for simulation have the following structure:

1. Working directory and packages are loaded.
2. A global seed and sub seeds for every data generation process are defined
3. Initialization of simulation conditions
4. Definition of blocks
5. For each of the 1000 repetitions
  - (a) Create and save data file
  - (b) Create syntax and run the analysis for the Thurstonian factor and IRT model for the different blocks (blocks 1/2/3, linked/unlinked, sample size 200/500/1000)

All files to invoke the simulation can be found in the folder 00\_sim\_syntaxes. The definition of a number to each condition (including 01-14 is for one negatively keyed item and 21-34 for two negatively keyed items per trait) is

1. CI\_01, CI\_21 = uncor\_1\_12 = uncorrelated, 1 trait, 12 items total (12 per trait)
2. CI\_02, CI\_22 = uncor\_1\_18 = uncorrelated, 1 trait, 18 items total (18 per trait)
3. CI\_03, CI\_23 = uncor\_3\_18 = uncorrelated, 3 traits, 18 items total (6 per trait)
4. CI\_04, CI\_24 = uncor\_3\_36 = uncorrelated, 3 traits, 36 items total (12 per trait)
5. CI\_05, CI\_25 = uncor\_3\_54 = uncorrelated, 3 traits, 54 items total (18 per trait)

6. CI\_06, CI\_26 = cor\_3\_18 = correlated, 3 traits, 18 items total (6 per trait)
7. CI\_07, CI\_27 = cor\_3\_36 = correlated, 3 traits, 36 items total (12 per trait)
8. CI\_08, CI\_28 = cor\_3\_54 = correlated, 3 traits, 54 items total (18 per trait)
9. CI\_09, CI\_29 = uncor\_5\_30 = uncorrelated, 5 traits, 30 items total (6 per trait)
10. CI\_10, CI\_30 = uncor\_5\_60 = uncorrelated, 5 traits, 60 items total (12 per trait)
11. CI\_11, CI\_31 = uncor\_5\_90 = uncorrelated, 5 traits, 90 items total (18 per trait)
12. CI\_12, CI\_32 = cor\_5\_30 = correlated, 5 traits, 30 items total (6 per trait)
13. CI\_13, CI\_33 = cor\_5\_60 = correlated, 5 traits, 60 items total (12 per trait)
14. CI\_14, CI\_34 = cor\_5\_90 = correlated, 5 traits, 90 items total (18 per trait)

All simulations used triplets as blocks. All simulations used a ranking design (transitive responding).

## 1.2 Results

After analysis of all simulation conditions, results are gathered. All simulated data and Mplus files combined are too large to share. Raw data of the results are shared in excel files which can be found in the folder 01\_raw. Steps for Thurstonian factor model:

1. Test for each repetition if results are plausible. Even for models where estimation converges, some results are not usable. Data is not usable if one of the following conditions apply
  - (a) The SD of the loading estimates is 0.
  - (b) The residual variance of a trait is negative.
  - (c) The SD of the utility estimates is 0.
  - (d) The correlation between traits is larger |1|.
  - (e) The relative bias of loadings is larger than 50
2. The number of converged data is counted (after exclusion of implausible data).
3. Results are gathered for
  - (a) Fit statistics
  - (b) Bias, recovery and bias of standard errors for loadings
  - (c) Bias, recovery and bias of standard errors for utilities
  - (d) Bias, recovery and bias of standard errors for thresholds
  - (e) Bias/Mean of trait correlations
  - (f) Bias of standard errors or SD of trait correlations

- (g) Latent trait recovery
- (h) empirical rejection rates

The relative bias for each estimation is calculated by

$$Bias = \frac{\hat{\theta} - \theta}{\theta}$$

where  $\theta$  is the true parameter and  $\hat{\theta}$  the estimation. Due to to identification constraints, true and estimated parameters need to be defined on the same scale. For loadings and utilities the unstandardized estimations are used. This is mainly the case, as for standardized results, some parameters that are defined to be equal, are not anymore. For thresholds no rescaling need to be done, when standardized results are used. Loading estimates that are reversed keyed (they are directly compared to one of the negative items) are rekeyed.

For unlinked blocks, the true parameters where regressed on the estimations and rescaled for each block. For linked blocks, the estimations where regressed on the true score and rescaled once for the whole data set. This procedure potentially favors the unlinked block design, therefore, also the recovery (correlation between true and estimated parameters) is reported.

Blocks are

1. Blocks 1: Items are ordered by their loadings. Items with similar loadings (e.g., items with the least three loadings) are combined into blocks.
2. Blocks 2: All blocks are purely multidimensional.
3. Blocks 3: Initial blocks are purely uni-dimensional.

All results are presented by a figure and a corresponding numeric table for reference. If any cell in a table is empty, there are no results available (no convergence, other reasons for exclusion, see above). For any figure, the x-axis has the number of traits per factor (6 vs. 12 vs. 18). The different lines represent the sample size (200 vs. 500 vs. 1000). For each result variable, a dashed reference line is included, which references the best value (e.g. 1000 for converged repetitions, 0 for bias), a lower limit (e.g. latent trait score recovery), or the corresponding alpha level for the rejection rates.

The y-axis per result variable was chosen to be equal across conditions. Therefore, it is

1. Convergence rates (Factor and IRT model): 0 to 1000
2. Bias (loadings, utilities, thresholds): -.6 to .6
3. Item parameter recovery (loadings, utilities): 0 to 1
4. Bias for SE (loadings, utilities): 0 to 5
5. Item parameter recovery (thresholds): .9 to 1
6. Bias for SE (thresholds): -.1 to .1
7. Bias or mean of trait correlation: -.6 to .6

8. Bias for SE or SD of trait correlation: 0 to .6
9. Recovery of latent trait scores: 0 to 1
10. empirical rejection rates for  $\alpha = .01$ : 0 to .20
11. empirical rejection rates for  $\alpha = .05$ : 0 to .30
12. empirical rejection rates for  $\alpha = .10$ : 0 to .40
13. empirical rejection rates for  $\alpha = .20$ : 0 to .50

## 2 Simulation Study with one trait

### 2.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: one trait
3. Number of items: 12 vs. 18 items per trait (12 vs. 18 total)

### 2.2 Definition of parameters

#### 2.2.1 Correlation matrix $\Phi$ of traits

	Trait 1
Trait 1	1

#### 2.2.2 Matrix of utilities $U$

	Trait 1
Items 1	-0.99865
Items 2	-0.97542
Items 3	0.55028
Items 4	0.25627
Items 5	0.27547
Items 6	-0.67215
Items 7	-0.87130
Items 8	-0.84986
Items 9	-0.66475
Items 10	0.58136
Items 11	-0.43110
Items 12	-0.71761
Items 13	0.09677
Items 14	-0.88366
Items 15	-0.07744
Items 16	0.57069
Items 17	0.30198
Items 18	0.47432

### 2.2.3 Matrix of loadings $\Lambda$ (one negatively keyed item)

	Trait 1
Items 1	-0.74498
Items 2	0.88550
Items 3	0.59527
Items 4	0.40189
Items 5	0.32282
Items 6	0.54599
Items 7	0.72332
Items 8	0.86733
Items 9	0.63353
Items 10	0.73280
Items 11	0.82750
Items 12	0.58275
Items 13	0.73265
Items 14	0.85592
Items 15	0.32795
Items 16	0.37976
Items 17	0.81138
Items 18	0.77367

### 2.2.4 Matrix of loadings $\Lambda$ (two negatively keyed items)

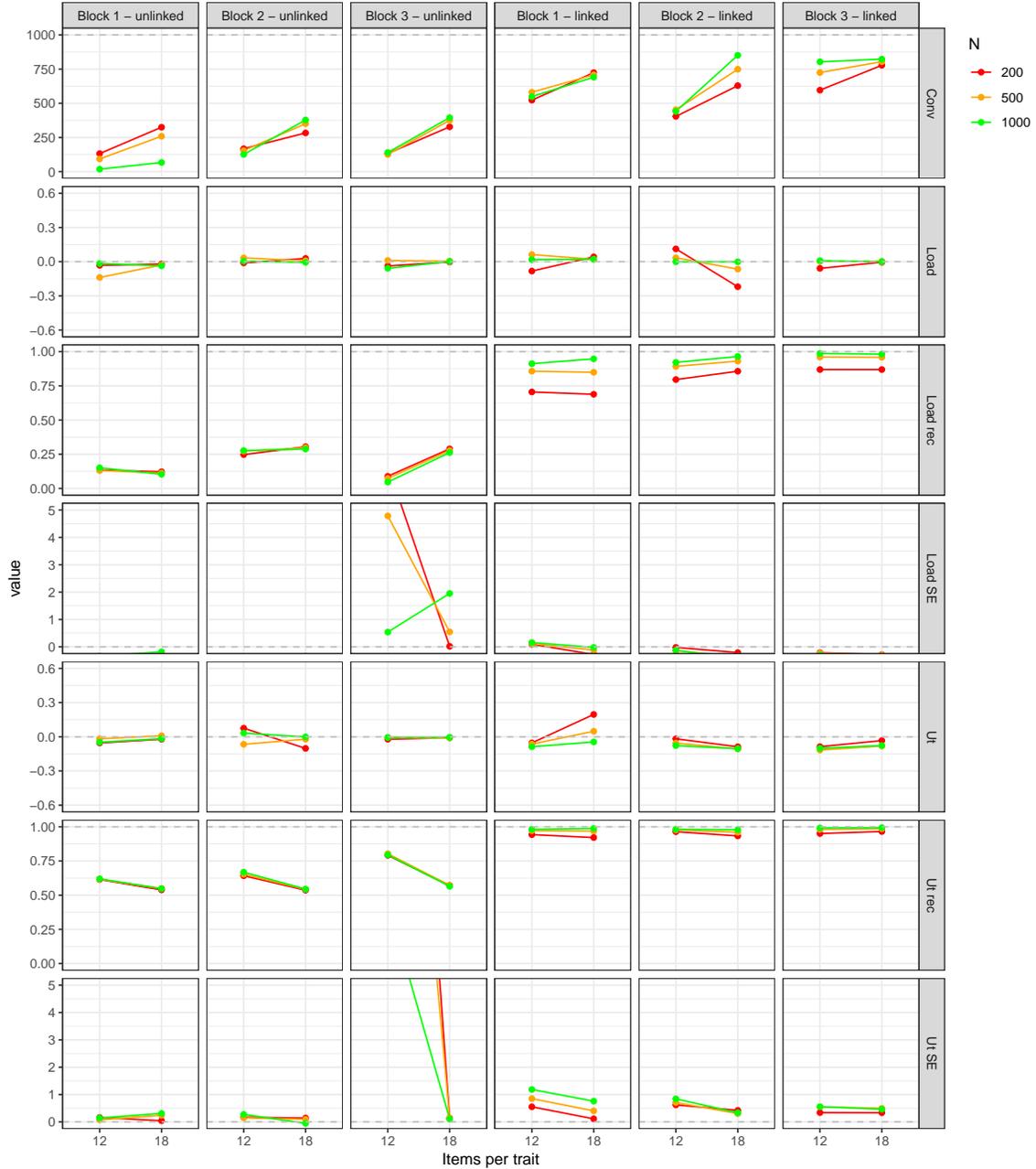
	Trait 1
Items 1	-0.74498
Items 2	-0.88550
Items 3	0.59527
Items 4	0.40189
Items 5	0.32282
Items 6	0.54599
Items 7	0.72332
Items 8	0.86733
Items 9	0.63353
Items 10	0.73280
Items 11	0.82750
Items 12	0.58275
Items 13	0.73265
Items 14	0.85592
Items 15	0.32795
Items 16	0.37976
Items 17	0.81138
Items 18	0.77367

## 2.3 Results

### 2.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 1

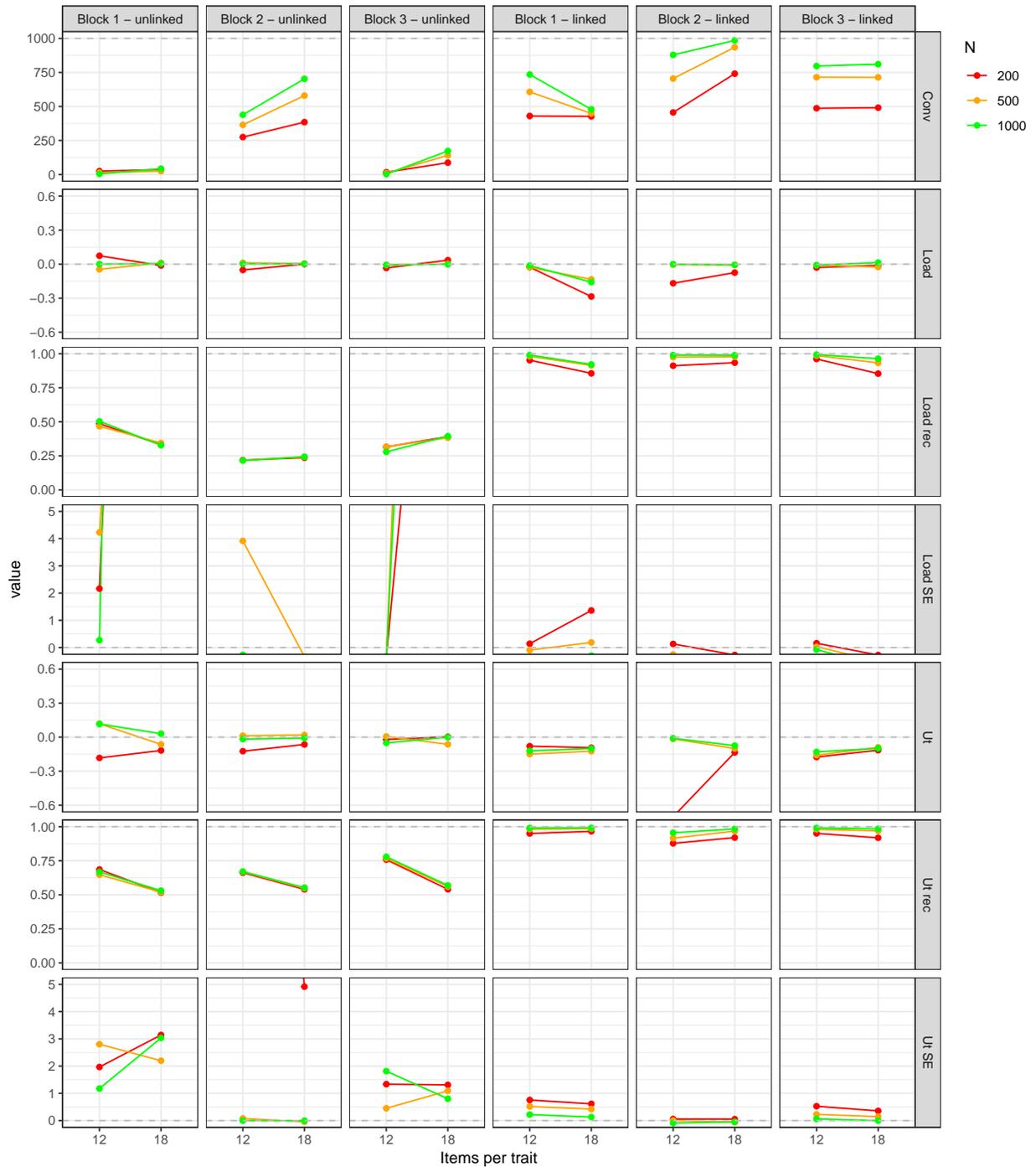
*Bias results for the Thurstonian factor model with one trait. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 2**

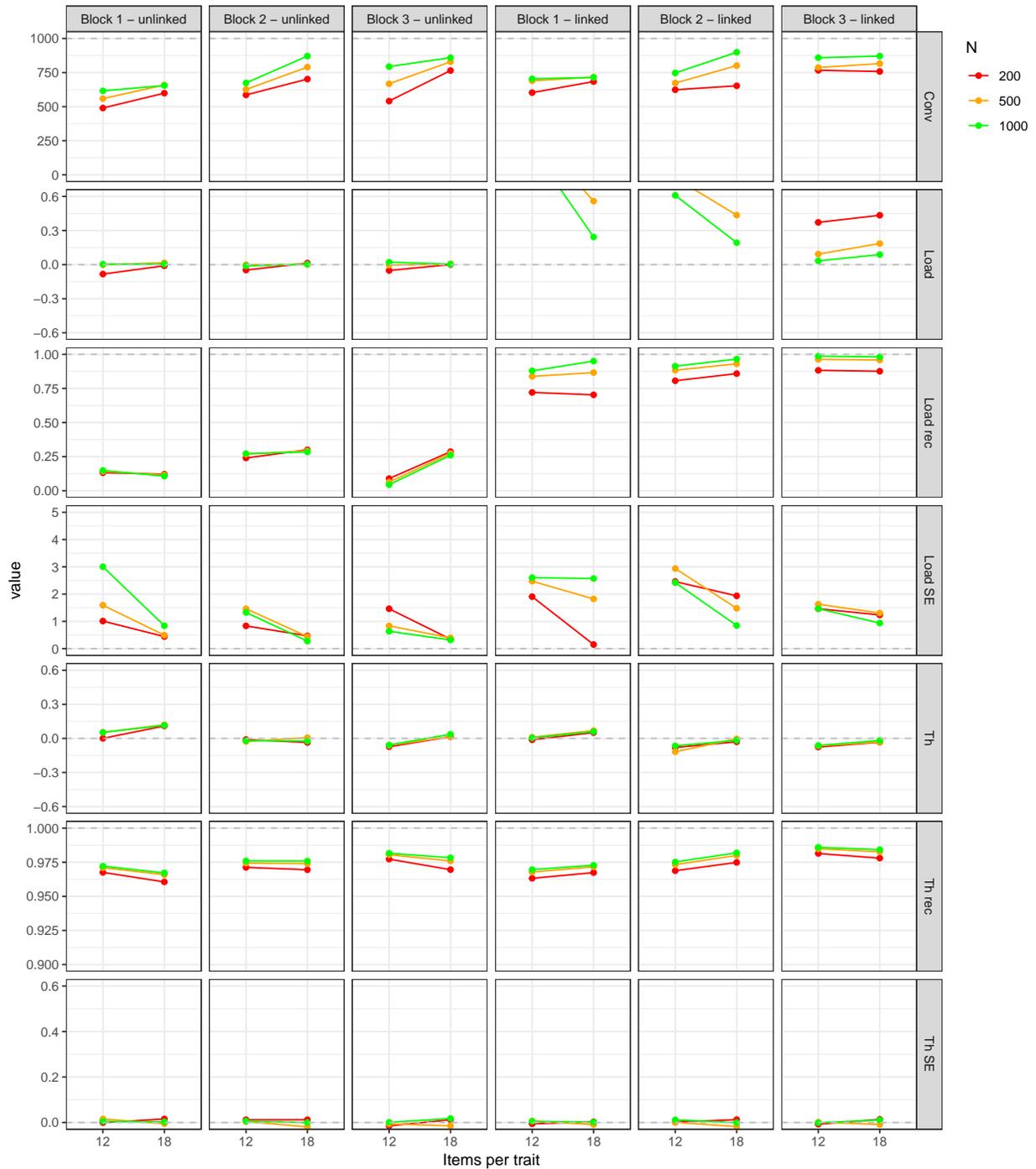
*Bias results for the Thurstonian factor model with one trait. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 3**

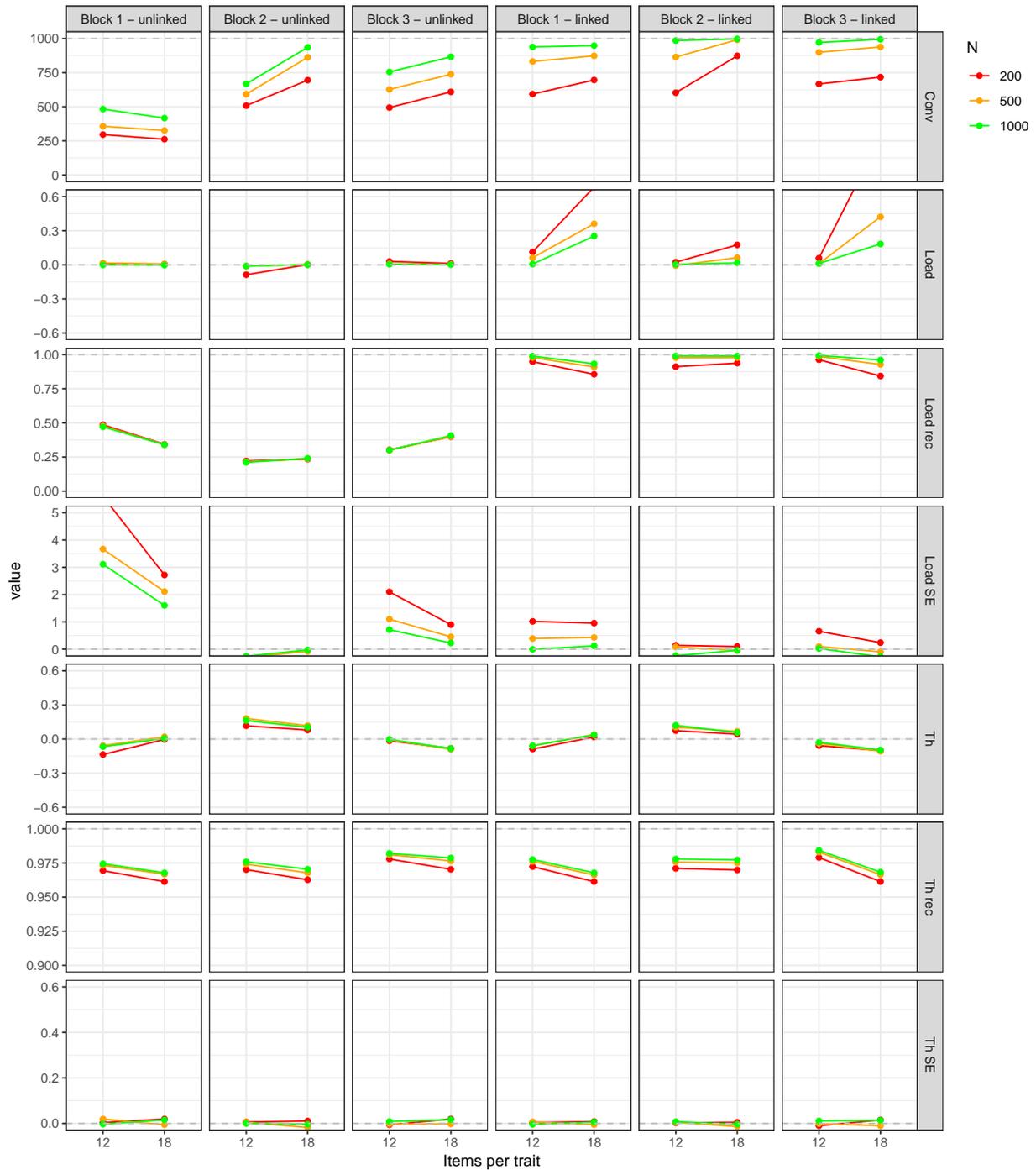
*Bias results for the Thurstonian IRT model with one trait. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Figure 4**

*Bias results for the Thurstonian IRT model with one trait. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 1**

*Bias results for the Thurstonian factor model with one trait. One negatively keyed item.*

	N	unlinked									linked																													
		Block1			Block2			Block3			Block1			Block2			Block3																							
		12	18	18	12	18	18	12	18	18	12	18	18	12	18	18	12	18	18																					
Conv	200	132	325	168	283	133	328	523	724	404	629	596	778	500	93	259	154	351	126	375	581	708	453	749	725	804	1000	18	67	126	378	140	395	549	691	441	851	803	823	
Load	200	-0.03	-0.02	-0.01	0.03	-0.04	0.00	-0.08	0.04	0.11	-0.22	-0.06	0.00	500	-0.14	-0.03	0.03	0.01	0.01	0.00	0.06	0.02	0.03	-0.07	0.00	0.00	1000	-0.02	-0.04	0.00	-0.01	-0.06	0.00	0.02	0.02	0.00	0.00	0.00	0.01	0.00
Load rec	200	0.13	0.12	0.25	0.31	0.09	0.29	0.71	0.69	0.80	0.86	0.87	0.87	500	0.13	0.11	0.27	0.30	0.07	0.28	0.86	0.85	0.89	0.93	0.96	0.96	1000	0.15	0.10	0.28	0.29	0.05	0.26	0.91	0.95	0.92	0.96	0.99	0.98	
Load SE	200	-0.35	-0.47	-0.37	-0.35	6.61	0.02	0.09	-0.28	-0.02	-0.21	-0.22	-0.29	500	-0.39	-0.38	-0.45	-0.45	4.78	0.55	0.11	-0.12	-0.15	-0.35	-0.23	-0.30	1000	-0.34	-0.18	-0.47	-0.47	0.54	1.95	0.15	-0.02	-0.13	-0.42	-0.30	-0.40	
Ut	200	-0.05	-0.02	0.08	-0.10	-0.02	-0.01	-0.05	0.20	-0.02	-0.09	-0.09	-0.03	500	-0.02	0.01	-0.07	-0.02	-0.01	-0.01	-0.06	0.05	-0.06	-0.11	-0.12	-0.08	1000	-0.05	-0.02	0.03	0.00	-0.01	0.00	-0.09	-0.04	-0.08	-0.10	-0.10	-0.08	
Ut rec	200	0.61	0.54	0.64	0.54	0.79	0.57	0.94	0.92	0.96	0.93	0.95	0.97	500	0.62	0.55	0.65	0.54	0.80	0.57	0.97	0.97	0.98	0.96	0.98	0.99	1000	0.62	0.55	0.67	0.54	0.79	0.56	0.98	0.99	0.98	0.98	0.99	0.99	
Ut SE	200	0.16	0.04	0.17	0.14	31.41	0.14	0.55	0.11	0.62	0.42	0.34	0.34	500	0.07	0.24	0.15	0.09	24.90	0.14	0.85	0.40	0.71	0.31	0.55	0.50	1000	0.14	0.31	0.27	-0.05	8.01	0.11	1.19	0.76	0.84	0.33	0.56	0.46	

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 2**

*Bias results for the Thurstonian factor model with one trait. Two negatively keyed items.*

	N	unlinked						linked					
		Block1		Block2		Block3		Block1		Block2		Block3	
		12	18	12	18	12	18	12	18	12	18	12	18
Conv	200	25	36	275	385	17	87	430	427	456	741	487	491
	500	14	25	365	580	10	141	607	449	705	935	715	714
	1000	5	43	439	703	3	173	735	479	879	985	797	811
Load	200	0.07	-0.01	-0.05	0.00	-0.03	0.04	-0.03	-0.29	-0.17	-0.07	-0.03	-0.01
	500	-0.05	0.01	0.01	0.01	-0.01	0.00	-0.03	-0.13	0.00	-0.01	-0.01	-0.03
	1000	0.00	0.01	0.00	0.00	-0.01	0.00	-0.01	-0.16	0.00	-0.01	-0.01	0.01
Load rec	200	0.49	0.33	0.22	0.24	0.32	0.39	0.95	0.86	0.91	0.93	0.96	0.85
	500	0.47	0.34	0.22	0.24	0.32	0.38	0.98	0.91	0.98	0.98	0.99	0.93
	1000	0.50	0.33	0.22	0.24	0.28	0.39	0.99	0.92	0.99	0.99	0.99	0.96
Load SE	200	2.16	53.62	130.99	32.15	-0.34	23.23	0.14	1.36	0.13	-0.27	0.16	-0.27
	500	4.23	34.69	3.92	-0.34	-0.44	56.52	-0.10	0.19	-0.26	-0.58	0.03	-0.50
	1000	0.27	98.52	-0.27	-0.45	-0.35	44.83	-0.40	-0.30	-0.53	-0.71	-0.08	-0.67
Ut	200	-0.18	-0.12	-0.12	-0.06	-0.02	0.00	-0.08	-0.09	-0.70	-0.14	-0.18	-0.12
	500	0.12	-0.06	0.01	0.02	0.01	-0.06	-0.15	-0.12	-0.02	-0.10	-0.16	-0.09
	1000	0.12	0.03	-0.02	-0.01	-0.05	0.00	-0.12	-0.10	-0.01	-0.07	-0.13	-0.10
Ut rec	200	0.69	0.52	0.66	0.54	0.76	0.54	0.95	0.97	0.88	0.92	0.95	0.92
	500	0.65	0.52	0.67	0.55	0.77	0.56	0.98	0.99	0.92	0.97	0.98	0.97
	1000	0.67	0.53	0.67	0.55	0.78	0.57	0.99	0.99	0.96	0.98	0.99	0.98
Ut SE	200	1.97	3.14	26.84	4.92	1.34	1.31	0.75	0.61	0.06	0.05	0.53	0.36
	500	2.80	2.20	0.08	-0.04	0.45	1.09	0.52	0.42	0.00	-0.06	0.23	0.14
	1000	1.18	3.03	0.00	0.00	1.81	0.80	0.22	0.13	-0.09	-0.05	0.06	0.00

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 3**

*Bias results for the Thurstonian IRT model with one trait. One negatively keyed item.*

	N	unlinked						linked					
		Block1		Block2		Block3		Block1		Block2		Block3	
		12	18	12	18	12	18	12	18	12	18	12	18
Conv	200	490	599	586	702	541	765	603	684	624	653	767	758
	500	559	659	626	790	668	829	690	716	673	801	787	816
	1000	616	655	674	871	794	859	704	715	747	899	859	871
Load	200	-0.08	-0.01	-0.05	0.01	-0.05	0.00	1.65	0.97	1.01	0.76	0.37	0.43
	500	0.00	0.02	0.00	0.00	0.00	0.01	1.16	0.56	0.76	0.44	0.09	0.19
	1000	0.00	0.00	-0.01	0.00	0.02	0.00	1.02	0.24	0.61	0.19	0.03	0.09
Load rec	200	0.13	0.12	0.24	0.30	0.09	0.29	0.72	0.70	0.81	0.86	0.88	0.88
	500	0.14	0.12	0.27	0.29	0.06	0.27	0.84	0.87	0.88	0.93	0.96	0.96
	1000	0.15	0.11	0.27	0.28	0.04	0.26	0.88	0.95	0.91	0.97	0.99	0.98
Load SE	200	1.01	0.44	0.83	0.47	1.46	0.35	1.91	0.15	2.46	1.93	1.46	1.24
	500	1.59	0.50	1.46	0.44	0.84	0.39	2.47	1.82	2.94	1.48	1.63	1.31
	1000	3.00	0.84	1.33	0.28	0.64	0.32	2.60	2.57	2.42	0.85	1.47	0.94
Th	200	0.00	0.11	-0.01	-0.04	-0.07	0.01	-0.01	0.05	-0.08	-0.03	-0.08	-0.03
	500	0.05	0.12	-0.03	0.01	-0.06	0.02	0.01	0.07	-0.12	0.00	-0.07	-0.03
	1000	0.05	0.12	-0.02	-0.02	-0.06	0.04	0.00	0.06	-0.07	-0.02	-0.06	-0.02
Th rec	200	0.97	0.96	0.97	0.97	0.98	0.97	0.96	0.97	0.97	0.97	0.98	0.98
	500	0.97	0.97	0.97	0.97	0.98	0.98	0.97	0.97	0.97	0.98	0.98	0.98
	1000	0.97	0.97	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.98	0.99	0.98
Th SE	200	0.00	0.02	0.01	0.01	-0.02	0.01	-0.01	0.00	0.00	0.01	-0.01	0.01
	500	0.02	-0.01	0.00	-0.02	-0.01	-0.01	0.01	-0.01	0.00	-0.02	0.00	-0.01
	1000	0.01	0.00	0.01	0.00	0.00	0.02	0.01	0.00	0.01	0.00	0.00	0.01

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 4**

*Bias results for the Thurstonian IRT model with one trait. Two negatively keyed items.*

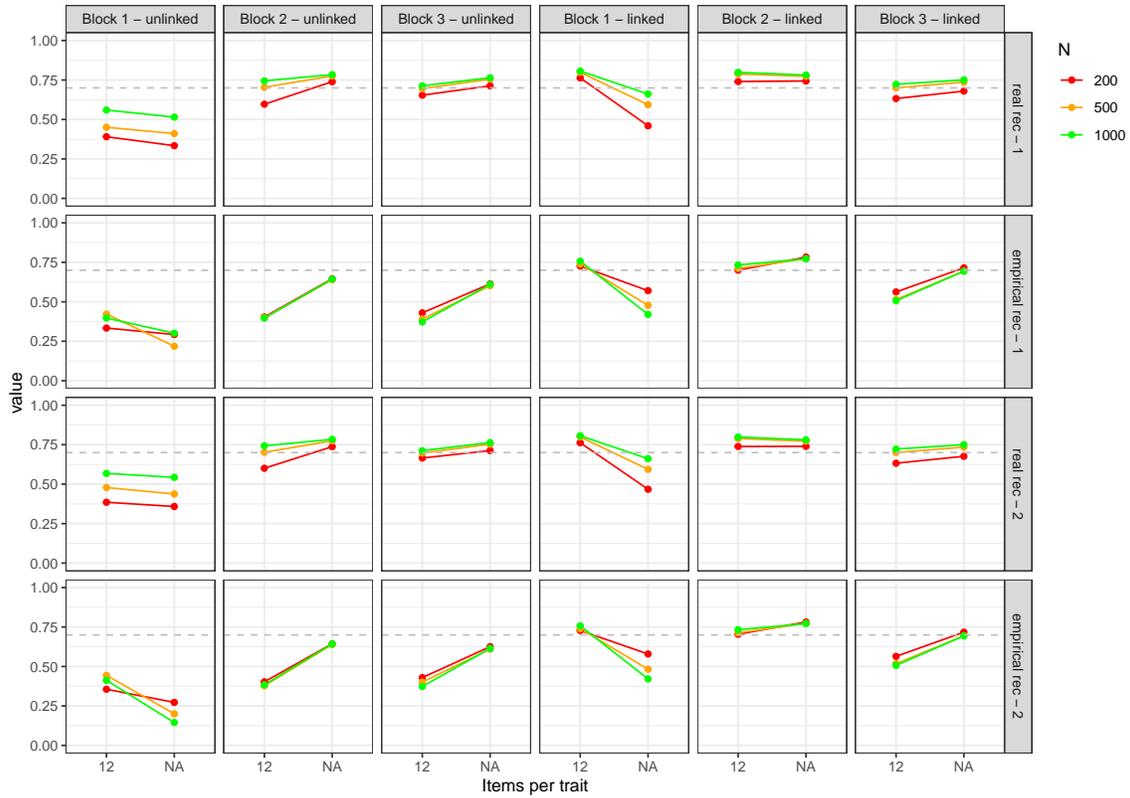
	N	unlinked						linked					
		Block1		Block2		Block3		Block1		Block2		Block3	
		12	18	12	18	12	18	12	18	12	18	12	18
Conv	200	296	262	508	695	494	609	593	696	603	873	667	717
	500	357	326	592	862	627	738	832	873	864	991	899	938
	1000	483	417	668	935	755	866	938	948	985	997	971	994
Load	200	0.01	0.01	-0.09	0.00	0.03	0.01	0.11	0.68	0.02	0.18	0.06	1.12
	500	0.02	0.01	-0.01	0.00	0.01	0.00	0.06	0.36	-0.01	0.06	0.01	0.42
	1000	0.00	0.00	-0.01	0.00	0.00	0.00	0.01	0.25	0.00	0.02	0.01	0.18
Load rec	200	0.49	0.34	0.22	0.23	0.30	0.40	0.95	0.86	0.91	0.94	0.96	0.84
	500	0.48	0.34	0.21	0.24	0.30	0.40	0.98	0.91	0.98	0.98	0.99	0.93
	1000	0.47	0.34	0.21	0.24	0.30	0.41	0.99	0.93	0.99	0.99	0.99	0.96
Load SE	200	5.60	2.72	-0.35	-0.45	2.10	0.90	1.02	0.95	0.14	0.10	0.66	0.24
	500	3.67	2.11	-0.29	-0.08	1.10	0.45	0.39	0.43	0.08	-0.05	0.10	-0.10
	1000	3.11	1.60	-0.26	-0.03	0.72	0.23	0.00	0.12	-0.24	-0.04	0.02	-0.28
Th	200	-0.14	0.00	0.12	0.08	-0.02	-0.08	-0.09	0.02	0.07	0.04	-0.06	-0.10
	500	-0.06	0.02	0.18	0.12	-0.01	-0.09	-0.06	0.04	0.10	0.07	-0.04	-0.11
	1000	-0.07	0.00	0.16	0.10	0.00	-0.08	-0.06	0.04	0.12	0.06	-0.03	-0.10
Th rec	200	0.97	0.96	0.97	0.96	0.98	0.97	0.97	0.96	0.97	0.97	0.98	0.96
	500	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.97	0.98	0.98	0.98	0.97
	1000	0.97	0.97	0.98	0.97	0.98	0.98	0.98	0.97	0.98	0.98	0.98	0.97
Th SE	200	0.00	0.02	0.01	0.01	-0.01	0.02	0.00	0.01	0.00	0.00	-0.01	0.02
	500	0.02	-0.01	0.01	-0.02	0.00	0.00	0.01	-0.01	0.00	-0.01	0.00	-0.01
	1000	0.00	0.02	0.00	0.00	0.01	0.02	0.00	0.01	0.01	0.00	0.01	0.01

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 2.3.2 Results for latent trait recovery

Figure 5

*Latent trait recovery results with one trait.*



*Note.* Abbreviations: rec: recovery. The first two rows come from simulation with one (- 1), the second two rows from simulation with two (- 2) negatively keyed items per trait.

**Table 5**

*Latent trait recovery results with one trait.*

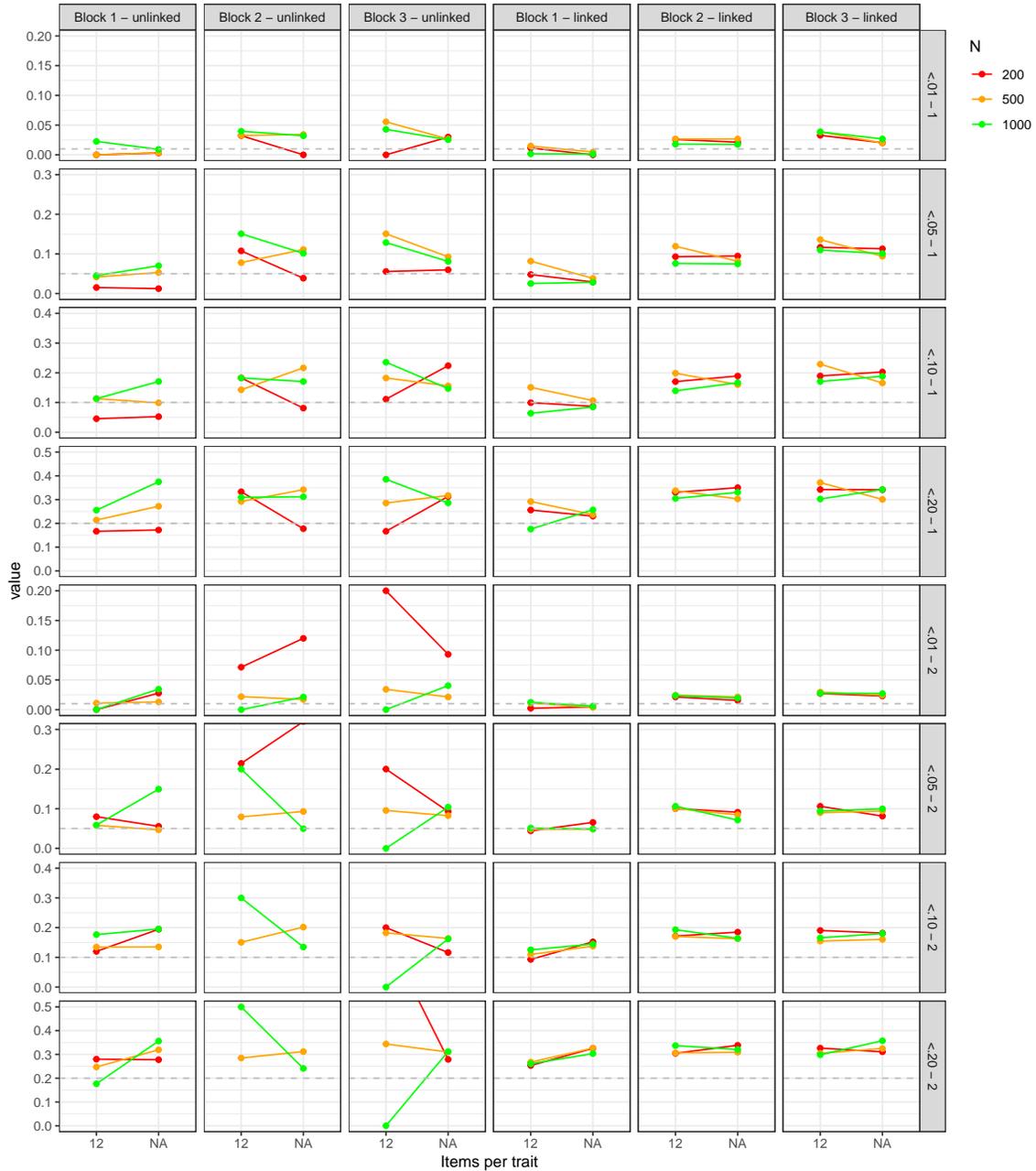
	N	unlinked									linked																												
		Block1			Block2			Block3			Block1			Block2			Block3																						
		12	18	18	12	18	18	12	18	18	12	18	18	12	18	18	12	18	18																				
real rec - 1	200	0.391	0.334	0.597	0.739	0.654	0.713	0.763	0.460	0.740	0.744	0.632	0.680	500	0.450	0.411	0.704	0.775	0.696	0.755	0.799	0.593	0.788	0.773	0.700	0.736	1000	0.560	0.515	0.744	0.784	0.713	0.764	0.807	0.661	0.798	0.781	0.723	0.751
empirical rec - 1	200	0.334	0.293	0.404	0.646	0.430	0.613	0.726	0.570	0.700	0.783	0.562	0.715	500	0.422	0.218	0.399	0.640	0.391	0.602	0.740	0.477	0.712	0.773	0.514	0.695	1000	0.397	0.301	0.397	0.644	0.373	0.611	0.757	0.420	0.733	0.772	0.507	0.694
real rec - 2	200	0.385	0.358	0.601	0.737	0.666	0.713	0.762	0.468	0.739	0.739	0.633	0.676	500	0.478	0.438	0.702	0.775	0.697	0.753	0.799	0.594	0.789	0.772	0.700	0.735	1000	0.568	0.543	0.743	0.784	0.712	0.763	0.807	0.661	0.798	0.781	0.722	0.750
empirical rec - 2	200	0.356	0.273	0.403	0.644	0.430	0.626	0.728	0.579	0.704	0.782	0.564	0.717	500	0.444	0.200	0.378	0.639	0.401	0.610	0.739	0.483	0.714	0.772	0.518	0.695	1000	0.413	0.145	0.384	0.643	0.373	0.614	0.757	0.421	0.733	0.772	0.507	0.694

*Note.* Abbreviations: rec: recovery.

### 2.3.3 Results for empirical rejection rates

Figure 6

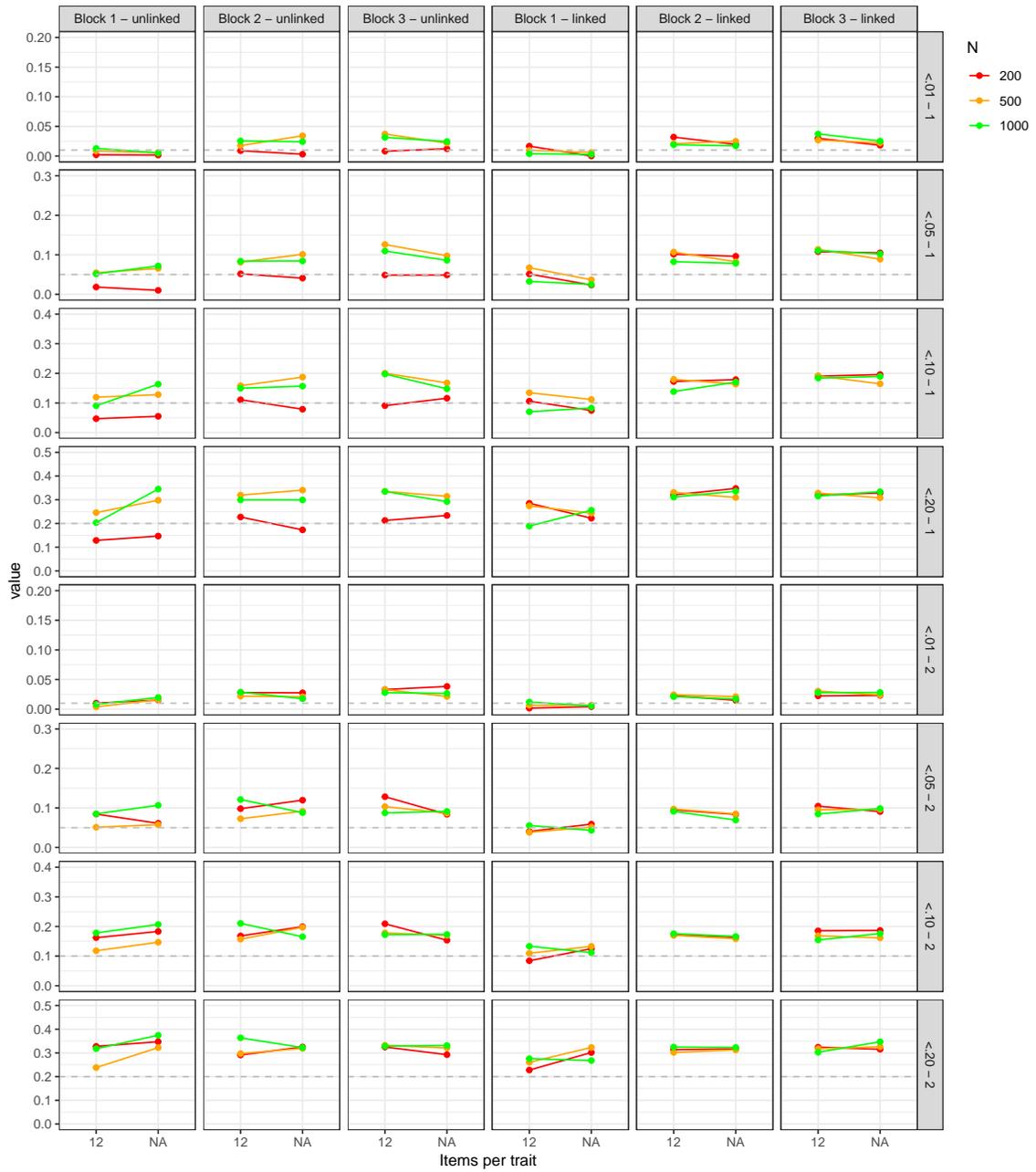
Empirical rejection rates for the Thurstonian factor model with one trait.



Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Figure 7**

*Empirical rejection rates for the Thurstonian IRT model with one trait.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 6**

*Empirical rejection rates for the Thurstonian factor model with one trait.*

	N	unlinked						linked					
		Block1		Block2		Block3		Block1		Block2		Block3	
		12	18	12	18	12	18	12	18	12	18	12	18
<.01 - 1	200	0.0000	0.0030	0.0320	0.0000	0.0000	0.0300	0.0110	0.0000	0.0260	0.0210	0.0330	0.0200
	500	0.0000	0.0040	0.0320	0.0340	0.0560	0.0260	0.0150	0.0050	0.0260	0.0270	0.0390	0.0200
	1000	0.0230	0.0090	0.0400	0.0320	0.0430	0.0250	0.0020	0.0010	0.0180	0.0170	0.0390	0.0270
<.05 - 1	200	0.0150	0.0120	0.1080	0.0390	0.0560	0.0600	0.0480	0.0290	0.0930	0.0950	0.1170	0.1130
	500	0.0420	0.0530	0.0780	0.1110	0.1510	0.0930	0.0820	0.0380	0.1190	0.0810	0.1360	0.0940
	1000	0.0450	0.0700	0.1510	0.1010	0.1290	0.0810	0.0250	0.0280	0.0760	0.0750	0.1100	0.1010
<.10 - 1	200	0.0450	0.0520	0.1830	0.0810	0.1110	0.2240	0.0990	0.0870	0.1700	0.1890	0.1890	0.2030
	500	0.1130	0.0990	0.1430	0.2170	0.1830	0.1560	0.1510	0.1070	0.1990	0.1600	0.2290	0.1660
	1000	0.1130	0.1710	0.1830	0.1710	0.2360	0.1470	0.0640	0.0850	0.1390	0.1670	0.1710	0.1880
<.20 - 1	200	0.1667	0.1723	0.3333	0.1776	0.1667	0.3134	0.2562	0.2307	0.3305	0.3503	0.3424	0.3415
	500	0.2143	0.2721	0.2922	0.3419	0.2857	0.3175	0.2921	0.2369	0.3377	0.3031	0.3719	0.3008
	1000	0.2556	0.3750	0.3095	0.3120	0.3857	0.2861	0.1762	0.2571	0.3048	0.3308	0.3026	0.3426
<.01 - 2	200	0.0000	0.0280	0.0710	0.1200	0.2000	0.0930	0.0020	0.0050	0.0210	0.0160	0.0270	0.0230
	500	0.0110	0.0130	0.0220	0.0170	0.0340	0.0210	0.0110	0.0040	0.0240	0.0210	0.0300	0.0240
	1000	0.0000	0.0340	0.0000	0.0210	0.0000	0.0400	0.0120	0.0060	0.0240	0.0200	0.0280	0.0270
<.05 - 2	200	0.0800	0.0560	0.2140	0.3200	0.2000	0.0930	0.0440	0.0660	0.1000	0.0910	0.1060	0.0810
	500	0.0580	0.0470	0.0790	0.0930	0.0960	0.0830	0.0480	0.0490	0.1010	0.0840	0.0900	0.0940
	1000	0.0590	0.1490	0.2000	0.0500	0.0000	0.1040	0.0510	0.0490	0.1060	0.0710	0.0940	0.1000
<.10 - 2	200	0.1200	0.1940	0.4290	0.4800	0.2000	0.1160	0.0930	0.1520	0.1710	0.1850	0.1900	0.1820
	500	0.1350	0.1350	0.1510	0.2020	0.1820	0.1640	0.1100	0.1380	0.1700	0.1630	0.1550	0.1600
	1000	0.1760	0.1950	0.3000	0.1350	0.0000	0.1620	0.1250	0.1450	0.1930	0.1640	0.1660	0.1800
<.20 - 2	200	0.2800	0.2778	0.5714	0.6000	0.8000	0.2791	0.2535	0.3255	0.3048	0.3385	0.3265	0.3111
	500	0.2473	0.3195	0.2849	0.3121	0.3440	0.3101	0.2675	0.3266	0.3064	0.3091	0.3049	0.3249
	1000	0.1765	0.3563	0.5000	0.2411	0.0000	0.3121	0.2608	0.3035	0.3371	0.3207	0.2986	0.3576

*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 7**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with one trait.*

	N	unlinked									linked																														
		Block1			Block2			Block3			Block1			Block2			Block3																								
		12	18	18	12	18	18	12	18	18	12	18	18	12	18	18	12	18	18																						
$\chi^2$ - 1	200	52	131	131	55	130	53	135	120	300	121	304	122	303	500	53	136	137	54	137	54	137	121	328	122	330	123	329	1000	53	138	137	56	136	136	301	140	303	140	304	
$\chi^2$ SD - 1	200	7	12	9	14	8	16	11	16	11	16	14	21	15	22	500	7	12	9	15	12	16	12	17	15	22	19	23	1000	8	13	10	15	10	17	11	15	14	20	16	21
df - 1	200	54	135	54	135	54	135	54	135	120	300	120	300	120	300	500	54	135	54	135	54	135	120	327	120	327	120	327	1000	54	135	54	135	54	135	138	300	138	300	138	300
Corrected df - 1	200	50	129	50	129	50	129	114	291	114	291	114	291	114	291	500	50	129	50	129	50	129	114	318	114	318	114	318	1000	50	129	50	129	50	129	132	291	132	291	132	291
$\chi^2$ - 2	200	55	138	63	148	63	139	138	333	140	331	139	330	500	53	137	53	137	54	135	137	331	138	330	138	329	1000	53	138	59	135	49	139	138	304	140	303	139	304		
$\chi^2$ SD - 2	200	8	13	10	22	13	27	11	17	15	22	16	23	500	9	12	10	15	11	16	13	17	16	22	16	23	1000	7	15	9	14	4	23	12	16	15	20	15	22		
df - 2	200	54	135	54	135	54	135	54	138	327	138	327	138	327	500	54	135	54	135	54	135	138	327	138	327	138	327	1000	54	135	54	135	54	135	138	300	138	300	138	300	
Corrected df - 2	200	50	129	50	129	50	129	132	318	132	318	132	318	500	50	129	50	129	50	129	132	318	132	318	132	318	1000	50	129	50	129	50	129	132	291	132	291	132	291		

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

Table 8

Empirical rejection rates for the Thurstonian IRT model with one trait.

N		unlinked						linked					
		Block1		Block2		Block3		Block1		Block2		Block3	
		12	18	12	18	12	18	12	18	12	18	12	18
<.01 - 1	200	0.0020	0.0020	0.0090	0.0030	0.0080	0.0120	0.0170	0.0000	0.0320	0.0200	0.0300	0.0180
	500	0.0090	0.0060	0.0180	0.0340	0.0370	0.0220	0.0100	0.0060	0.0210	0.0250	0.0270	0.0220
	1000	0.0130	0.0050	0.0250	0.0240	0.0310	0.0240	0.0040	0.0030	0.0190	0.0170	0.0370	0.0250
<.05 - 1	200	0.0180	0.0100	0.0520	0.0410	0.0490	0.0490	0.0510	0.0230	0.1010	0.0960	0.1080	0.1050
	500	0.0550	0.0660	0.0810	0.1010	0.1260	0.0980	0.0670	0.0370	0.1070	0.0820	0.1140	0.0890
	1000	0.0520	0.0720	0.0840	0.0840	0.1100	0.0860	0.0330	0.0250	0.0830	0.0780	0.1110	0.1020
<.10 - 1	200	0.0470	0.0550	0.1110	0.0790	0.0910	0.1160	0.1060	0.0750	0.1720	0.1790	0.1900	0.1960
	500	0.1190	0.1280	0.1580	0.1870	0.2000	0.1680	0.1350	0.1120	0.1800	0.1640	0.1910	0.1650
	1000	0.0910	0.1630	0.1500	0.1570	0.1980	0.1480	0.0700	0.0830	0.1390	0.1700	0.1840	0.1890
<.20 - 1	200	0.1286	0.1469	0.2272	0.1730	0.2127	0.2336	0.2852	0.2222	0.3203	0.3478	0.3196	0.3287
	500	0.2457	0.2977	0.3195	0.3405	0.3353	0.3146	0.2740	0.2435	0.3314	0.3096	0.3280	0.3081
	1000	0.2033	0.3451	0.2994	0.2992	0.3350	0.2922	0.1890	0.2559	0.3113	0.3358	0.3155	0.3341
<.01 - 2	200	0.0100	0.0150	0.0280	0.0280	0.0330	0.0380	0.0020	0.0040	0.0230	0.0150	0.0220	0.0230
	500	0.0040	0.0160	0.0220	0.0210	0.0330	0.0210	0.0070	0.0070	0.0240	0.0210	0.0300	0.0240
	1000	0.0080	0.0200	0.0290	0.0180	0.0280	0.0270	0.0120	0.0060	0.0210	0.0170	0.0280	0.0280
<.05 - 2	200	0.0840	0.0610	0.0980	0.1200	0.1280	0.0840	0.0400	0.0590	0.0950	0.0840	0.1040	0.0910
	500	0.0510	0.0580	0.0730	0.0920	0.1030	0.0870	0.0380	0.0520	0.0970	0.0850	0.0950	0.0960
	1000	0.0850	0.1070	0.1210	0.0880	0.0870	0.0910	0.0550	0.0430	0.0910	0.0690	0.0840	0.0990
<.10 - 2	200	0.1620	0.1830	0.1680	0.1990	0.2090	0.1530	0.0840	0.1250	0.1720	0.1640	0.1860	0.1870
	500	0.1180	0.1470	0.1570	0.1970	0.1780	0.1700	0.1090	0.1330	0.1700	0.1580	0.1690	0.1610
	1000	0.1780	0.2070	0.2110	0.1650	0.1720	0.1730	0.1330	0.1120	0.1760	0.1660	0.1540	0.1760
<.20 - 2	200	0.3277	0.3473	0.2913	0.3252	0.3251	0.2926	0.2277	0.3017	0.3137	0.3173	0.3241	0.3154
	500	0.2382	0.3223	0.2973	0.3190	0.3323	0.3209	0.2604	0.3230	0.3021	0.3128	0.3178	0.3260
	1000	0.3178	0.3744	0.3636	0.3225	0.3285	0.3314	0.2759	0.2678	0.3248	0.3230	0.3028	0.3471

Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 9**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with one trait.*

	N	unlinked									linked								
		Block1			Block2			Block3			Block1			Block2			Block3		
		12	18	18	12	18	18	12	18	18	12	18	18	12	18	18	12	18	18
$\chi^2$ - 1	200	46	124	47	124	48	126	114	290	115	294	115	294	115	294	115	294	115	294
	500	49	130	50	131	51	130	114	318	115	320	115	319	115	319	115	319	115	319
	1000	49	131	50	130	51	129	130	291	133	294	133	294	133	295	133	295	133	295
$\chi^2$ SD - 1	200	7	11	9	14	8	14	11	15	14	20	14	22	14	22	14	22	14	22
	500	8	11	9	15	10	15	12	17	14	21	14	23	14	23	14	23	14	23
	1000	7	12	8	14	10	15	11	15	13	20	15	21	13	20	15	21	13	20
df - 1	200	50	129	50	129	50	129	114	291	114	291	114	291	114	291	114	291	114	291
	500	50	129	50	129	50	129	114	317	114	317	114	317	114	317	114	317	114	317
	1000	50	129	50	129	50	129	131	291	131	291	131	291	131	291	131	291	131	291
Corrected df - 1	200	46	123	46	123	46	123	108	282	108	282	108	282	108	282	108	282	108	282
	500	46	123	46	123	46	123	108	308	108	308	108	308	108	308	108	308	108	308
	1000	46	123	46	123	46	123	125	282	125	282	125	282	125	282	125	282	125	282
$\chi^2$ - 2	200	51	132	50	130	51	130	131	321	133	320	132	319	133	320	132	319	133	320
	500	49	131	49	130	50	129	130	321	131	319	131	319	131	319	131	319	131	319
	1000	50	132	51	130	50	130	132	293	132	294	131	294	132	294	131	294	132	294
$\chi^2$ SD - 2	200	8	12	10	16	10	15	10	16	14	21	15	23	14	21	15	23	14	21
	500	8	12	9	15	10	15	12	17	16	22	16	23	16	22	16	23	16	22
	1000	8	13	10	14	10	15	12	15	15	20	15	22	15	20	15	22	15	20
df - 2	200	50	129	50	129	50	129	131	317	131	317	131	317	131	317	131	317	131	317
	500	50	129	50	129	50	129	131	317	131	317	131	317	131	317	131	317	131	317
	1000	50	129	50	129	50	129	131	291	131	291	131	291	131	291	131	291	131	291
Corrected df - 2	200	46	123	46	123	46	123	125	308	125	308	125	308	125	308	125	308	125	308
	500	46	123	46	123	46	123	125	308	125	308	125	308	125	308	125	308	125	308
	1000	46	123	46	123	46	123	125	282	125	282	125	282	125	282	125	282	125	282

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

### 3 Simulation Study with three uncorrelated traits

#### 3.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: three traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (18 vs. 36 vs. 54 total)

#### 3.2 Definition of parameters

##### 3.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3
Trait 1	1	0	0
Trait 2	0	1	0
Trait 3	0	0	1

##### 3.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.86717	0.33740	0.55793
Items 4/5/6	-0.55555	-0.51616	0.27310
Items 7/8/9	0.53219	-0.40178	-0.12323
Items 10/11/12	-0.94921	0.28081	0.14628
Items 13/14/15	-0.29657	0.99159	-0.06612
Items 16/17/18	0.24752	-0.89646	0.49086
Items 19/20/21	0.59121	0.72194	0.38019
Items 22/23/24	-0.44583	0.72294	-0.43986
Items 25/26/27	0.13031	-0.66098	0.17700
Items 28/29/30	-0.76699	0.55198	0.69346
Items 31/32/33	0.52706	-0.12255	-0.47498
Items 34/35/36	-0.17488	-0.15692	0.09644
Items 37/38/39	-0.39531	0.15195	-0.60847
Items 40/41/42	-0.33474	0.37905	-0.88619
Items 43/44/45	-0.43994	0.27589	0.36878
Items 46/47/48	-0.95536	0.87734	-0.49639
Items 49/50/51	0.36323	0.36314	-0.03288
Items 52/53/54	0.04349	0.36513	-0.19464

### 3.2.3 Matrix of loadings $\Lambda$ (one negatively keyed item)

	Trait 1	Trait 2	Trait 3
Items 1/2/3	-0.89072	-0.50382	-0.48484
Items 4/5/6	0.30902	0.87724	0.38009
Items 7/8/9	0.69719	0.86641	0.75646
Items 10/11/12	0.43638	0.47083	0.45057
Items 13/14/15	0.89740	0.87764	0.64048
Items 16/17/18	0.75581	0.44547	0.68639
Items 19/20/21	0.64361	0.74375	0.85357
Items 22/23/24	0.78769	0.46937	0.58859
Items 25/26/27	0.44965	0.49089	0.44026
Items 28/29/30	0.31203	0.36859	0.87470
Items 31/32/33	0.86588	0.74518	0.38074
Items 34/35/36	0.86170	0.42299	0.52370
Items 37/38/39	0.76982	0.85155	0.78476
Items 40/41/42	0.50312	0.39656	0.40667
Items 43/44/45	0.31833	0.63527	0.35355
Items 46/47/48	0.82372	0.46375	0.79862
Items 49/50/51	0.41252	0.69088	0.70690
Items 52/53/54	0.31835	0.69240	0.66172

### 3.2.4 Matrix of loadings $\Lambda$ (two negatively keyed items)

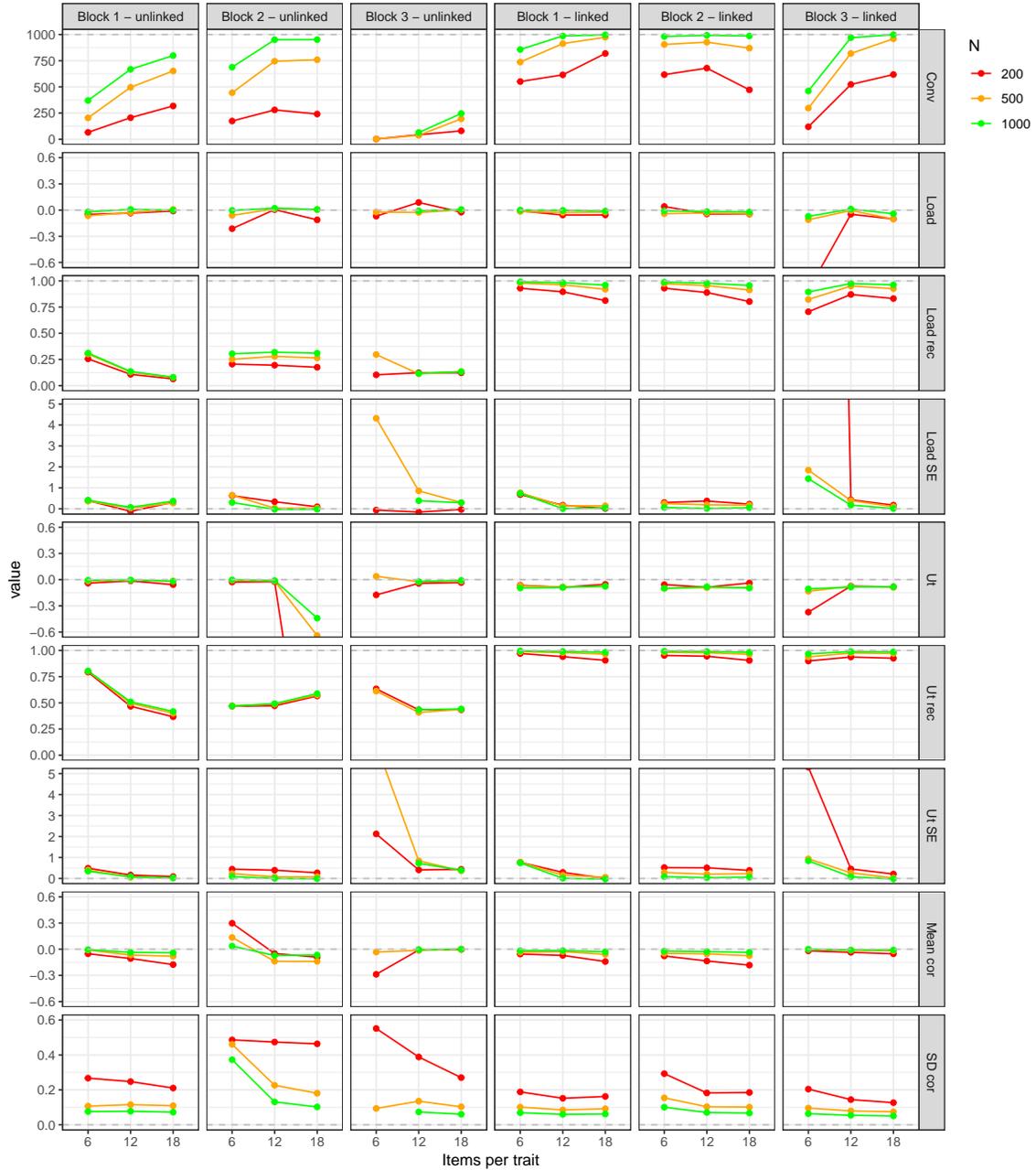
	Trait 1	Trait 2	Trait 3
Items 1/2/3	-0.89072	-0.50382	-0.48484
Items 4/5/6	-0.30902	-0.87724	-0.38009
Items 7/8/9	0.69719	0.86641	0.75646
Items 10/11/12	0.43638	0.47083	0.45057
Items 13/14/15	0.89740	0.87764	0.64048
Items 16/17/18	0.75581	0.44547	0.68639
Items 19/20/21	0.64361	0.74375	0.85357
Items 22/23/24	0.78769	0.46937	0.58859
Items 25/26/27	0.44965	0.49089	0.44026
Items 28/29/30	0.31203	0.36859	0.87470
Items 31/32/33	0.86588	0.74518	0.38074
Items 34/35/36	0.86170	0.42299	0.52370
Items 37/38/39	0.76982	0.85155	0.78476
Items 40/41/42	0.50312	0.39656	0.40667
Items 43/44/45	0.31833	0.63527	0.35355
Items 46/47/48	0.82372	0.46375	0.79862
Items 49/50/51	0.41252	0.69088	0.70690
Items 52/53/54	0.31835	0.69240	0.66172

### 3.3 Results

#### 3.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 8

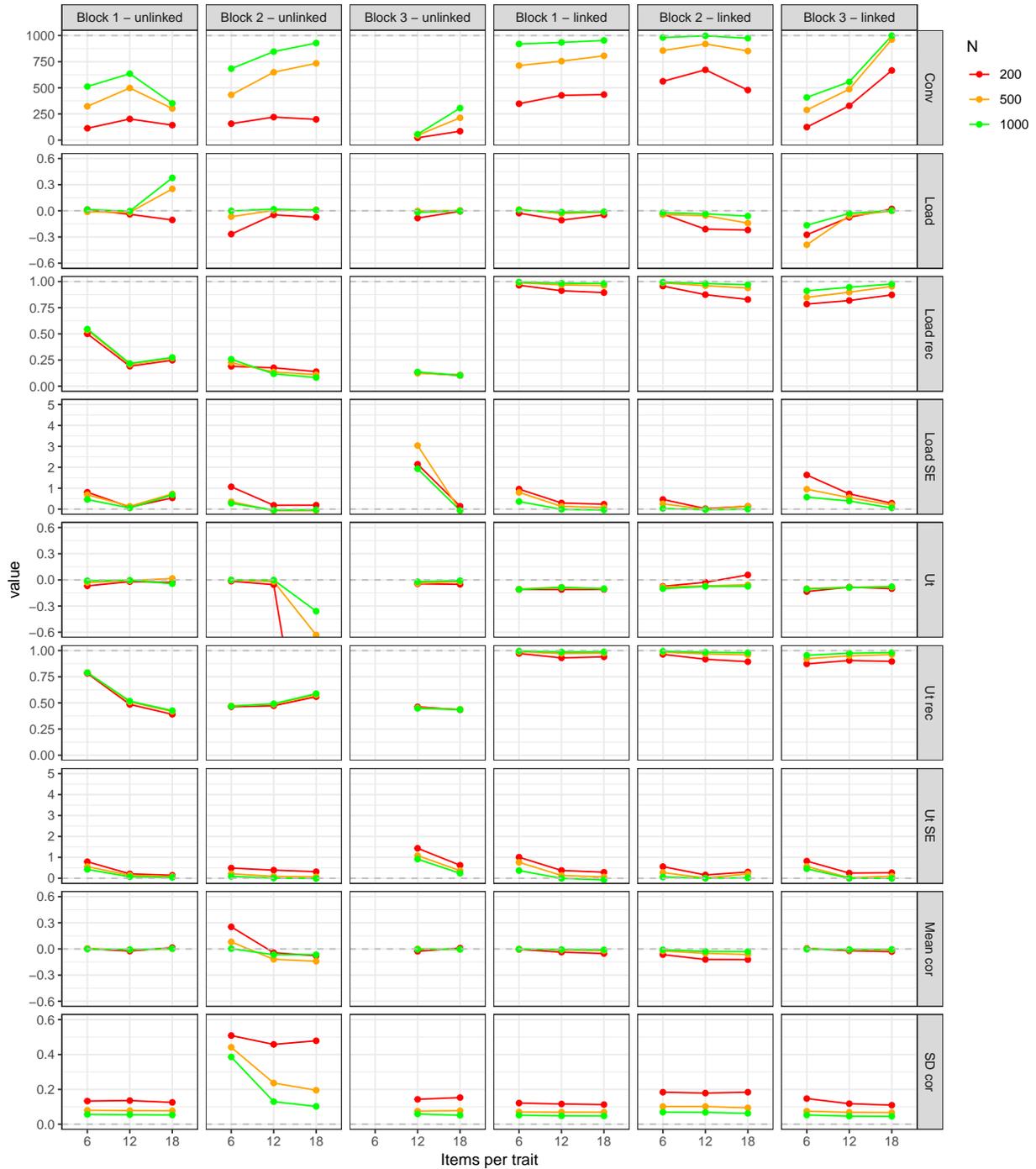
*Bias results for the Thurstonian factor model with three uncorrelated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 9**

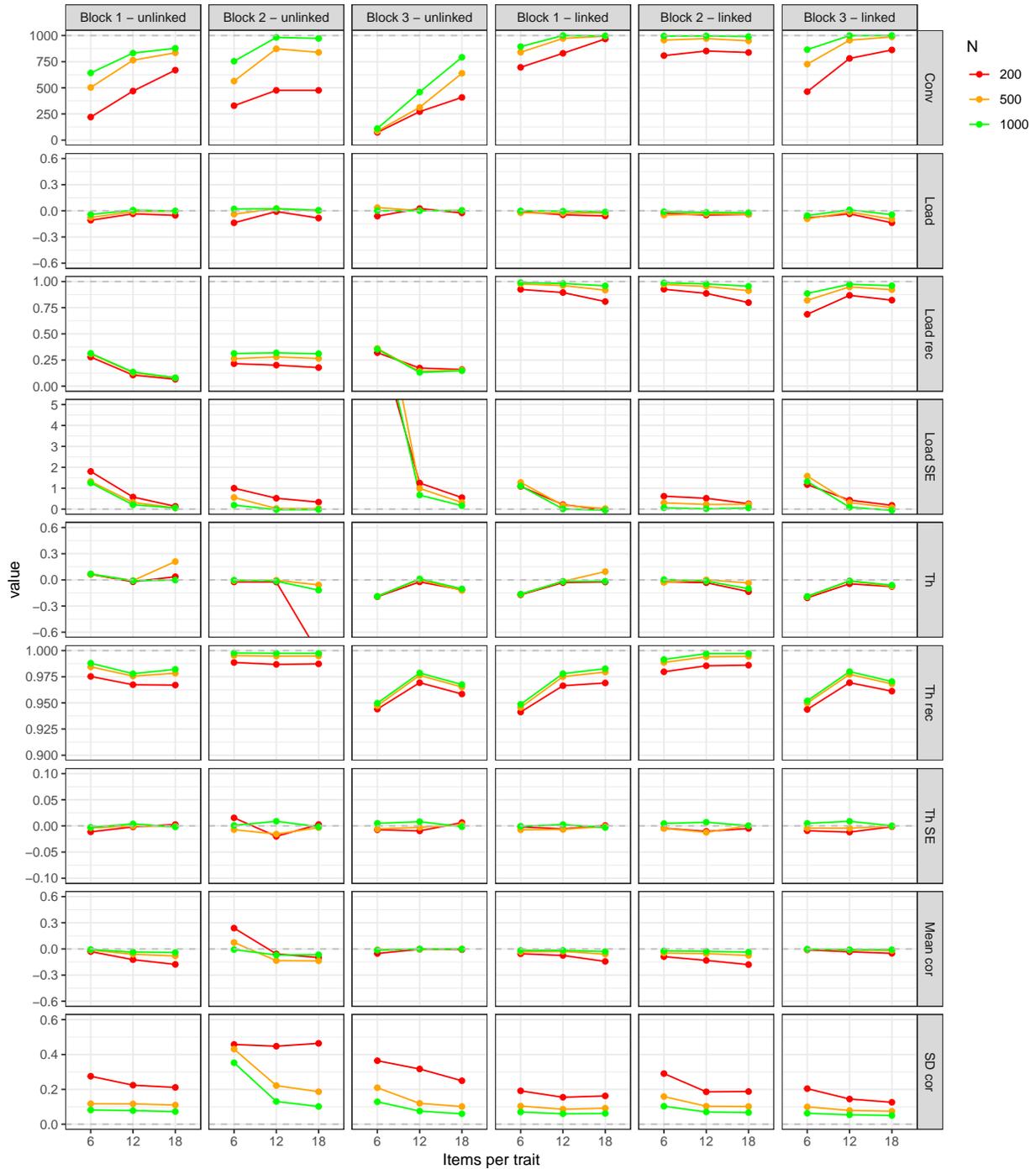
*Bias results for the Thurstonian factor model with three uncorrelated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 10**

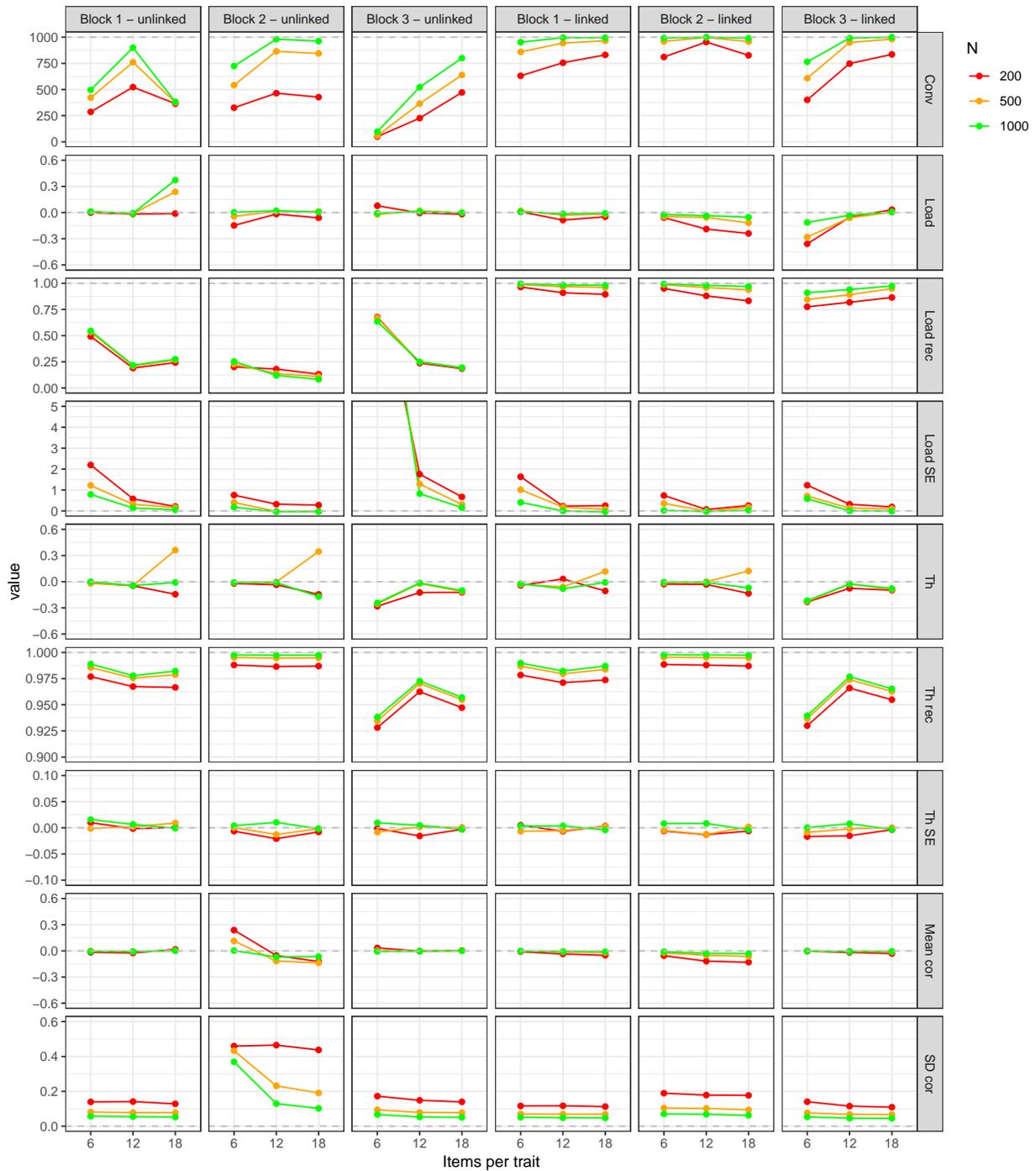
*Bias results for the Thurstonian IRT model with three uncorrelated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Figure 11**

*Bias results for the Thurstonian IRT model with three uncorrelated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 10**

*Bias results for the Thurstonian factor model with three uncorrelated traits. One negatively keyed item.*

	unlinked																		linked																																							
	Block1						Block2						Block3						Block1						Block2						Block3																											
	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18																							
Conv	200	66	206	318	174	280	241	2	43	80	551	615	819	617	679	472	472	119	523	618	500	204	496	652	444	745	759	2	38	195	737	913	974	905	927	870	297	819	959	1000	370	668	799	688	950	951	64	246	857	986	997	981	992	986	461	970	998	
Load	200	-0.05	-0.03	-0.01	-0.21	0.01	-0.11	-0.07	0.09	-0.02	-0.01	-0.06	-0.06	0.04	-0.04	-0.05	-0.05	-0.92	-0.05	-0.10	500	-0.07	-0.03	0.01	-0.06	0.02	0.01	-0.02	-0.03	0.00	-0.02	-0.03	-0.02	-0.04	-0.03	-0.04	-0.11	0.00	-0.10	1000	-0.02	0.01	0.00	0.00	0.02	0.01	-0.01	0.01	0.00	0.01	0.00	-0.01	-0.01	-0.02	-0.02	-0.07	0.01	-0.04
Load rec	200	0.26	0.11	0.06	0.21	0.20	0.18	0.10	0.12	0.12	0.93	0.90	0.81	0.93	0.89	0.80	0.80	0.71	0.87	0.83	500	0.30	0.13	0.08	0.25	0.28	0.26	0.30	0.11	0.14	0.98	0.96	0.92	0.97	0.95	0.91	0.82	0.95	0.93	1000	0.31	0.14	0.08	0.30	0.32	0.31	0.12	0.13	0.99	0.98	0.96	0.96	0.99	0.98	0.96	0.89	0.97	0.96
Load SE	200	0.39	-0.12	0.31	0.63	0.34	0.10	-0.06	-0.15	-0.03	0.68	0.17	0.02	0.30	0.37	0.23	94.18	0.43	0.18	500	0.37	0.04	0.28	0.65	0.03	0.03	4.32	0.86	0.30	0.77	0.14	0.15	0.25	0.19	0.17	1.85	0.39	0.10	1000	0.41	0.07	0.37	0.30	-0.03	-0.02	0.39	0.39	0.29	0.74	0.01	0.04	0.07	0.02	0.06	1.44	0.18	0.02	
Ut	200	-0.04	-0.01	-0.06	-0.03	-0.02	-3.20	-0.17	-0.04	-0.03	-0.06	-0.09	-0.05	-0.06	-0.09	-0.04	-0.37	-0.07	-0.08	500	-0.02	-0.01	-0.02	-0.01	-0.02	-0.64	0.04	-0.02	-0.01	-0.07	-0.08	-0.08	-0.10	-0.09	-0.09	-0.13	-0.08	-0.09	1000	-0.01	0.00	-0.02	0.00	-0.01	-0.44	-0.02	-0.02	-0.01	-0.10	-0.09	-0.08	-0.10	-0.08	-0.10	-0.11	-0.08	-0.08	
Ut rec	200	0.79	0.47	0.37	0.47	0.47	0.57	0.63	0.43	0.43	0.97	0.94	0.91	0.95	0.94	0.90	0.90	0.90	0.94	0.93	500	0.80	0.50	0.40	0.47	0.49	0.57	0.61	0.41	0.44	0.99	0.98	0.96	0.98	0.98	0.96	0.94	0.98	0.97	1000	0.80	0.51	0.42	0.47	0.49	0.59	0.43	0.43	0.44	0.99	0.99	0.98	0.99	0.99	0.98	0.97	0.99	0.99
Ut SE	200	0.49	0.16	0.09	0.44	0.39	0.27	2.12	0.41	0.43	0.77	0.29	0.00	0.52	0.51	0.38	5.31	0.45	0.21	500	0.40	0.06	0.03	0.23	0.08	0.08	6.41	0.84	0.37	0.78	0.16	0.06	0.28	0.20	0.23	0.94	0.26	0.02	1000	0.34	0.08	0.03	0.09	0.00	-0.01	0.72	0.72	0.39	0.74	0.01	-0.03	0.09	0.04	0.06	0.84	0.08	-0.02	
Mean cor	200	-0.05	-0.11	-0.18	0.30	-0.05	-0.09	-0.29	-0.01	0.00	-0.06	-0.07	-0.14	-0.08	-0.14	-0.18	-0.02	-0.04	-0.05	500	-0.01	-0.07	-0.08	0.13	-0.14	-0.14	-0.03	-0.01	0.00	-0.03	-0.03	-0.06	-0.05	-0.05	-0.07	0.00	-0.01	-0.02	1000	-0.01	-0.04	-0.04	0.04	-0.07	-0.07	0.00	0.00	0.00	-0.02	-0.02	-0.03	-0.02	-0.03	-0.04	0.00	-0.01	-0.01	
SD cor	200	0.27	0.25	0.21	0.49	0.47	0.46	0.55	0.39	0.27	0.19	0.15	0.16	0.29	0.18	0.18	0.20	0.14	0.13	500	0.11	0.12	0.11	0.46	0.23	0.18	0.09	0.14	0.10	0.10	0.08	0.09	0.15	0.10	0.10	0.10	0.08	0.07	1000	0.08	0.08	0.07	0.37	0.13	0.10	0.07	0.07	0.06	0.07	0.06	0.06	0.10	0.07	0.07	0.06	0.05	0.05	

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

Table 11

*Bias results for the Thurstonian factor model with three uncorrelated traits. Two negatively keyed items.*

	unlinked																		linked																																				
	Block1						Block2						Block3						Block1						Block2						Block3																								
	N	6	12	18	6	12	N	6	12	18	6	12	N	6	12	18	6	12	N	6	12	18	6	12	N	6	12	18	6	12	N	6	12	18	6	12																			
Conv	200	113	202	143	157	220	198	198	22	85	348	428	435	562	672	478	124	328	666	500	323	498	302	433	649	734	46	213	713	755	806	856	919	852	288	487	962	1000	512	635	352	683	846	928	56	306	919	934	953	981	997	974	408	557	998
Load	200	0.01	-0.04	-0.10	-0.27	-0.05	-0.07	-0.08	0.00	-0.03	-0.11	-0.05	-0.04	-0.21	-0.22	-0.27	-0.07	0.02	500	-0.01	-0.02	0.25	-0.07	0.01	0.01	0.00	0.01	0.02	-0.03	-0.01	-0.04	-0.05	-0.14	-0.39	-0.06	0.00	1000	0.01	0.00	0.38	0.00	0.02	0.01	-0.02	0.00	0.01	-0.02	-0.01	-0.02	-0.03	-0.06	-0.17	-0.03	0.00	
Load rec	200	0.50	0.19	0.25	0.19	0.18	0.14	0.13	0.10	0.96	0.91	0.89	0.96	0.87	0.83	0.78	0.82	0.87	500	0.54	0.21	0.27	0.23	0.14	0.11	0.12	0.11	0.99	0.97	0.96	0.99	0.96	0.94	0.85	0.90	0.95	1000	0.55	0.22	0.27	0.26	0.12	0.08	0.14	0.10	0.99	0.98	0.98	0.98	0.99	0.98	0.97	0.91	0.95	0.98
Load SE	200	0.81	0.10	0.54	1.07	0.19	0.19	2.15	0.14	0.96	0.30	0.24	0.47	0.03	0.14	1.63	0.73	0.28	500	0.69	0.14	0.73	0.36	-0.07	-0.07	3.04	-0.06	0.80	0.13	0.08	0.25	0.00	0.14	0.95	0.55	0.20	1000	0.46	0.06	0.68	0.28	-0.05	-0.03	1.93	-0.06	0.37	-0.01	-0.04	0.03	-0.02	0.00	0.58	0.39	0.06	
Ut	200	-0.07	-0.02	-0.03	-0.01	-0.06	-2.97	-0.05	-0.05	-0.11	-0.11	-0.11	-0.07	-0.03	0.06	-0.13	-0.08	-0.10	500	-0.03	-0.01	0.02	0.00	-0.02	-0.63	-0.04	-0.02	-0.11	-0.09	-0.10	-0.08	-0.07	-0.06	-0.10	-0.09	-0.08	1000	-0.01	0.00	-0.04	0.00	0.00	-0.36	-0.02	-0.01	-0.11	-0.09	-0.10	-0.10	-0.07	-0.07	-0.11	-0.09	-0.08	
Ut rec	200	0.78	0.49	0.39	0.46	0.47	0.56	0.46	0.43	0.97	0.93	0.94	0.96	0.92	0.89	0.87	0.91	0.90	500	0.79	0.51	0.42	0.47	0.49	0.58	0.45	0.44	0.99	0.97	0.98	0.99	0.97	0.96	0.92	0.95	0.96	1000	0.79	0.52	0.43	0.47	0.49	0.59	0.45	0.44	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.96	0.97	0.98
Ut SE	200	0.79	0.21	0.14	0.49	0.39	0.31	1.43	0.62	1.00	0.37	0.29	0.55	0.15	0.30	0.82	0.25	0.26	500	0.55	0.14	0.08	0.21	0.09	0.06	1.08	0.36	0.76	0.13	0.06	0.27	0.01	0.20	0.56	0.02	0.10	1000	0.42	0.07	0.04	0.08	0.01	-0.02	0.91	0.24	0.37	-0.01	-0.08	0.06	0.00	0.02	0.45	-0.01	-0.01	
Mean cor	200	0.00	-0.02	0.01	0.25	-0.04	-0.08	-0.03	0.01	0.00	-0.04	-0.05	-0.07	-0.12	-0.12	0.01	-0.02	-0.03	500	0.00	-0.01	0.00	0.08	-0.12	-0.14	0.00	0.00	0.00	-0.01	-0.02	-0.02	-0.05	-0.06	0.00	-0.01	-0.01	1000	0.00	-0.01	0.00	0.00	-0.07	-0.07	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.03	0.00	-0.01	-0.01	
SD cor	200	0.13	0.14	0.13	0.51	0.46	0.48	0.14	0.15	0.12	0.12	0.11	0.18	0.18	0.18	0.15	0.12	0.11	500	0.08	0.08	0.08	0.44	0.24	0.20	0.08	0.08	0.07	0.07	0.07	0.10	0.10	0.09	0.08	0.07	0.07	1000	0.06	0.05	0.05	0.39	0.13	0.10	0.06	0.05	0.05	0.05	0.05	0.07	0.07	0.06	0.05	0.05	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 12**

*Bias results for the Thurstonian IRT model with three uncorrelated traits. One negatively keyed item.*

	unlinked																		linked																																																		
	Block1						Block2						Block3						Block1						Block2						Block3																																						
	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18																																		
Conv	200	220	469	669	329	476	476	476	329	476	476	74	272	408	696	830	967	808	852	838	463	781	862	500	503	764	835	564	873	839	86	313	639	840	972	994	956	971	949	726	955	987	1000	642	832	878	754	982	972	111	459	792	894	1000	997	995	996	990	865	999	1000								
Load	200	-0.11	-0.03	-0.05	-0.14	-0.01	-0.08	-0.01	-0.08	-0.06	0.03	-0.02	-0.01	-0.05	-0.06	-0.03	-0.05	-0.03	-0.05	-0.04	-0.08	-0.03	-0.14	500	-0.08	-0.01	0.00	-0.04	0.03	0.00	0.04	0.00	0.00	-0.02	-0.03	-0.03	-0.05	-0.03	-0.04	-0.04	-0.09	-0.01	-0.10	1000	-0.04	0.01	0.00	0.02	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.02	-0.02	-0.05	0.01	-0.04					
Load rec	200	0.28	0.11	0.07	0.21	0.20	0.18	0.32	0.17	0.16	0.17	0.32	0.17	0.16	0.92	0.89	0.81	0.93	0.89	0.80	0.69	0.87	0.82	500	0.31	0.13	0.08	0.26	0.28	0.26	0.36	0.14	0.15	0.98	0.96	0.92	0.97	0.95	0.91	0.82	0.95	0.92	0.95	0.92	1000	0.31	0.14	0.08	0.31	0.32	0.31	0.35	0.13	0.15	0.99	0.98	0.96	0.99	0.98	0.96	0.89	0.96	0.96	0.96	0.89	0.97	0.96		
Load SE	200	1.80	0.58	0.14	1.00	0.52	0.34	8.42	1.25	0.55	1.09	0.22	-0.03	0.62	0.52	0.26	1.17	0.43	0.19	500	1.33	0.32	0.06	0.56	0.03	0.03	11.52	0.99	0.32	1.28	0.19	0.03	0.30	0.23	0.23	1.58	0.31	0.07	1000	1.26	0.21	0.06	0.19	-0.02	-0.02	9.53	0.67	0.16	1.11	0.01	-0.05	0.07	0.02	0.06	1.32	0.09	-0.05												
Th	200	0.06	-0.02	0.04	-0.02	-0.02	-0.84	-0.19	-0.02	-0.11	-0.17	-0.03	-0.02	-0.02	-0.03	-0.13	-0.21	-0.05	-0.08	500	0.06	-0.01	0.21	-0.01	0.00	-0.06	-0.19	0.00	-0.12	-0.17	-0.02	0.09	-0.03	0.00	-0.04	-0.19	-0.01	-0.07	1000	0.07	-0.01	0.00	0.00	-0.01	-0.12	-0.19	0.01	-0.10	-0.16	-0.02	-0.02	0.00	-0.01	-0.10	-0.19	-0.01	-0.06												
Th rec	200	0.98	0.97	0.97	0.99	0.99	0.99	0.94	0.97	0.96	0.94	0.97	0.96	0.97	0.98	0.97	0.98	0.99	0.99	0.96	0.94	0.97	0.96	500	0.98	0.98	0.98	1.00	0.99	0.99	0.95	0.98	0.97	0.95	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.98	0.95	0.98	0.97	1000	0.99	0.98	0.98	1.00	1.00	1.00	0.95	0.98	0.97	0.95	0.98	0.98	0.98	0.99	1.00	1.00	0.95	0.98	0.97				
Th SE	200	-0.01	0.00	0.00	0.02	-0.02	0.00	-0.01	-0.01	0.01	0.00	-0.01	0.01	0.00	-0.01	0.00	-0.01	0.00	-0.01	0.00	-0.01	-0.01	0.00	500	0.00	0.00	0.00	-0.01	-0.02	0.00	-0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1000	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00
Mean cor	200	-0.03	-0.12	-0.18	0.24	-0.06	-0.10	-0.05	0.00	-0.01	-0.05	0.00	-0.01	-0.07	-0.14	-0.09	-0.13	-0.18	-0.01	-0.03	-0.05	500	-0.01	-0.06	-0.08	0.07	-0.13	-0.14	-0.01	0.00	0.00	-0.03	-0.03	-0.06	-0.05	-0.05	-0.07	0.00	-0.01	-0.02	1000	-0.01	-0.04	-0.04	-0.01	-0.07	-0.07	-0.01	0.00	0.00	-0.02	-0.02	-0.03	-0.02	-0.03	-0.04	0.00	-0.01	-0.01										
SD cor	200	0.28	0.22	0.21	0.46	0.45	0.46	0.36	0.32	0.25	0.19	0.15	0.16	0.29	0.19	0.19	0.20	0.14	0.13	500	0.12	0.12	0.11	0.43	0.22	0.19	0.21	0.12	0.10	0.10	0.09	0.09	0.16	0.10	0.10	0.08	0.07	1000	0.08	0.08	0.07	0.35	0.13	0.10	0.13	0.08	0.06	0.07	0.06	0.06	0.10	0.07	0.07	0.06	0.05														

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 13**

*Bias results for the Thurstonian IRT model with three uncorrelated traits. Two negatively keyed items.*

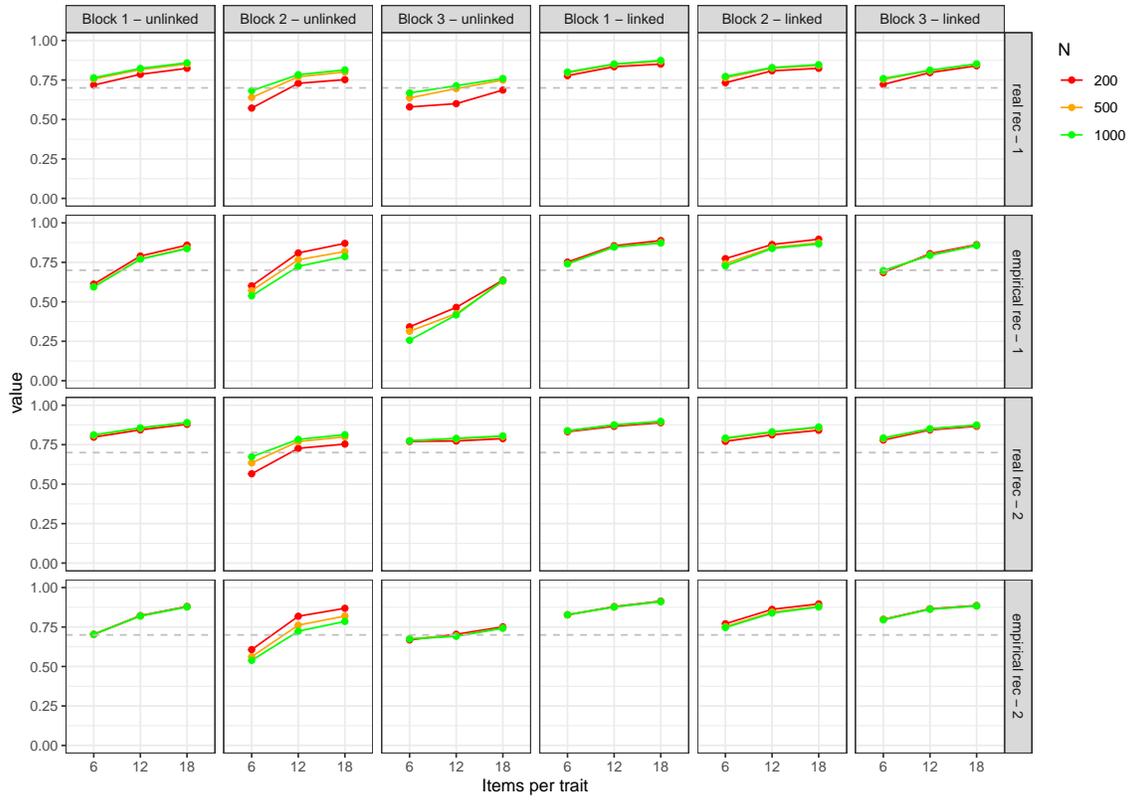
	unlinked																		linked																			
	Block1						Block2						Block3						Block1						Block2						Block3							
	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18			
Conv	200	286	523	365	326	465	427	50	226	472	631	756	831	811	954	827	401	748	836	200	286	523	365	326	465	427	50	226	472	631	756	831	811	954	827	401	748	836
	500	422	761	376	542	865	845	58	365	639	860	944	966	960	998	958	608	949	982	500	422	761	376	542	865	845	58	365	639	860	944	966	960	998	958	608	949	982
	1000	497	900	382	724	981	963	97	522	800	952	996	995	992	1000	990	765	991	1000	1000	497	900	382	724	981	963	97	522	800	952	996	995	992	1000	990	765	991	1000
Load	200	0.00	-0.02	-0.01	-0.15	-0.02	-0.06	0.08	0.00	-0.02	0.01	-0.09	-0.05	-0.06	-0.19	-0.24	-0.36	-0.05	0.03	200	0.00	-0.02	-0.01	-0.15	-0.02	-0.06	0.08	0.00	-0.02	0.01	-0.09	-0.05	-0.06	-0.19	-0.24	-0.36	-0.05	0.03
	500	0.01	-0.01	0.24	-0.04	0.02	0.01	-0.02	0.02	0.00	0.02	-0.03	-0.02	-0.05	-0.05	-0.12	-0.28	-0.06	0.00	500	0.01	-0.01	0.24	-0.04	0.02	0.01	-0.02	0.02	0.00	0.02	-0.03	-0.02	-0.05	-0.05	-0.12	-0.28	-0.06	0.00
	1000	0.01	-0.01	0.37	0.00	0.02	0.01	-0.01	0.01	0.00	0.01	-0.01	-0.01	-0.02	-0.04	-0.05	-0.11	-0.03	0.01	1000	0.01	-0.01	0.37	0.00	0.02	0.01	-0.01	0.01	0.00	0.01	-0.01	-0.01	-0.02	-0.04	-0.05	-0.11	-0.03	0.01
Load rec	200	0.49	0.19	0.24	0.20	0.18	0.13	0.68	0.24	0.19	0.96	0.91	0.89	0.95	0.88	0.83	0.78	0.82	0.86	200	0.49	0.19	0.24	0.20	0.18	0.13	0.68	0.24	0.19	0.96	0.91	0.89	0.95	0.88	0.83	0.78	0.82	0.86
	500	0.53	0.21	0.27	0.23	0.14	0.11	0.67	0.25	0.19	0.99	0.97	0.96	0.99	0.96	0.94	0.84	0.89	0.95	500	0.53	0.21	0.27	0.23	0.14	0.11	0.67	0.25	0.19	0.99	0.97	0.96	0.99	0.96	0.94	0.84	0.89	0.95
	1000	0.54	0.22	0.27	0.25	0.12	0.08	0.63	0.25	0.19	0.99	0.98	0.98	0.99	0.98	0.97	0.91	0.94	0.97	1000	0.54	0.22	0.27	0.25	0.12	0.08	0.63	0.25	0.19	0.99	0.98	0.98	0.99	0.98	0.97	0.91	0.94	0.97
Load SE	200	2.20	0.58	0.21	0.76	0.33	0.28	11.64	1.76	0.67	1.64	0.23	0.25	0.74	0.07	0.26	1.23	0.32	0.19	200	2.20	0.58	0.21	0.76	0.33	0.28	11.64	1.76	0.67	1.64	0.23	0.25	0.74	0.07	0.26	1.23	0.32	0.19
	500	1.22	0.32	0.16	0.41	-0.01	-0.02	13.39	1.29	0.31	1.02	0.18	0.08	0.36	0.00	0.18	0.71	0.14	0.10	500	1.22	0.32	0.16	0.41	-0.01	-0.02	13.39	1.29	0.31	1.02	0.18	0.08	0.36	0.00	0.18	0.71	0.14	0.10
	1000	0.79	0.14	0.06	0.18	-0.03	-0.02	14.32	0.83	0.16	0.41	0.00	-0.04	0.03	-0.02	0.04	0.58	0.01	-0.01	1000	0.79	0.14	0.06	0.18	-0.03	-0.02	14.32	0.83	0.16	0.41	0.00	-0.04	0.03	-0.02	0.04	0.58	0.01	-0.01
Th	200	-0.01	-0.05	-0.14	-0.02	-0.04	-0.15	-0.28	-0.12	-0.12	-0.04	0.03	-0.11	-0.03	-0.03	-0.13	-0.23	-0.08	-0.10	200	-0.01	-0.05	-0.14	-0.02	-0.04	-0.15	-0.28	-0.12	-0.12	-0.04	0.03	-0.11	-0.03	-0.03	-0.13	-0.23	-0.08	-0.10
	500	-0.02	-0.04	0.36	-0.01	0.00	0.35	-0.25	-0.02	-0.11	-0.03	-0.06	0.12	-0.01	0.00	0.12	-0.23	-0.03	-0.09	500	-0.02	-0.04	0.36	-0.01	0.00	0.35	-0.25	-0.02	-0.11	-0.03	-0.06	0.12	-0.01	0.00	0.12	-0.23	-0.03	-0.09
	1000	0.00	-0.04	-0.01	-0.01	-0.01	-0.17	-0.24	-0.02	-0.10	-0.02	-0.08	-0.01	-0.01	-0.01	-0.07	-0.22	-0.03	-0.08	1000	0.00	-0.04	-0.01	-0.01	-0.01	-0.17	-0.24	-0.02	-0.10	-0.02	-0.08	-0.01	-0.01	-0.01	-0.07	-0.22	-0.03	-0.08
Th rec	200	0.98	0.97	0.97	0.99	0.99	0.99	0.93	0.96	0.95	0.98	0.97	0.97	0.99	0.99	0.99	0.93	0.97	0.95	200	0.98	0.97	0.97	0.99	0.99	0.99	0.93	0.96	0.95	0.98	0.97	0.97	0.99	0.99	0.99	0.93	0.97	0.95
	500	0.99	0.98	0.98	1.00	0.99	0.99	0.93	0.97	0.95	0.99	0.98	0.98	1.00	1.00	0.99	0.94	0.97	0.96	500	0.99	0.98	0.98	1.00	0.99	0.99	0.93	0.97	0.95	0.99	0.98	0.98	1.00	1.00	0.99	0.94	0.97	0.96
	1000	0.99	0.98	0.98	1.00	1.00	1.00	0.94	0.97	0.96	0.99	0.98	0.99	1.00	1.00	1.00	0.94	0.97	0.96	1000	0.99	0.98	0.98	1.00	1.00	1.00	0.94	0.97	0.96	0.99	0.98	0.99	1.00	1.00	1.00	0.94	0.97	0.96
Th SE	200	0.01	0.00	0.00	-0.01	-0.02	-0.01	0.00	-0.02	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.02	-0.02	0.00	200	0.01	0.00	0.00	-0.01	-0.02	-0.01	0.00	-0.02	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	0.00
	500	0.00	0.00	0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00	500	0.00	0.00	0.01	0.00	-0.01	0.00	-0.01	0.00	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	-0.01	0.00	0.00
	1000	0.02	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.00	1000	0.02	0.01	0.00	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.01	0.00
Mean cor	200	-0.02	-0.02	0.02	0.24	-0.05	-0.12	0.03	0.00	0.00	-0.01	-0.04	-0.05	-0.06	-0.12	-0.13	0.00	-0.02	-0.03	200	-0.02	-0.02	0.02	0.24	-0.05	-0.12	0.03	0.00	0.00	-0.01	-0.04	-0.05	-0.06	-0.12	-0.13	0.00	-0.02	-0.03
	500	0.00	-0.01	0.00	0.11	-0.12	-0.14	0.00	0.00	0.00	0.00	-0.01	-0.02	-0.02	-0.05	-0.06	0.00	-0.01	-0.01	500	0.00	-0.01	0.00	0.11	-0.12	-0.14	0.00	0.00	0.00	0.00	-0.01	-0.02	-0.02	-0.05	-0.06	0.00	-0.01	-0.01
	1000	-0.01	-0.01	0.00	0.00	-0.07	-0.07	-0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.03	-0.03	0.00	-0.01	-0.01	1000	-0.01	-0.01	0.00	0.00	-0.07	-0.07	-0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.03	-0.03	0.00	-0.01	-0.01
SD cor	200	0.14	0.14	0.13	0.46	0.47	0.44	0.17	0.15	0.14	0.12	0.12	0.11	0.19	0.18	0.18	0.14	0.12	0.11	200	0.14	0.14	0.13	0.46	0.47	0.44	0.17	0.15	0.14	0.12	0.12	0.11	0.19	0.18	0.18	0.14	0.12	0.11
	500	0.08	0.08	0.08	0.43	0.23	0.19	0.09	0.08	0.08	0.07	0.07	0.07	0.10	0.10	0.09	0.08	0.07	0.07	500	0.08	0.08	0.08	0.43	0.23	0.19	0.09	0.08	0.08	0.07	0.07	0.07	0.10	0.10	0.09	0.08	0.07	0.07
	1000	0.06	0.06	0.05	0.37	0.13	0.10	0.07	0.05	0.05	0.05	0.05	0.05	0.07	0.07	0.06	0.05	0.05	0.05	1000	0.06	0.06	0.05	0.37	0.13	0.10	0.07	0.05	0.05	0.05	0.05	0.05	0.07	0.07	0.06	0.05	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 3.3.2 Results for latent trait recovery

Figure 12

*Latent trait recovery results with three uncorrelated traits.*



*Note.* Abbreviations: rec: recovery. The first two rows come from simulation with one (- 1), the second two rows from simulation with two (- 2) negatively keyed items per trait.

**Table 14**

*Latent trait recovery results with three uncorrelated traits.*

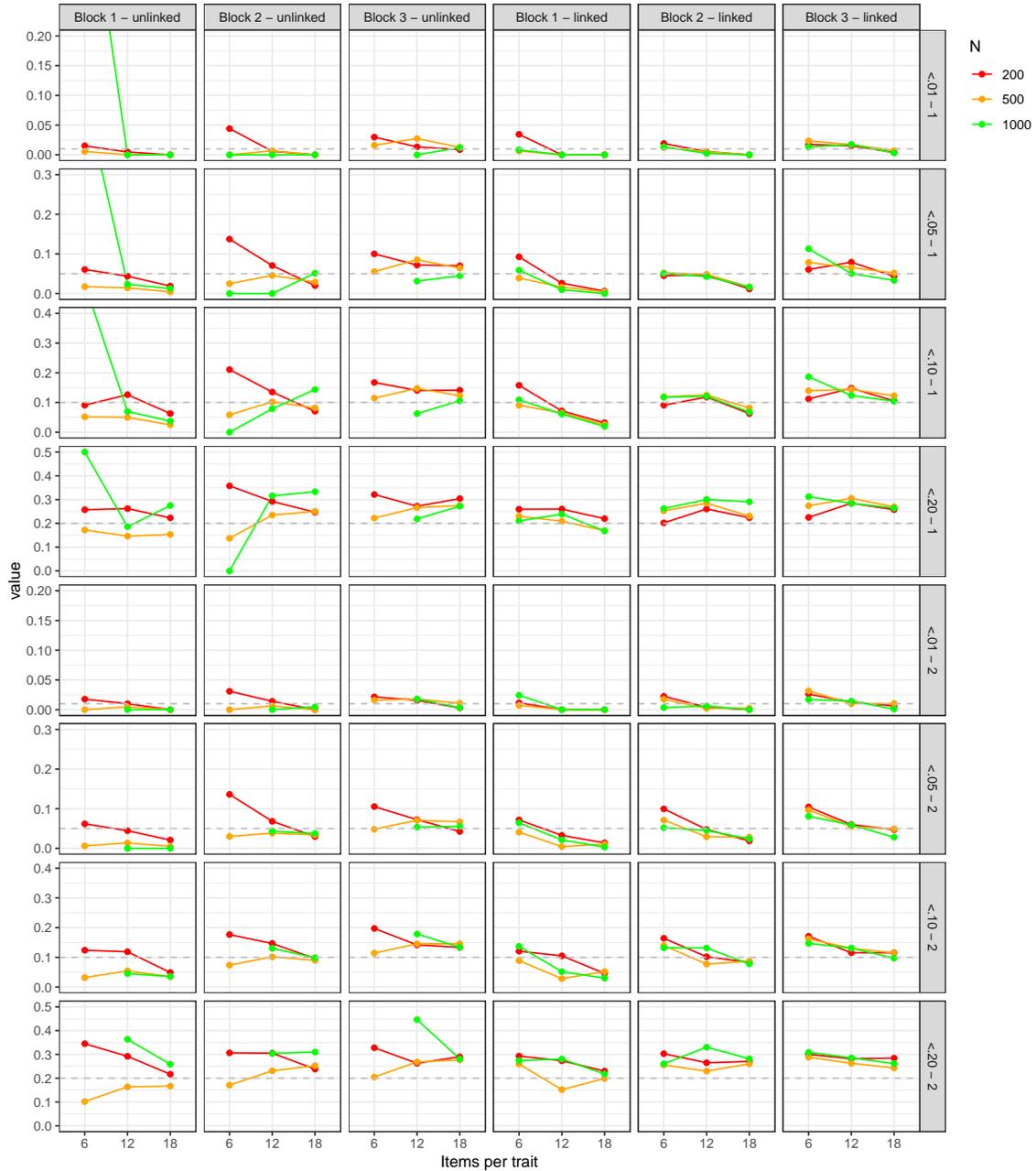
	N	unlinked									linked								
		Block1			Block2			Block3			Block1			Block2			Block3		
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
real rec - 1	200	0.718	0.786	0.824	0.572	0.729	0.752	0.579	0.600	0.686	0.777	0.834	0.851	0.733	0.808	0.824	0.722	0.797	0.840
	500	0.756	0.816	0.851	0.639	0.770	0.801	0.636	0.696	0.749	0.795	0.848	0.869	0.763	0.825	0.842	0.754	0.809	0.849
	1000	0.764	0.823	0.859	0.682	0.784	0.814	0.668	0.714	0.759	0.801	0.851	0.874	0.773	0.829	0.847	0.759	0.812	0.852
empirical rec - 1	200	0.612	0.790	0.859	0.601	0.809	0.870	0.341	0.465	0.637	0.751	0.855	0.888	0.774	0.863	0.896	0.685	0.804	0.862
	500	0.596	0.773	0.841	0.571	0.766	0.818	0.313	0.425	0.630	0.742	0.848	0.876	0.741	0.844	0.874	0.694	0.797	0.857
	1000	0.594	0.770	0.836	0.538	0.724	0.786	0.256	0.417	0.634	0.741	0.847	0.872	0.727	0.838	0.867	0.698	0.794	0.856
real rec - 2	200	0.798	0.844	0.879	0.566	0.727	0.754	0.771	0.773	0.788	0.832	0.866	0.889	0.771	0.812	0.841	0.779	0.843	0.867
	500	0.809	0.854	0.888	0.635	0.770	0.800	0.774	0.787	0.801	0.837	0.874	0.896	0.787	0.827	0.857	0.788	0.849	0.872
	1000	0.812	0.857	0.890	0.673	0.784	0.813	0.775	0.790	0.805	0.839	0.876	0.898	0.792	0.832	0.862	0.793	0.851	0.874
empirical rec - 2	200	0.704	0.822	0.880	0.607	0.818	0.868	0.704	0.751	0.827	0.879	0.914	0.771	0.862	0.896	0.798	0.865	0.886	
	500	0.702	0.820	0.878	0.562	0.762	0.819	0.676	0.694	0.744	0.828	0.877	0.912	0.752	0.845	0.882	0.797	0.863	0.884
	1000	0.704	0.820	0.877	0.539	0.724	0.785	0.674	0.691	0.742	0.828	0.877	0.911	0.747	0.838	0.877	0.795	0.862	0.883

*Note.* Abbreviations: rec: recovery.

### 3.3.3 Results for empirical rejection rates

Figure 13

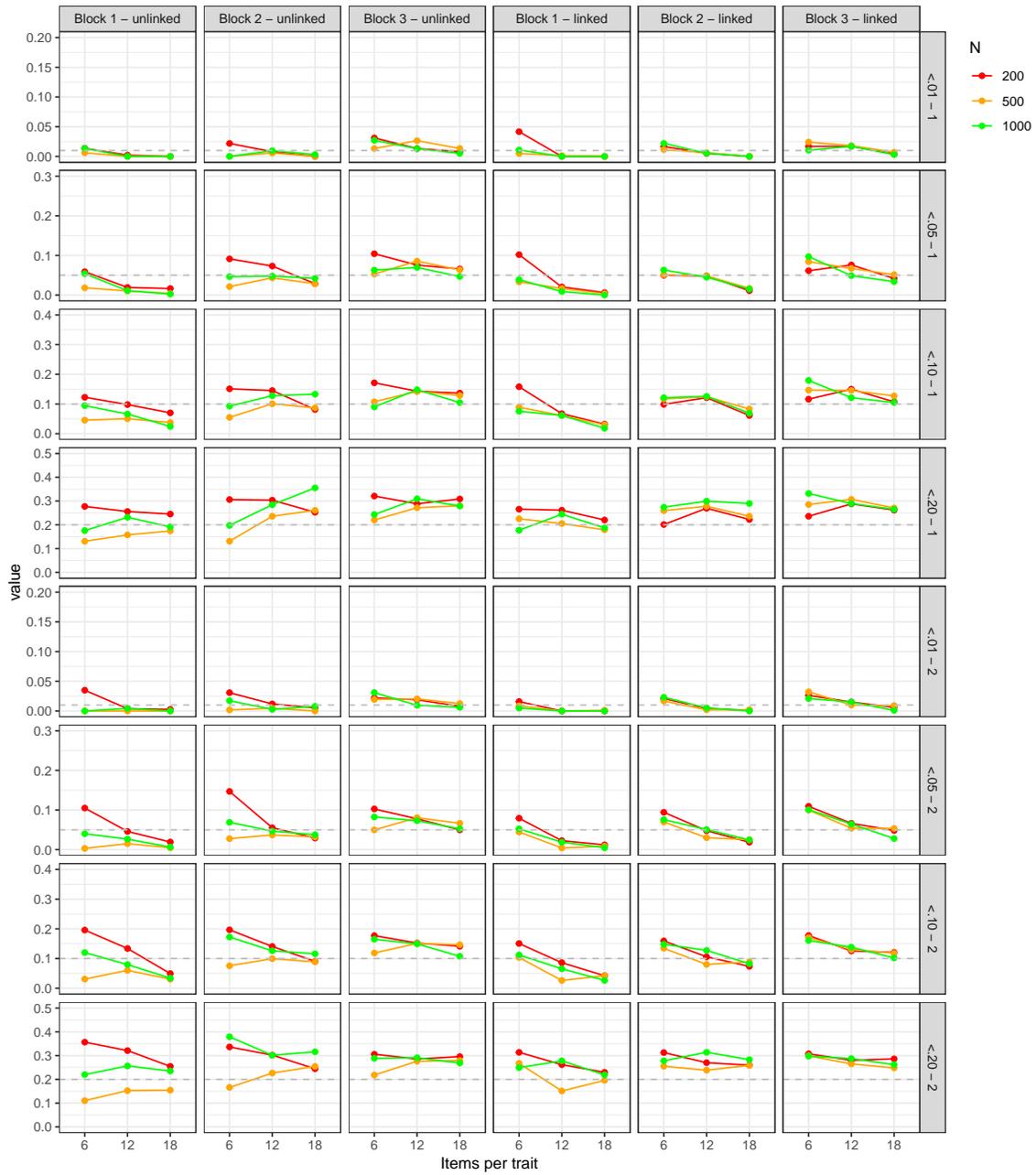
Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits.



Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Figure 14**

*Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 15**

*Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits.*

N	unlinked									linked									
	Block1			Block2			Block3			Block1			Block2			Block3			
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	
<.01 - 1	200	0.015	0.005	0.000	0.044	0.006	0.000	0.030	0.013	0.009	0.034	0.000	0.000	0.019	0.005	0.000	0.018	0.015	0.004
	500	0.006	0.000	0.000	0.000	0.007	0.000	0.016	0.027	0.013	0.006	0.000	0.000	0.012	0.005	0.000	0.023	0.017	0.007
	1000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.012	0.008	0.000	0.000	0.013	0.002	0.000	0.013	0.018	0.003
<.05 - 1	200	0.061	0.044	0.019	0.137	0.071	0.020	0.100	0.072	0.070	0.093	0.026	0.006	0.045	0.047	0.011	0.061	0.079	0.043
	500	0.017	0.014	0.004	0.025	0.046	0.029	0.055	0.085	0.064	0.039	0.016	0.004	0.052	0.049	0.016	0.078	0.066	0.052
	1000	0.500	0.023	0.013	0.000	0.000	0.051	0.031	0.045	0.045	0.059	0.010	0.000	0.051	0.043	0.017	0.113	0.051	0.033
<.10 - 1	200	0.091	0.126	0.063	0.211	0.135	0.071	0.168	0.141	0.141	0.158	0.072	0.032	0.091	0.118	0.063	0.112	0.148	0.105
	500	0.052	0.050	0.025	0.059	0.102	0.082	0.115	0.147	0.123	0.091	0.065	0.025	0.119	0.125	0.082	0.140	0.144	0.123
	1000	0.500	0.070	0.038	0.000	0.079	0.144	0.062	0.106	0.106	0.109	0.061	0.019	0.118	0.121	0.068	0.187	0.124	0.104
<.20 - 1	200	0.258	0.262	0.223	0.358	0.292	0.247	0.322	0.272	0.304	0.260	0.260	0.220	0.202	0.261	0.224	0.225	0.285	0.258
	500	0.172	0.146	0.154	0.137	0.235	0.250	0.222	0.266	0.275	0.230	0.209	0.169	0.253	0.285	0.231	0.274	0.305	0.270
	1000	0.500	0.186	0.275	0.000	0.316	0.333	0.219	0.272	0.210	0.239	0.168	0.263	0.300	0.291	0.312	0.285	0.265	0.265
<.01 - 2	200	0.018	0.010	0.000	0.031	0.014	0.000	0.021	0.016	0.003	0.011	0.000	0.000	0.022	0.004	0.000	0.026	0.013	0.006
	500	0.000	0.005	0.000	0.000	0.006	0.000	0.016	0.018	0.011	0.007	0.000	0.000	0.018	0.002	0.002	0.032	0.010	0.010
	1000	0.000	0.000	0.000	0.000	0.000	0.005	0.018	0.018	0.003	0.024	0.000	0.000	0.003	0.006	0.000	0.017	0.014	0.001
<.05 - 2	200	0.062	0.045	0.021	0.136	0.068	0.030	0.105	0.072	0.043	0.072	0.033	0.014	0.100	0.048	0.019	0.104	0.060	0.047
	500	0.006	0.014	0.005	0.030	0.039	0.034	0.048	0.071	0.067	0.041	0.004	0.010	0.071	0.029	0.028	0.097	0.055	0.049
	1000	0.000	0.000	0.000	0.043	0.038	0.043	0.054	0.054	0.056	0.065	0.021	0.003	0.052	0.045	0.024	0.081	0.059	0.028
<.10 - 2	200	0.124	0.119	0.049	0.176	0.147	0.096	0.197	0.142	0.134	0.121	0.105	0.046	0.164	0.102	0.083	0.171	0.116	0.115
	500	0.032	0.055	0.035	0.074	0.102	0.090	0.114	0.145	0.145	0.089	0.028	0.052	0.140	0.077	0.087	0.162	0.130	0.115
	1000	0.045	0.035	0.045	0.130	0.099	0.130	0.099	0.179	0.134	0.137	0.052	0.030	0.132	0.131	0.078	0.147	0.131	0.097
<.20 - 2	200	0.345	0.292	0.217	0.307	0.305	0.238	0.328	0.263	0.290	0.293	0.273	0.230	0.303	0.265	0.270	0.300	0.282	0.284
	500	0.102	0.164	0.167	0.171	0.231	0.252	0.205	0.268	0.277	0.260	0.152	0.199	0.256	0.230	0.261	0.288	0.263	0.243
	1000	0.364	0.259	0.304	0.310	0.304	0.310	0.446	0.281	0.274	0.280	0.218	0.260	0.331	0.282	0.309	0.285	0.262	0.262

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 16**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three uncorrelated traits.*

	N	unlinked												linked												
		Block1				Block2				Block3				Block1				Block2				Block3				
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6
$\chi^2 - 1$	200	125	585	1363	128	582	1360	127	580	1362	327	1384	3170	324	1381	3163	324	1379	3163	324	1379	3163	324	1379	3163	
	500	121	575	1358	120	578	1360	123	579	1359	322	1380	3167	322	1380	3167	322	1379	3167	322	1379	3167	322	1379	3165	
	1000	153	590	1387	123	598	1387	589	1379	296	1383	3175	296	1384	3175	296	1384	3175	298	1380	3167	298	1380	3167		
$\chi^2$ SD - 1	200	12	22	26	15	27	32	15	28	41	20	28	34	18	37	43	19	44	55	19	44	55	19	44	55	
	500	12	19	24	12	26	33	14	30	40	18	28	33	21	38	44	24	44	58	24	44	58	24	44	58	
	1000	25	21	22	3	19	31	22	38	16	25	26	18	34	40	21	40	50	21	40	50	21	40	50		
df - 1	200	126	579	1357	126	579	1357	126	579	1357	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	
	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	
	1000	132	591	1374	132	591	1374	591	1374	296	1376	3158	296	1376	3158	296	1376	3158	296	1376	3158	296	1376	3158		
Corrected df - 1	200	120	567	1339	120	567	1339	120	567	1339	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	
	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	
	1000	126	579	1356	126	579	1356	579	1356	287	1358	3131	287	1358	3131	287	1358	3131	287	1358	3131	287	1358	3131		
$\chi^2 - 2$	200	126	585	1366	127	584	1361	127	580	1360	325	1387	3175	325	1379	3169	324	1378	3165	324	1378	3165	324	1378	3165	
	500	119	577	1359	121	577	1360	123	579	1360	323	1374	3172	322	1376	3168	324	1376	3163	324	1376	3163	324	1376	3163	
	1000	599	1386	594	1385	594	1385	598	1379	298	1386	3176	297	1384	3175	297	1384	3175	297	1380	3165	297	1380	3165		
$\chi^2$ SD - 2	200	14	21	24	15	27	32	15	29	40	20	29	34	23	36	45	24	42	57	24	42	57	24	42	57	
	500	11	19	24	13	25	33	14	29	41	18	25	34	22	35	47	24	43	56	24	43	56	24	43	56	
	1000	18	19	19	26	31	26	31	28	37	16	26	28	17	36	41	21	40	51	21	40	51	21	40	51	
df - 2	200	126	579	1357	126	579	1357	126	579	1357	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	
	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	323	1376	3158	
	1000	591	1374	591	1374	591	1374	591	1374	296	1376	3158	296	1376	3158	296	1376	3158	296	1376	3158	296	1376	3158		
Corrected df - 2	200	120	567	1339	120	567	1339	120	567	1339	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	
	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	314	1358	3131	
	1000	579	1356	579	1356	579	1356	579	1356	287	1358	3131	287	1358	3131	287	1358	3131	287	1358	3131	287	1358	3131		

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

Table 17

*Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits.*

N	unlinked																		linked								
	Block1						Block2						Block3						Block1			Block2			Block3		
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
<.01 - 1	200	0.014	0.002	0.000	0.022	0.008	0.000	0.000	0.031	0.013	0.008	0.042	0.000	0.000	0.017	0.005	0.000	0.017	0.005	0.000	0.017	0.005	0.000	0.017	0.017	0.004	
500	0.006	0.000	0.000	0.000	0.006	0.000	0.013	0.026	0.013	0.005	0.001	0.000	0.012	0.006	0.000	0.024	0.018	0.007									
1000	0.014	0.000	0.000	0.000	0.010	0.003	0.027	0.013	0.005	0.011	0.000	0.000	0.022	0.005	0.000	0.010	0.017	0.003									
<.05 - 1	200	0.059	0.019	0.016	0.091	0.073	0.029	0.104	0.076	0.066	0.102	0.020	0.006	0.050	0.048	0.011	0.062	0.076	0.042								
500	0.018	0.011	0.004	0.021	0.044	0.029	0.053	0.086	0.064	0.033	0.016	0.004	0.051	0.048	0.017	0.084	0.067	0.052									
1000	0.054	0.011	0.002	0.047	0.048	0.042	0.063	0.070	0.047	0.039	0.009	0.000	0.063	0.045	0.015	0.097	0.049	0.034									
<.10 - 1	200	0.123	0.098	0.070	0.151	0.145	0.081	0.171	0.143	0.137	0.158	0.067	0.032	0.099	0.121	0.061	0.116	0.150	0.107								
500	0.046	0.050	0.038	0.055	0.101	0.087	0.107	0.143	0.129	0.089	0.061	0.030	0.117	0.125	0.083	0.147	0.146	0.127									
1000	0.095	0.066	0.025	0.093	0.128	0.133	0.090	0.148	0.105	0.076	0.061	0.019	0.121	0.126	0.069	0.179	0.121	0.105									
<.20 - 1	200	0.277	0.256	0.245	0.306	0.304	0.253	0.321	0.288	0.309	0.266	0.261	0.220	0.201	0.270	0.222	0.236	0.288	0.262								
500	0.131	0.158	0.174	0.131	0.236	0.261	0.220	0.272	0.280	0.225	0.205	0.179	0.259	0.278	0.236	0.285	0.307	0.271									
1000	0.176	0.232	0.191	0.198	0.284	0.355	0.243	0.309	0.279	0.177	0.245	0.187	0.274	0.299	0.290	0.332	0.289	0.265									
<.01 - 2	200	0.035	0.004	0.003	0.031	0.012	0.005	0.022	0.019	0.008	0.016	0.000	0.000	0.021	0.004	0.001	0.026	0.015	0.006								
500	0.000	0.000	0.000	0.002	0.005	0.000	0.019	0.020	0.012	0.009	0.000	0.001	0.017	0.002	0.002	0.032	0.010	0.009									
1000	0.000	0.004	0.000	0.017	0.003	0.008	0.031	0.010	0.006	0.005	0.000	0.000	0.023	0.005	0.000	0.021	0.015	0.001									
<.05 - 2	200	0.105	0.046	0.019	0.147	0.055	0.029	0.103	0.078	0.050	0.079	0.022	0.012	0.094	0.048	0.019	0.109	0.066	0.048								
500	0.003	0.015	0.005	0.028	0.037	0.032	0.050	0.081	0.066	0.044	0.004	0.008	0.070	0.030	0.025	0.100	0.054	0.054									
1000	0.040	0.027	0.006	0.069	0.047	0.038	0.082	0.073	0.052	0.019	0.005	0.052	0.076	0.051	0.024	0.101	0.065	0.028									
<.10 - 2	200	0.196	0.134	0.049	0.197	0.141	0.090	0.177	0.152	0.141	0.151	0.086	0.042	0.159	0.106	0.073	0.178	0.126	0.121								
500	0.031	0.060	0.030	0.076	0.099	0.089	0.119	0.151	0.146	0.104	0.026	0.042	0.134	0.080	0.089	0.170	0.130	0.118									
1000	0.120	0.080	0.034	0.172	0.126	0.116	0.165	0.149	0.108	0.112	0.066	0.026	0.148	0.128	0.082	0.161	0.138	0.102									
<.20 - 2	200	0.357	0.321	0.255	0.336	0.302	0.245	0.306	0.286	0.296	0.314	0.262	0.230	0.313	0.270	0.260	0.308	0.286									
500	0.110	0.153	0.155	0.166	0.227	0.254	0.218	0.276	0.279	0.268	0.151	0.196	0.255	0.238	0.259	0.265	0.248										
1000	0.220	0.257	0.235	0.379	0.301	0.316	0.289	0.291	0.269	0.249	0.278	0.219	0.278	0.314	0.283	0.298	0.288	0.262									

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 18**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with three uncorrelated traits.*

	N	unlinked																		linked																	
		Block1						Block2						Block3						Block1						Block2						Block3					
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18						
$\chi^2 - 1$	200	119	571	1347	120	571	1343	121	568	1344	318	1364	314	3142	314	1361	315	314	1359	3135	314	1359	3135	314	1359	3135											
	500	114	564	1341	113	566	1342	116	567	1341	312	1361	311	3140	311	1361	313	312	1360	3137	312	1360	3137	312	1360	3137											
	1000	125	582	1365	125	583	1368	126	581	1359	286	1365	288	3144	288	1364	3145	290	1360	3137	290	1360	3137	290	1360	3137											
$\chi^2$ SD - 1	200	13	20	26	14	26	32	15	28	41	20	27	33	18	36	43	19	43	54	36	43	19	43	36	43	19	43	54									
	500	11	19	24	12	25	32	14	30	40	18	27	33	21	38	44	24	43	57	38	44	24	43	38	44	24	43	57									
	1000	12	18	20	11	25	32	13	29	38	15	25	27	18	34	40	20	40	50	34	40	20	40	34	40	20	40	50									
df - 1	200	120	567	1339	120	567	1339	120	567	1339	313	1356	313	3130	313	1356	3130	313	1356	3130	313	1356	3130	313	1356	3130											
	500	120	567	1338	120	567	1338	120	567	1338	313	1357	313	3130	313	1357	3130	313	1357	3130	313	1357	3130	313	1357	3130											
	1000	126	579	1356	126	579	1356	126	579	1356	287	1356	287	3128	287	1356	3128	287	1356	3128	287	1356	3128	287	1356	3128											
Corrected df - 1	200	114	555	1321	114	555	1321	114	555	1321	304	1338	304	3103	304	1338	3103	304	1338	3103	304	1338	3103	304	1338	3103											
	500	114	555	1320	114	555	1320	114	555	1320	304	1339	304	3103	304	1339	3103	304	1339	3103	304	1339	3103	304	1339	3103											
	1000	120	567	1338	120	567	1338	120	567	1338	278	1338	278	3101	278	1338	3101	278	1338	3101	278	1338	3101	278	1338	3101											
$\chi^2 - 2$	200	122	574	1348	122	571	1344	120	569	1343	316	1367	314	3146	314	1361	319	314	1359	3137	314	1359	3137	314	1359	3137											
	500	113	564	1339	114	565	1342	117	567	1342	314	1355	312	3142	312	1357	3140	314	1357	3136	312	1357	3136	312	1357	3136											
	1000	125	583	1366	128	581	1365	126	581	1360	288	1365	289	3145	289	1364	3145	289	1360	3135	289	1360	3135	289	1360	3135											
$\chi^2$ SD - 2	200	15	22	26	15	26	32	15	29	40	19	27	34	23	36	45	24	42	56	36	45	24	42	36	45	24	42	56									
	500	11	19	23	12	25	32	14	29	41	18	25	34	22	34	46	24	42	56	34	46	24	42	34	46	24	42	56									
	1000	11	19	22	13	25	33	14	29	37	16	26	28	19	36	41	21	41	51	36	41	21	41	36	41	21	41	51									
df - 2	200	120	567	1339	120	567	1339	120	567	1339	313	1357	313	3130	313	1357	3130	313	1357	3130	313	1357	3130	313	1357	3130											
	500	120	567	1338	120	567	1338	120	567	1338	313	1357	313	3130	313	1357	3130	313	1357	3130	313	1357	3130	313	1357	3130											
	1000	126	579	1356	126	579	1356	126	579	1356	287	1356	287	3128	287	1356	3128	287	1356	3128	287	1356	3128	287	1356	3128											
Corrected df - 2	200	114	555	1321	114	555	1321	114	555	1321	304	1339	304	3103	304	1339	3103	304	1339	3103	304	1339	3103	304	1339	3103											
	500	114	555	1320	114	555	1320	114	555	1320	304	1339	304	3103	304	1339	3103	304	1339	3103	304	1339	3103	304	1339	3103											
	1000	120	567	1338	120	567	1338	120	567	1338	278	1338	278	3101	278	1338	3101	278	1338	3101	278	1338	3101	278	1338	3101											

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

## 4 Simulation Study with three correlated traits

### 4.1 Conditions

1. Correlation of traits: correlated traits
2. Number of traits: three traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (18 vs. 36 vs. 54 total)

### 4.2 Definition of parameters

#### 4.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3
Trait 1	1.0	0.2	0.3
Trait 2	0.2	1.0	0.5
Trait 3	0.3	0.5	1.0

#### 4.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3
Items 1/2/3	0.65220	0.38612	0.65354
Items 4/5/6	0.21798	-0.56033	-0.19856
Items 7/8/9	0.59442	0.03511	0.46618
Items 10/11/12	0.66429	-0.90733	-0.85052
Items 13/14/15	0.83475	-0.85749	0.43123
Items 16/17/18	0.52782	-0.59783	-0.37149
Items 19/20/21	-0.67013	0.62960	-0.14779
Items 22/23/24	-0.53568	0.50710	-0.78256
Items 25/26/27	-0.05393	0.38237	-0.60397
Items 28/29/30	-0.95819	0.85352	0.30527
Items 31/32/33	-0.33310	-0.18085	-0.88838
Items 34/35/36	0.86469	0.48681	-0.61221
Items 37/38/39	0.09419	0.45168	0.69948
Items 40/41/42	-0.98371	-0.09699	-0.02899
Items 43/44/45	0.98126	-0.52369	0.23588
Items 46/47/48	-0.36027	0.75411	-0.88957
Items 49/50/51	-0.80400	0.04564	-0.06666
Items 52/53/54	-0.36997	0.23126	0.78271

#### 4.2.3 Matrix of loadings $\Lambda$ (one negatively keyed item)

	Trait 1	Trait 2	Trait 3
Items 1/2/3	-0.83025	-0.50260	-0.38240
Items 4/5/6	0.48222	0.84543	0.53571
Items 7/8/9	0.42997	0.40555	0.44442
Items 10/11/12	0.86891	0.62249	0.56245
Items 13/14/15	0.87654	0.86188	0.69397
Items 16/17/18	0.46127	0.78081	0.57389
Items 19/20/21	0.77608	0.72651	0.72443
Items 22/23/24	0.85391	0.60987	0.56380
Items 25/26/27	0.31874	0.82240	0.89856
Items 28/29/30	0.58987	0.60792	0.51911
Items 31/32/33	0.37888	0.59224	0.74367
Items 34/35/36	0.55469	0.73592	0.64431
Items 37/38/39	0.60338	0.54275	0.86486
Items 40/41/42	0.88784	0.37312	0.61986
Items 43/44/45	0.75505	0.85499	0.70387
Items 46/47/48	0.52191	0.89989	0.84919
Items 49/50/51	0.51131	0.75749	0.38001
Items 52/53/54	0.31936	0.66787	0.33693

#### 4.2.4 Matrix of loadings $\Lambda$ (two negatively keyed items)

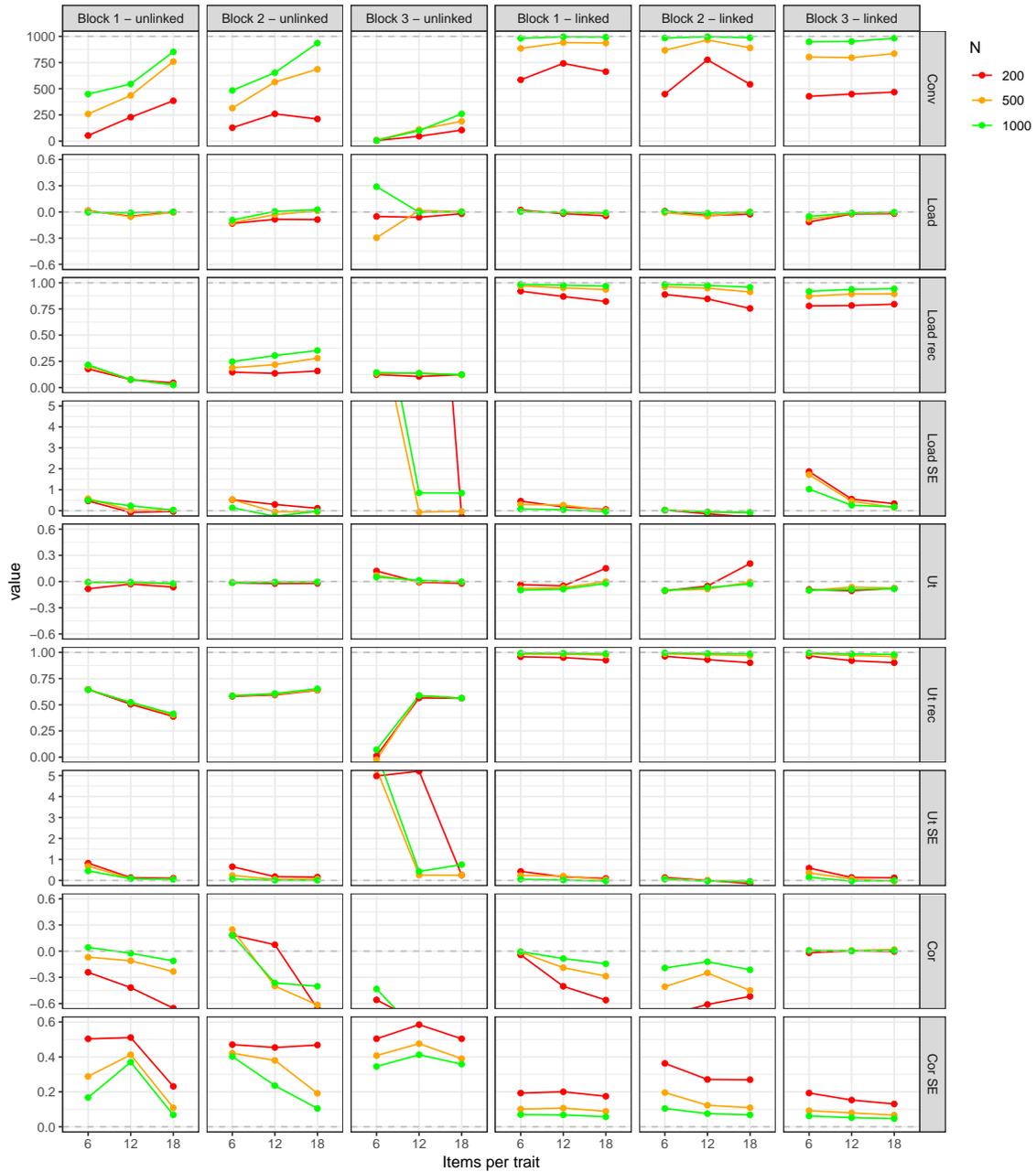
	Trait 1	Trait 2	Trait 3
Items 1/2/3	-0.83025	-0.50260	-0.38240
Items 4/5/6	-0.48222	-0.84543	-0.53571
Items 7/8/9	0.42997	0.40555	0.44442
Items 10/11/12	0.86891	0.62249	0.56245
Items 13/14/15	0.87654	0.86188	0.69397
Items 16/17/18	0.46127	0.78081	0.57389
Items 19/20/21	0.77608	0.72651	0.72443
Items 22/23/24	0.85391	0.60987	0.56380
Items 25/26/27	0.31874	0.82240	0.89856
Items 28/29/30	0.58987	0.60792	0.51911
Items 31/32/33	0.37888	0.59224	0.74367
Items 34/35/36	0.55469	0.73592	0.64431
Items 37/38/39	0.60338	0.54275	0.86486
Items 40/41/42	0.88784	0.37312	0.61986
Items 43/44/45	0.75505	0.85499	0.70387
Items 46/47/48	0.52191	0.89989	0.84919
Items 49/50/51	0.51131	0.75749	0.38001
Items 52/53/54	0.31936	0.66787	0.33693

## 4.3 Results

### 4.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 15

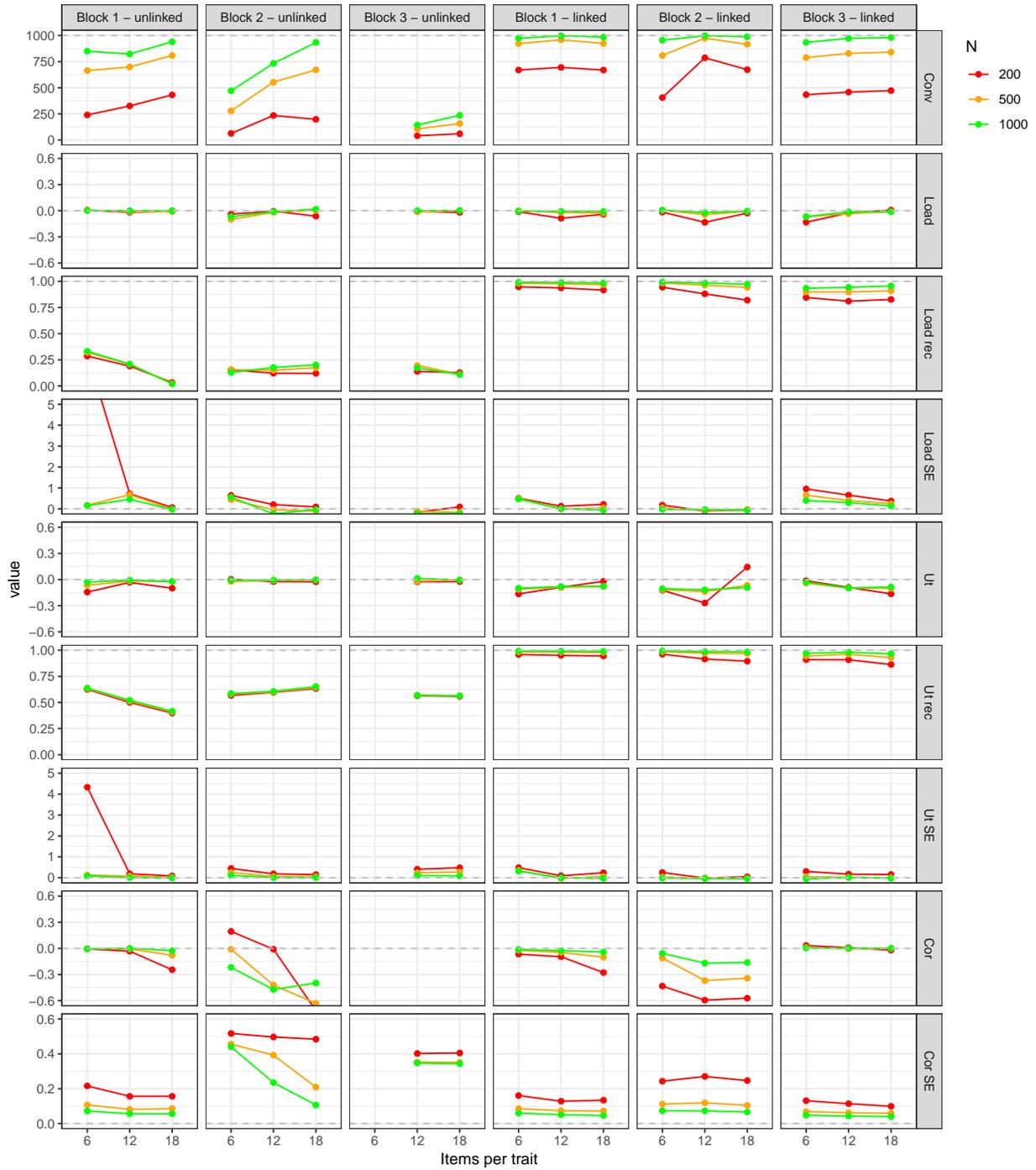
*Bias results for the Thurstonian factor model with three correlated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 16**

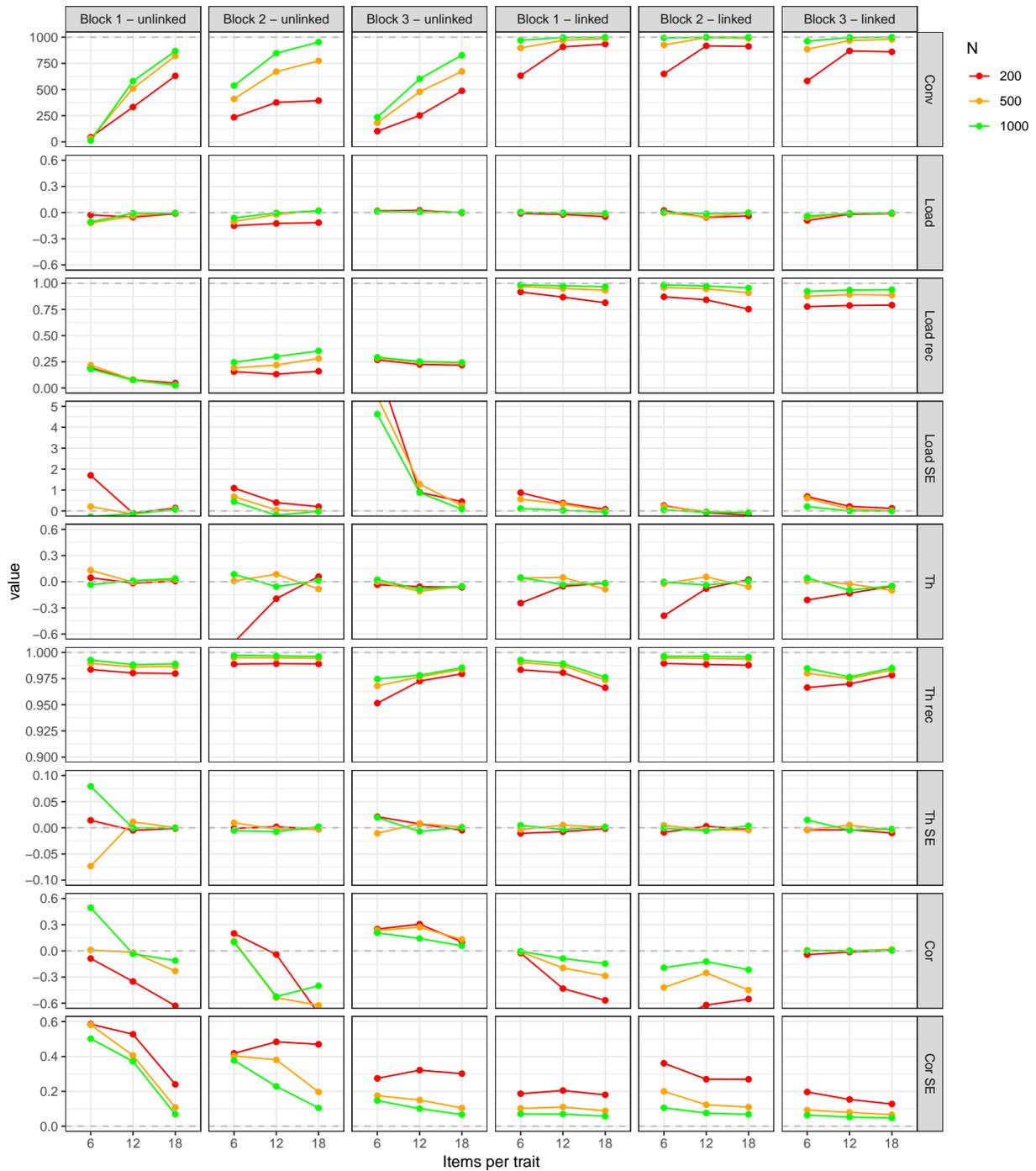
*Bias results for the Thurstonian factor model with three correlated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 17**

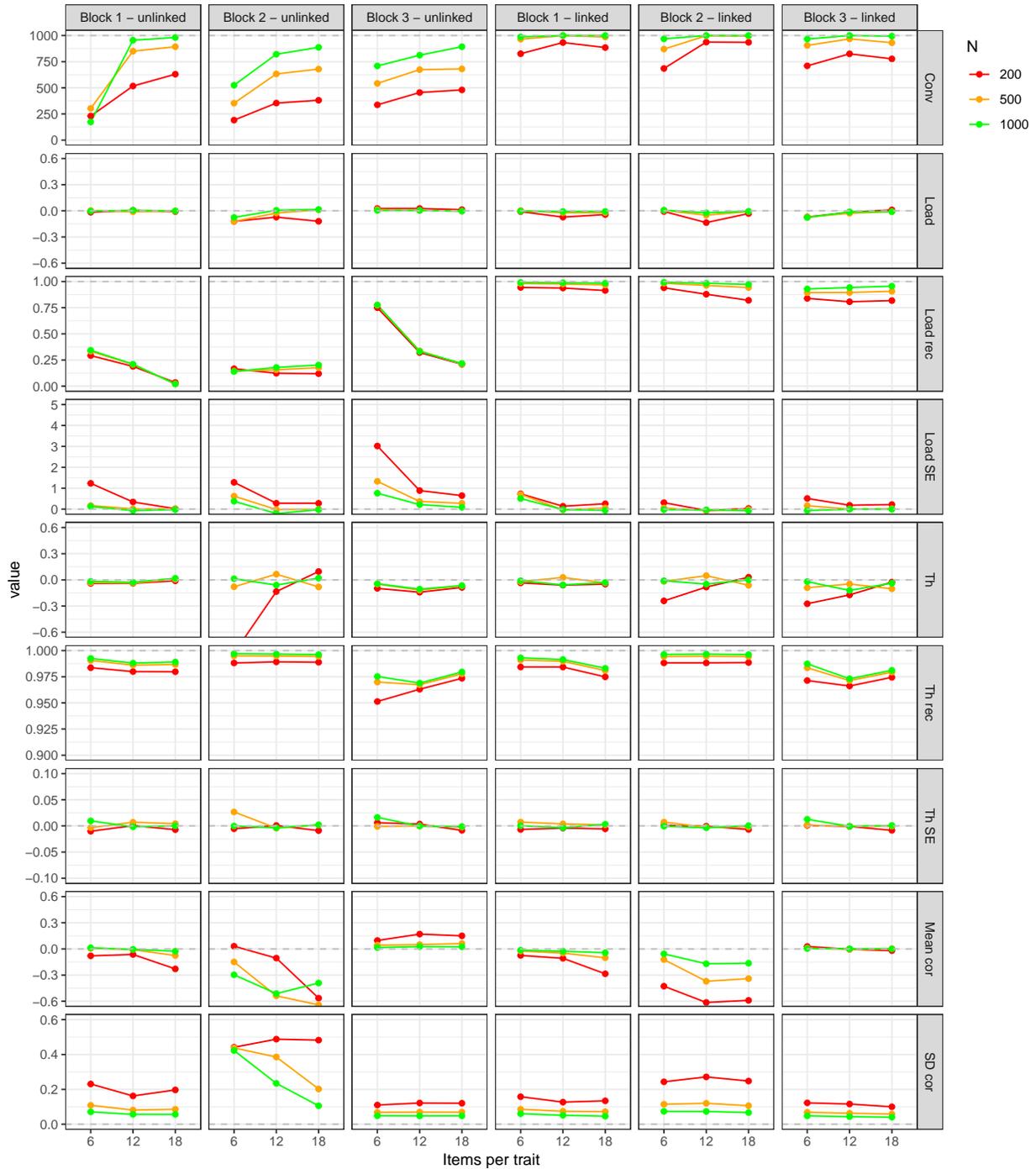
*Bias results for the Thurstonian IRT model with three correlated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Figure 18**

*Bias results for the Thurstonian IRT model with three correlated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 19**

*Bias results for the Thurstonian factor model with three correlated traits. One negatively keyed item.*

	unlinked									linked								
	Block1			Block2			Block3			Block1			Block2			Block3		
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
N	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000
Conv	53	228	385	128	260	211	6	46	105	585	742	663	449	776	542	428	449	468
	259	436	759	315	564	686	9	111	189	885	941	936	867	966	890	803	797	835
	449	545	852	484	653	936	6	100	260	981	996	992	984	996	987	948	951	982
Load	0.01	-0.05	0.00	-0.13	-0.08	-0.09	-0.05	-0.06	-0.02	0.02	-0.02	-0.04	0.01	-0.04	-0.03	-0.12	-0.02	-0.02
	0.02	-0.05	-0.01	-0.12	-0.03	0.01	-0.30	0.02	0.01	0.01	-0.01	-0.02	-0.01	-0.05	0.00	-0.08	-0.02	-0.01
	-0.01	-0.01	0.00	-0.09	0.01	0.03	0.29	0.00	0.00	0.00	0.00	-0.01	0.01	-0.01	0.00	-0.05	-0.01	0.00
Load rec	0.18	0.08	0.04	0.15	0.14	0.16	0.12	0.11	0.12	0.92	0.87	0.82	0.89	0.85	0.75	0.78	0.78	0.80
	0.20	0.08	0.03	0.19	0.22	0.28	0.13	0.14	0.12	0.97	0.95	0.94	0.96	0.95	0.91	0.87	0.89	0.89
	0.22	0.08	0.02	0.25	0.31	0.35	0.14	0.14	0.12	0.98	0.98	0.97	0.98	0.97	0.96	0.92	0.94	0.94
Load SE	0.47	-0.08	-0.03	0.52	0.30	0.12	6.22	25.73	-0.28	0.46	0.18	0.06	0.03	-0.14	-0.30	1.87	0.56	0.34
	0.58	0.03	0.03	0.53	-0.05	-0.05	9.49	-0.06	-0.03	0.29	0.27	0.02	0.04	-0.05	-0.09	1.72	0.45	0.16
	0.50	0.24	0.03	0.14	-0.26	-0.02	11.38	0.85	0.84	0.08	0.05	-0.04	0.03	-0.06	-0.08	1.03	0.26	0.18
Ut	-0.08	-0.03	-0.06	-0.01	-0.02	-0.02	0.12	-0.01	-0.02	-0.04	-0.05	0.15	-0.11	-0.05	0.21	-0.09	-0.11	-0.08
	-0.01	-0.01	-0.03	-0.01	-0.01	-0.01	0.07	0.00	-0.01	-0.08	-0.07	0.00	-0.10	-0.09	-0.01	-0.10	-0.06	-0.07
	-0.01	-0.01	-0.02	-0.01	-0.01	0.00	0.05	0.01	0.00	-0.10	-0.09	-0.02	-0.10	-0.07	-0.03	-0.10	-0.09	-0.08
Ut rec	0.65	0.51	0.39	0.58	0.59	0.64	0.01	0.57	0.56	0.96	0.95	0.93	0.96	0.93	0.90	0.97	0.92	0.90
	0.65	0.52	0.40	0.59	0.60	0.64	-0.02	0.59	0.56	0.98	0.98	0.97	0.99	0.98	0.97	0.99	0.97	0.96
	0.64	0.52	0.41	0.59	0.61	0.65	0.07	0.59	0.56	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98
Ut SE	0.82	0.13	0.10	0.65	0.18	0.16	4.98	5.21	0.25	0.43	0.17	0.09	0.14	0.00	-0.16	0.59	0.14	0.12
	0.68	0.09	0.06	0.23	0.05	0.07	5.31	0.25	0.25	0.23	0.20	0.04	0.08	0.00	-0.06	0.36	0.07	-0.04
	0.45	0.08	0.05	0.07	0.01	0.01	6.12	0.42	0.75	0.06	0.02	-0.03	0.06	-0.04	-0.05	0.15	-0.03	0.00
Cor	-0.24	-0.42	-0.65	0.18	0.07	-0.66	-0.56	-0.81	-0.95	-0.04	-0.40	-0.56	-0.74	-0.61	-0.52	-0.02	0.00	0.00
	-0.07	-0.11	-0.23	0.25	-0.40	-0.62	-0.69	-0.92	-0.96	-0.01	-0.19	-0.29	-0.41	-0.25	-0.45	0.01	0.01	0.02
	0.04	-0.02	-0.11	0.18	-0.36	-0.40	-0.43	-0.94	-0.98	-0.01	-0.08	-0.14	-0.19	-0.12	-0.21	0.01	0.00	0.01
Cor SE	0.50	0.51	0.23	0.47	0.45	0.47	0.50	0.58	0.50	0.19	0.20	0.17	0.36	0.27	0.27	0.19	0.15	0.13
	0.29	0.41	0.11	0.42	0.38	0.19	0.41	0.48	0.39	0.10	0.11	0.09	0.20	0.12	0.11	0.09	0.08	0.07
	0.17	0.37	0.07	0.40	0.24	0.10	0.35	0.41	0.36	0.07	0.07	0.06	0.10	0.08	0.07	0.06	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 20**

*Bias results for the Thurstonian factor model with three correlated traits. Two negatively keyed items.*

	unlinked															linked																																							
	Block1					Block2					Block3					Block1					Block2					Block3																													
	N	6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18																							
Conv	200	240	326	432	63	235	198	41	60	670	695	670	406	787	673	434	458	473	500	664	699	809	279	554	672	106	158	922	959	924	809	974	916	789	829	841	1000	850	823	940	471	734	934	144	237	974	997	985	955	998	988	934	973	980	
Load	200	0.01	-0.02	0.00	-0.04	-0.01	-0.06	-0.01	-0.02	-0.01	-0.09	-0.04	-0.02	-0.13	-0.03	-0.13	-0.02	0.01	500	0.01	-0.01	-0.10	-0.02	0.00	0.00	-0.01	0.00	0.00	-0.02	-0.03	0.01	-0.05	-0.01	-0.07	-0.03	-0.01	1000	0.00	0.00	0.00	-0.07	-0.01	0.02	0.00	0.00	-0.01	-0.01	-0.01	0.01	-0.02	-0.01	-0.07	-0.01	-0.01	-0.01
Load rec	200	0.29	0.19	0.03	0.15	0.12	0.12	0.14	0.13	0.95	0.94	0.92	0.94	0.88	0.82	0.85	0.81	0.83	500	0.32	0.21	0.02	0.16	0.15	0.17	0.20	0.11	0.98	0.98	0.97	0.98	0.96	0.94	0.90	0.90	0.91	1000	0.33	0.21	0.02	0.13	0.18	0.20	0.17	0.11	0.99	0.99	0.98	0.99	0.98	0.97	0.93	0.94	0.96	
Load SE	200	7.42	0.72	0.07	0.65	0.20	0.10	-0.16	0.09	0.51	0.13	0.22	0.18	-0.10	-0.04	0.96	0.66	0.38	500	0.18	0.68	-0.01	0.44	-0.04	-0.11	-0.12	-0.16	0.50	-0.01	0.04	0.04	-0.08	-0.03	0.65	0.40	0.24	1000	0.16	0.46	-0.02	0.56	-0.24	-0.03	-0.22	-0.21	0.47	0.01	-0.06	-0.02	-0.03	-0.07	0.40	0.30	0.14	
Ut	200	-0.14	-0.03	-0.10	0.00	-0.02	-0.03	-0.03	-0.02	-0.16	-0.09	-0.02	-0.12	-0.27	0.14	-0.01	-0.09	-0.16	500	-0.07	-0.02	-0.02	-0.02	-0.01	-0.01	-0.02	-0.01	-0.11	-0.09	-0.08	-0.12	-0.14	-0.07	-0.04	-0.10	-0.10	1000	-0.03	-0.01	-0.02	-0.01	-0.01	0.00	0.01	0.00	-0.10	-0.08	-0.08	-0.10	-0.12	-0.09	-0.03	-0.10	-0.09	
Ut rec	200	0.62	0.50	0.40	0.57	0.60	0.63	0.56	0.56	0.96	0.95	0.94	0.96	0.92	0.90	0.91	0.91	0.86	500	0.63	0.51	0.41	0.58	0.60	0.64	0.57	0.56	0.98	0.98	0.98	0.99	0.97	0.97	0.94	0.96	0.93	1000	0.64	0.52	0.42	0.59	0.61	0.65	0.57	0.56	0.99	0.99	0.99	0.99	0.99	0.98	0.97	0.98	0.97	
Ut SE	200	4.33	0.19	0.09	0.44	0.18	0.15	0.40	0.48	0.48	0.09	0.24	0.25	-0.02	0.04	0.30	0.17	0.15	500	0.12	0.08	0.02	0.25	0.05	0.05	0.24	0.27	0.35	-0.03	0.05	0.03	-0.02	-0.03	0.05	0.02	0.00	1000	0.08	0.01	-0.01	0.11	0.01	0.01	0.11	0.08	0.31	0.00	-0.03	-0.02	-0.04	-0.04	-0.05	0.01	-0.02	
Cor	200	-0.01	-0.03	-0.25	0.20	-0.01	-0.71	-0.80	-0.96	-0.07	-0.10	-0.28	-0.43	-0.59	-0.57	0.03	0.01	-0.02	500	0.00	-0.01	-0.08	-0.01	-0.42	-0.63	-0.90	-0.96	-0.02	-0.05	-0.10	-0.11	-0.37	-0.34	0.01	0.00	0.00	1000	0.00	0.00	-0.03	-0.22	-0.47	-0.40	-0.95	-0.95	-0.01	-0.03	-0.04	-0.06	-0.17	-0.16	0.00	0.00	0.00	
Cor SE	200	0.22	0.16	0.16	0.52	0.50	0.48	0.40	0.41	0.16	0.13	0.13	0.24	0.27	0.25	0.13	0.11	0.10	500	0.11	0.08	0.09	0.46	0.39	0.21	0.35	0.35	0.09	0.07	0.07	0.11	0.12	0.10	0.07	0.06	0.06	1000	0.07	0.06	0.06	0.44	0.24	0.11	0.35	0.34	0.06	0.05	0.05	0.07	0.07	0.07	0.05	0.04	0.04	

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 21**

*Bias results for the Thurstonian IRT model with three correlated traits. One negatively keyed item.*

	unlinked									linked									
	Block1			Block2			Block3			Block1			Block2			Block3			
	N	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
Conv	200	44	332	630	234	376	394	102	252	487	632	907	934	649	917	912	582	869	860
	500	32	511	821	410	671	774	183	478	673	897	970	986	926	996	988	885	968	979
	1000	12	580	868	537	847	953	235	602	828	971	997	999	993	1000	999	962	997	999
Load	200	-0.03	-0.05	-0.01	-0.15	-0.12	-0.12	0.02	0.02	0.00	-0.01	-0.02	-0.05	0.02	-0.05	-0.04	-0.09	-0.02	-0.01
	500	-0.12	-0.04	0.00	-0.10	-0.02	0.02	0.02	0.01	0.00	0.01	-0.01	-0.02	0.00	-0.05	0.00	-0.06	-0.01	-0.01
	1000	-0.11	-0.01	-0.01	-0.06	0.00	0.02	0.01	0.01	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.04	-0.01	0.00
Load rec	200	0.19	0.08	0.05	0.16	0.13	0.16	0.27	0.22	0.22	0.92	0.87	0.81	0.87	0.84	0.75	0.78	0.79	0.79
	500	0.22	0.08	0.03	0.19	0.22	0.28	0.28	0.25	0.24	0.97	0.95	0.93	0.96	0.95	0.91	0.88	0.89	0.89
	1000	0.18	0.07	0.02	0.25	0.30	0.35	0.29	0.25	0.24	0.98	0.98	0.97	0.98	0.97	0.96	0.92	0.93	0.94
Load SE	200	1.70	-0.11	0.14	1.09	0.40	0.20	6.96	0.89	0.45	0.88	0.38	0.08	0.26	-0.08	-0.20	0.69	0.22	0.13
	500	0.21	-0.15	0.10	0.68	0.05	-0.01	5.44	1.28	0.26	0.57	0.33	0.01	0.22	-0.02	-0.09	0.61	0.09	0.02
	1000	-0.27	-0.13	0.07	0.45	-0.20	-0.02	4.62	0.88	0.08	0.12	0.03	-0.05	0.05	-0.06	-0.08	0.21	0.00	0.00
Th	200	0.05	-0.02	0.01	-0.69	-0.19	0.06	-0.03	-0.06	-0.06	-0.25	-0.05	-0.02	-0.39	-0.08	0.02	-0.21	-0.13	-0.05
	500	0.13	-0.01	0.02	0.01	0.09	-0.08	0.00	-0.11	-0.05	0.04	0.05	-0.09	-0.02	0.06	-0.06	0.01	-0.02	-0.10
	1000	-0.03	0.01	0.04	0.09	-0.06	0.01	0.02	-0.09	-0.05	0.05	-0.03	-0.02	0.00	-0.04	0.01	0.04	-0.10	-0.05
Th rec	200	0.98	0.98	0.98	0.99	0.99	0.99	0.95	0.97	0.98	0.98	0.98	0.97	0.99	0.99	0.99	0.97	0.97	0.98
	500	0.99	0.99	0.99	1.00	0.99	0.99	0.97	0.98	0.98	0.99	0.99	0.97	0.99	0.99	0.99	0.98	0.98	0.98
	1000	0.99	0.99	0.99	1.00	1.00	1.00	0.97	0.98	0.99	0.99	0.99	0.98	1.00	1.00	1.00	0.98	0.98	0.99
Th SE	200	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.00	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01
	500	-0.07	0.01	0.00	0.01	0.00	0.00	-0.01	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.00
	1000	0.08	0.00	0.00	-0.01	-0.01	0.00	0.02	-0.01	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.01	0.00	0.00
Cor	200	-0.09	-0.35	-0.63	0.20	-0.04	-0.72	0.25	0.31	0.10	-0.02	-0.43	-0.57	-0.82	-0.62	-0.55	-0.04	-0.01	0.01
	500	0.01	-0.02	-0.23	0.11	-0.54	-0.62	0.24	0.27	0.13	-0.01	-0.20	-0.29	-0.42	-0.25	-0.45	0.01	0.00	0.02
	1000	0.49	-0.03	-0.11	0.10	-0.52	-0.40	0.21	0.14	0.06	0.00	-0.09	-0.15	-0.19	-0.12	-0.22	0.00	0.00	0.01
Cor SE	200	0.59	0.53	0.24	0.42	0.48	0.47	0.27	0.32	0.30	0.19	0.20	0.18	0.36	0.27	0.27	0.20	0.15	0.13
	500	0.58	0.41	0.11	0.40	0.38	0.20	0.18	0.15	0.10	0.10	0.11	0.09	0.20	0.12	0.11	0.09	0.08	0.07
	1000	0.50	0.37	0.07	0.38	0.23	0.10	0.15	0.10	0.07	0.07	0.07	0.06	0.11	0.08	0.07	0.06	0.05	0.05

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 22**

*Bias results for the Thurstonian IRT model with three correlated traits. Two negatively keyed items.*

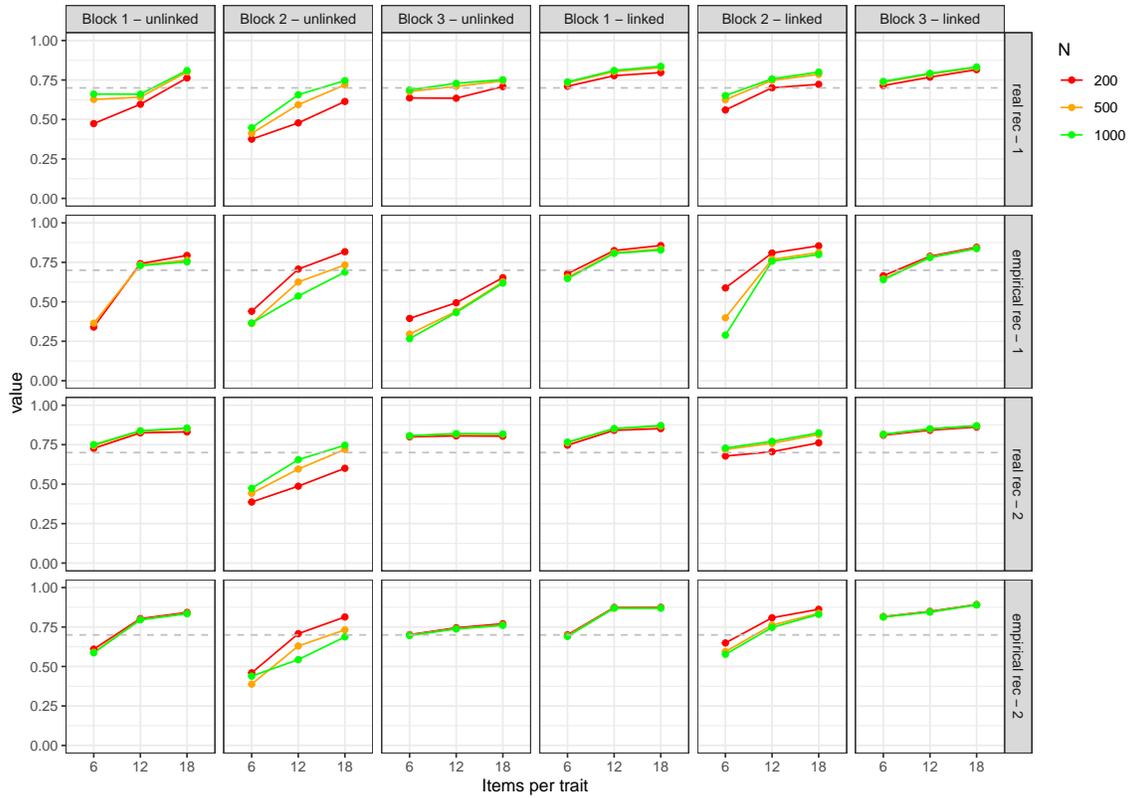
	unlinked									linked									
	Block1			Block2			Block3			Block1			Block2			Block3			
	N	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
Conv	200	230	517	630	191	354	381	337	455	480	826	933	885	685	937	935	710	825	777
	500	303	850	893	353	633	679	542	674	681	965	999	986	870	998	994	905	968	931
	1000	173	955	981	525	821	887	709	812	893	987	1000	1000	968	1000	1000	966	1000	994
Load	200	-0.02	0.01	-0.01	-0.12	-0.07	-0.12	0.03	0.03	0.01	-0.01	-0.07	-0.04	-0.01	-0.14	-0.03	-0.07	-0.02	0.01
	500	0.00	-0.01	0.00	-0.12	-0.03	0.01	0.01	0.00	0.00	0.00	-0.02	-0.02	0.01	-0.05	-0.01	-0.07	-0.03	-0.01
	1000	0.00	0.01	0.00	-0.08	0.01	0.02	0.00	0.01	0.00	0.00	-0.01	-0.01	0.01	-0.02	-0.01	-0.08	-0.01	-0.01
Load rec	200	0.29	0.19	0.03	0.17	0.12	0.12	0.75	0.32	0.21	0.94	0.94	0.91	0.94	0.88	0.82	0.84	0.81	0.82
	500	0.33	0.21	0.02	0.14	0.16	0.18	0.77	0.34	0.21	0.98	0.98	0.97	0.98	0.96	0.94	0.89	0.90	0.91
	1000	0.34	0.21	0.02	0.14	0.18	0.20	0.78	0.33	0.22	0.99	0.99	0.98	0.99	0.98	0.97	0.93	0.94	0.96
Load SE	200	1.24	0.34	0.02	1.28	0.28	0.28	3.02	0.89	0.65	0.73	0.14	0.26	0.31	-0.06	0.03	0.51	0.19	0.22
	500	0.17	0.02	0.02	0.62	-0.02	-0.01	1.33	0.37	0.28	0.70	-0.03	0.05	0.06	-0.07	-0.02	0.17	0.00	0.02
	1000	0.13	-0.07	-0.02	0.38	-0.21	-0.02	0.76	0.22	0.09	0.51	-0.02	-0.06	-0.02	-0.03	-0.07	0.00	0.00	0.00
Th	200	-0.04	-0.04	-0.01	-0.84	-0.13	0.10	-0.10	-0.14	-0.09	-0.03	-0.06	-0.05	-0.24	-0.08	0.03	-0.27	-0.17	-0.03
	500	-0.03	-0.04	0.01	-0.08	0.07	-0.08	-0.05	-0.12	-0.07	-0.02	0.03	-0.04	-0.02	0.05	-0.06	-0.09	-0.05	-0.10
	1000	-0.02	-0.03	0.02	0.01	-0.06	0.02	-0.04	-0.11	-0.07	-0.01	-0.06	-0.03	-0.01	-0.05	0.00	-0.02	-0.12	-0.04
Th rec	200	0.98	0.98	0.98	0.99	0.99	0.99	0.95	0.96	0.97	0.98	0.98	0.97	0.99	0.99	0.99	0.97	0.97	0.97
	500	0.99	0.99	0.99	1.00	0.99	0.99	0.97	0.97	0.98	0.99	0.99	0.98	0.99	0.99	0.99	0.98	0.97	0.98
	1000	0.99	0.99	0.99	1.00	1.00	1.00	0.98	0.97	0.98	0.99	0.99	0.98	1.00	1.00	1.00	0.99	0.97	0.98
Th SE	200	-0.01	0.00	-0.01	-0.01	0.00	-0.01	0.01	0.00	-0.01	-0.01	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	-0.01
	500	0.00	0.01	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	1000	0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Cor	200	-0.08	-0.06	-0.23	0.03	-0.11	-0.56	0.10	0.17	0.15	-0.07	-0.11	-0.29	-0.43	-0.61	-0.59	0.03	0.00	-0.02
	500	0.01	-0.01	-0.08	-0.15	-0.54	-0.64	0.04	0.05	0.06	-0.02	-0.05	-0.10	-0.12	-0.37	-0.34	0.01	0.00	0.00
	1000	0.01	0.00	-0.03	-0.30	-0.51	-0.39	0.02	0.03	0.02	-0.01	-0.03	-0.04	-0.06	-0.17	-0.16	0.00	0.00	0.00
Cor SE	200	0.23	0.16	0.20	0.44	0.49	0.48	0.11	0.12	0.12	0.16	0.13	0.13	0.24	0.27	0.25	0.12	0.12	0.10
	500	0.11	0.08	0.09	0.44	0.39	0.20	0.07	0.07	0.07	0.09	0.07	0.07	0.11	0.12	0.11	0.07	0.06	0.06
	1000	0.07	0.06	0.06	0.42	0.23	0.11	0.05	0.05	0.05	0.06	0.05	0.05	0.07	0.07	0.07	0.05	0.04	0.04

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 4.3.2 Results for latent trait recovery

Figure 19

*Latent trait recovery results with three correlated traits.*



*Note.* Abbreviations: rec: recovery. The first two rows come from simulation with one (- 1), the second two rows from simulation with two (- 2) negatively keyed items per trait.

**Table 23**

*Latent trait recovery results with three correlated traits.*

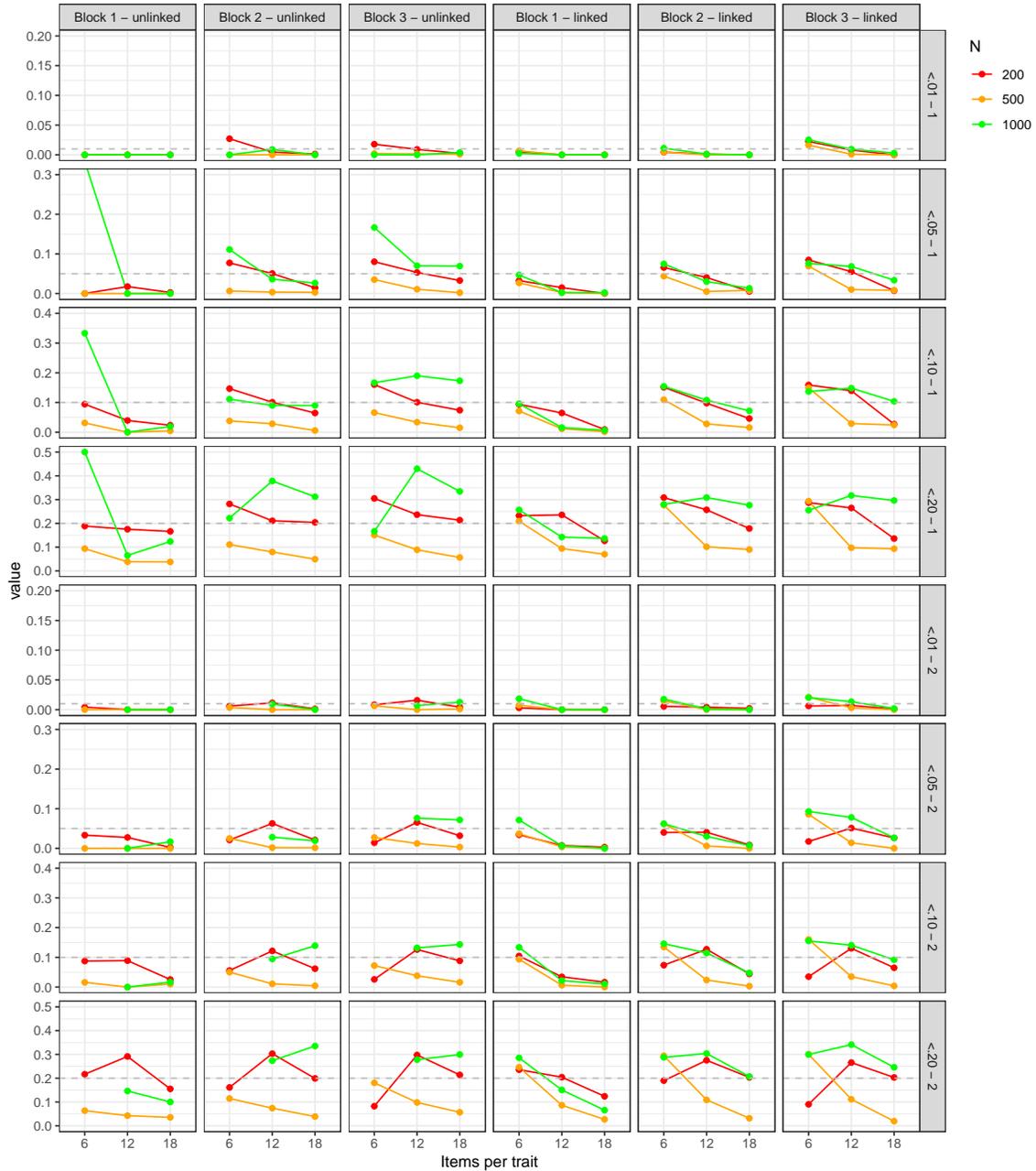
	N	unlinked									linked								
		Block1			Block2			Block3			Block1			Block2			Block3		
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
real rec - 1	200	0.474	0.596	0.763	0.375	0.478	0.614	0.636	0.635	0.708	0.710	0.777	0.797	0.560	0.701	0.722	0.715	0.768	0.816
	500	0.627	0.641	0.801	0.412	0.593	0.719	0.677	0.710	0.744	0.733	0.803	0.828	0.625	0.748	0.786	0.736	0.786	0.829
	1000	0.661	0.660	0.810	0.447	0.657	0.746	0.687	0.729	0.752	0.738	0.810	0.837	0.651	0.757	0.800	0.741	0.792	0.832
empirical rec - 1	200	0.340	0.742	0.794	0.440	0.707	0.817	0.395	0.494	0.653	0.678	0.825	0.856	0.588	0.809	0.855	0.665	0.789	0.845
	500	0.364	0.734	0.763	0.362	0.626	0.733	0.296	0.440	0.626	0.655	0.811	0.835	0.399	0.769	0.812	0.645	0.783	0.839
	1000	0.729	0.753	0.366	0.536	0.688	0.266	0.432	0.619	0.647	0.807	0.828	0.289	0.757	0.799	0.640	0.780	0.837	
real rec - 2	200	0.727	0.825	0.831	0.387	0.487	0.600	0.800	0.806	0.804	0.747	0.841	0.852	0.678	0.705	0.762	0.810	0.841	0.862
	500	0.744	0.835	0.851	0.442	0.596	0.720	0.805	0.817	0.815	0.762	0.850	0.867	0.720	0.758	0.814	0.815	0.848	0.869
	1000	0.750	0.838	0.855	0.474	0.655	0.746	0.807	0.820	0.818	0.766	0.852	0.871	0.729	0.771	0.825	0.816	0.851	0.870
empirical rec - 2	200	0.610	0.803	0.842	0.460	0.708	0.814	0.701	0.745	0.771	0.700	0.874	0.875	0.649	0.809	0.862	0.816	0.849	0.892
	500	0.590	0.798	0.836	0.388	0.630	0.732	0.698	0.740	0.763	0.694	0.871	0.871	0.595	0.762	0.837	0.814	0.845	0.891
	1000	0.587	0.796	0.834	0.439	0.543	0.687	0.696	0.738	0.761	0.690	0.870	0.869	0.577	0.747	0.830	0.813	0.845	0.890

*Note.* Abbreviations: rec: recovery.

### 4.3.3 Results for empirical rejection rates

Figure 20

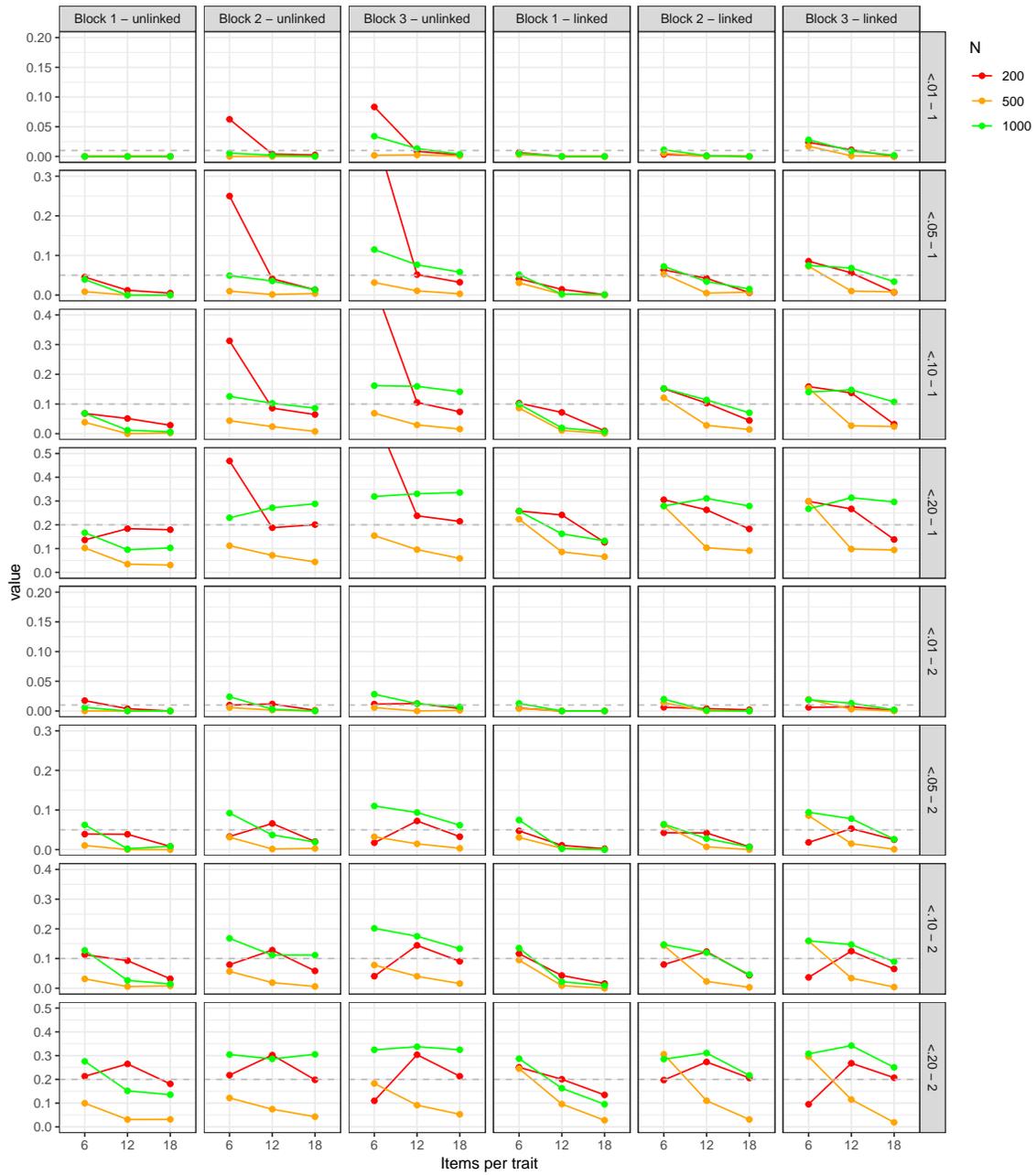
Empirical rejection rates for the Thurstonian factor model with three correlated traits.



Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Figure 21**

*Empirical rejection rates for the Thurstonian IRT model with three correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 24**

*Empirical rejection rates for the Thurstonian factor model with three correlated traits.*

	unlinked																		linked																																																																																																																																																																																																																																																																																																																																																																																																																																																																	
	Block1						Block2						Block3						Block1						Block2						Block3																																																																																																																																																																																																																																																																																																																																																																																																																																																					
	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18	N	6	12	18	6	12	18																																																																																																																																																																																																																																																																																																																																																																																																																																																	
<.01 - 1	200	0.000	0.000	0.000	0.027	0.005	0.001	0.018	0.009	0.002	0.005	0.000	0.000	0.000	0.005	0.001	0.000	0.000	0.000	0.005	0.001	0.000	0.000	0.022	0.008	0.000	500	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.007	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.016	0.001	0.000	1000	0.000	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.004	0.002	0.000	0.000	0.011	0.001	0.000	0.000	0.025	0.009	0.003	<.05 - 1	200	0.000	0.018	0.003	0.077	0.050	0.014	0.080	0.053	0.033	0.032	0.015	0.000	0.066	0.040	0.005	0.085	0.055	0.007	500	0.000	0.000	0.000	0.006	0.004	0.003	0.035	0.011	0.002	0.027	0.003	0.000	0.044	0.005	0.008	0.069	0.010	0.008	1000	0.333	0.000	0.000	0.111	0.036	0.026	0.167	0.070	0.069	0.047	0.002	0.002	0.075	0.030	0.013	0.076	0.068	0.034	<.10 - 1	200	0.094	0.039	0.023	0.147	0.101	0.065	0.160	0.101	0.074	0.094	0.065	0.009	0.151	0.098	0.046	0.159	0.140	0.027	500	0.031	0.000	0.005	0.038	0.028	0.006	0.066	0.034	0.015	0.071	0.012	0.002	0.110	0.028	0.016	0.149	0.029	0.024	1000	0.333	0.000	0.000	0.019	0.111	0.090	0.090	0.167	0.190	0.173	0.096	0.016	0.006	0.154	0.108	0.072	0.137	0.148	0.104	<.20 - 1	200	0.189	0.175	0.166	0.282	0.211	0.204	0.305	0.237	0.214	0.232	0.236	0.127	0.308	0.257	0.178	0.287	0.265	0.136	500	0.094	0.038	0.111	0.080	0.050	0.151	0.089	0.057	0.209	0.094	0.070	0.277	0.101	0.090	0.294	0.097	0.093	1000	0.500	0.065	0.124	0.222	0.378	0.312	0.167	0.430	0.335	0.257	0.143	0.137	0.280	0.309	0.277	0.255	0.318	0.296	<.01 - 2	200	0.004	0.000	0.000	0.006	0.011	0.001	0.008	0.016	0.004	0.003	0.000	0.000	0.005	0.004	0.002	0.006	0.007	0.001	500	0.000	0.000	0.000	0.004	0.000	0.000	0.006	0.000	0.001	0.007	0.000	0.000	0.015	0.000	0.000	0.021	0.003	0.000	1000	0.000	0.000	0.000	0.000	0.009	0.000	0.007	0.013	0.018	0.000	0.000	0.000	0.018	0.001	0.000	0.020	0.013	0.002	<.05 - 2	200	0.033	0.028	0.002	0.021	0.063	0.021	0.014	0.066	0.032	0.034	0.007	0.003	0.040	0.041	0.009	0.017	0.051	0.026	500	0.000	0.000	0.000	0.025	0.002	0.001	0.028	0.012	0.003	0.037	0.004	0.000	0.062	0.000	0.000	0.086	0.014	0.000	1000	0.000	0.000	0.017	0.028	0.019	0.028	0.076	0.072	0.071	0.007	0.000	0.000	0.062	0.030	0.007	0.093	0.078	0.027	<.10 - 2	200	0.088	0.089	0.025	0.056	0.122	0.062	0.026	0.126	0.088	0.104	0.035	0.016	0.074	0.127	0.044	0.035	0.130	0.065	500	0.016	0.000	0.010	0.050	0.011	0.004	0.072	0.038	0.016	0.094	0.006	0.000	0.135	0.024	0.003	0.160	0.035	0.004	1000	0.000	0.000	0.017	0.094	0.139	0.132	0.143	0.134	0.022	0.011	0.146	0.115	0.048	0.155	0.141	0.092	<.20 - 2	200	0.217	0.291	0.155	0.161	0.303	0.199	0.082	0.298	0.214	0.236	0.204	0.124	0.190	0.275	0.203	0.090	0.266	0.203	500	0.063	0.043	0.035	0.115	0.074	0.039	0.180	0.098	0.057	0.246	0.086	0.027	0.294	0.109	0.032	0.301	0.111	0.019	1000	0.146	0.100	0.100	0.274	0.335	0.278	0.300	0.286	0.151	0.066	0.288	0.304	0.207	0.300	0.341	0.246

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 25**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three correlated traits.*

	N	unlinked												linked																								
		Block1				Block2				Block3				Block1				Block2				Block3																
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18													
$\chi^2 - 1$	200	124	580	1360	126	579	1356	126	577	1352	323	1384	3167	323	1381	3161	323	1379	3147	200	124	580	1360	126	579	1356	126	577	1352	323	1384	3167	323	1381	3161	323	1379	3147
	500	119	567	1348	119	568	1340	120	565	1336	323	1371	3156	323	1363	3147	323	1360	3138	500	119	567	1348	119	568	1340	120	565	1336	323	1371	3156	323	1363	3147	323	1360	3138
	1000	139	586	1377	131	600	1387	129	598	1387	324	1380	3170	322	1386	3176	322	1386	3173	1000	139	586	1377	131	600	1387	129	598	1387	324	1380	3170	322	1386	3176	322	1386	3173
$\chi^2$ SD - 1	200	11	19	22	14	24	30	15	27	38	17	26	26	21	33	40	23	40	45	200	11	19	22	14	24	30	15	27	38	17	26	26	21	33	40	23	40	45
	500	10	14	19	11	20	24	13	22	28	16	22	27	20	29	37	22	32	45	500	10	14	19	11	20	24	13	22	28	16	22	27	20	29	37	22	32	45
	1000	15	16	19	13	21	28	17	27	38	18	21	25	23	31	36	23	38	48	1000	15	16	19	13	21	28	17	27	38	18	21	25	23	31	36	23	38	48
df - 1	200	126	580	1356	126	580	1356	126	580	1356	323	1376	3158	323	1376	3158	323	1376	3158	200	126	580	1356	126	580	1356	126	580	1356	323	1376	3158	323	1376	3158	323	1376	3158
	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3158	323	1376	3158	323	1376	3158	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3158	323	1376	3158	323	1376	3158
	1000	132	591	1374	132	591	1374	132	591	1374	323	1376	3158	323	1376	3158	323	1376	3158	1000	132	591	1374	132	591	1374	132	591	1374	323	1376	3158	323	1376	3158	323	1376	3158
Corrected df - 1	200	120	568	1338	120	568	1338	120	568	1338	314	1358	3131	314	1358	3131	314	1358	3131	200	120	568	1338	120	568	1338	120	568	1338	314	1358	3131	314	1358	3131	314	1358	3131
	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3131	314	1358	3131	314	1358	3131	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3131	314	1358	3131	314	1358	3131
	1000	126	579	1356	126	579	1356	126	579	1356	314	1358	3131	314	1358	3131	314	1358	3131	1000	126	579	1356	126	579	1356	126	579	1356	314	1358	3131	314	1358	3131	314	1358	3131
$\chi^2 - 2$	200	125	585	1360	121	584	1356	118	582	1352	296	1381	3166	291	1383	3165	286	1380	3156	200	125	585	1360	121	584	1356	118	582	1352	296	1381	3166	291	1383	3165	286	1380	3156
	500	119	566	1347	118	566	1339	120	566	1336	325	1371	3069	323	1365	3054	324	1360	3043	500	119	566	1347	118	566	1339	120	566	1336	325	1371	3069	323	1365	3054	324	1360	3043
	1000	592	1378	1378	594	1386	594	1385	594	1385	325	1379	3163	323	1386	3169	324	1386	3167	1000	592	1378	1378	594	1386	594	1385	594	1385	325	1379	3163	323	1386	3169	324	1386	3167
$\chi^2$ SD - 2	200	11	20	22	12	25	31	10	29	38	16	24	27	19	34	39	16	39	50	200	11	20	22	12	25	31	10	29	38	16	24	27	19	34	39	16	39	50
	500	9	15	18	13	19	23	14	22	29	17	22	24	21	30	31	23	34	34	500	9	15	18	13	19	23	14	22	29	17	22	24	21	30	31	23	34	34
	1000	13	20	20	22	22	31	27	37	37	20	23	23	22	31	37	24	40	47	1000	13	20	20	22	22	31	27	37	37	20	23	23	22	31	37	24	40	47
df - 2	200	126	580	1356	126	580	1356	126	580	1356	296	1376	3158	296	1376	3158	296	1376	3158	200	126	580	1356	126	580	1356	126	580	1356	296	1376	3158	296	1376	3158	296	1376	3158
	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3077	323	1376	3077	323	1376	3077	500	126	579	1356	126	579	1356	126	579	1356	323	1376	3077	323	1376	3077	323	1376	3077
	1000	591	1374	1374	591	1374	591	1374	591	1374	323	1376	3158	323	1376	3158	323	1376	3158	1000	591	1374	1374	591	1374	591	1374	591	1374	323	1376	3158	323	1376	3158	323	1376	3158
Corrected df - 2	200	120	568	1338	120	568	1338	120	568	1338	287	1358	3131	287	1358	3131	287	1358	3131	200	120	568	1338	120	568	1338	120	568	1338	287	1358	3131	287	1358	3131	287	1358	3131
	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3050	314	1358	3050	314	1358	3050	500	120	567	1338	120	567	1338	120	567	1338	314	1358	3050	314	1358	3050	314	1358	3050
	1000	579	1356	1356	579	1356	579	1356	579	1356	314	1358	3131	314	1358	3131	314	1358	3131	1000	579	1356	1356	579	1356	579	1356	579	1356	314	1358	3131	314	1358	3131	314	1358	3131

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.





## 5 Simulation Study with five uncorrelated traits

### 5.1 Conditions

1. Correlation of traits: uncorrelated traits
2. Number of traits: five traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (30 vs. 60 vs. 90 total)

### 5.2 Definition of parameters

#### 5.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Trait 1	1	0	0	0	0
Trait 2	0	1	0	0	0
Trait 3	0	0	1	0	0
Trait 4	0	0	0	1	0
Trait 5	0	0	0	0	1

#### 5.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.95271	-0.08450	0.03661	0.99755	0.11453
Items 6/7/8/9/10	-0.00577	-0.07635	-0.82547	0.24548	-0.20875
Items 11/12/13/14/15	0.42432	0.14416	-0.69830	-0.05606	-0.47702
Items 16/17/18/19/20	0.69325	-0.26406	-0.68138	-0.64580	-0.00606
Items 21/22/23/24/25	0.41826	0.09705	0.54650	-0.93027	-0.79421
Items 26/27/28/29/30	-0.93967	0.48942	-0.14667	-0.03280	0.63186
Items 31/32/33/34/35	-0.36005	-0.36504	0.88438	-0.98132	-0.32827
Items 36/37/38/39/40	0.67003	-0.63486	-0.45544	0.82732	0.13186
Items 41/42/43/44/45	-0.77568	0.01249	-0.91345	-0.54948	-0.26117
Items 46/47/48/49/50	-0.78664	0.71576	0.67864	0.92435	0.44364
Items 51/52/53/54/55	0.90839	0.42937	-0.90831	-0.82834	-0.93920
Items 56/57/58/59/60	0.89888	0.95876	0.17385	-0.71155	0.31524
Items 61/62/63/64/65	0.74331	0.55715	-0.34055	0.67006	0.26112
Items 66/67/68/69/70	-0.58839	0.12237	-0.12430	0.98932	-0.11381
Items 71/72/73/74/75	-0.94113	-0.24670	-0.02040	-0.05027	0.88589
Items 76/77/78/79/80	0.45167	-0.10183	-0.20565	-0.22893	0.09161
Items 81/82/83/84/85	-0.32107	0.43259	-0.12775	0.78885	-0.11082
Items 86/87/88/89/90	0.69538	-0.49471	-0.10395	-0.83398	0.57443

### 5.2.3 Matrix of loadings $\Lambda$ (one negatively keyed item)

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.81089	-0.58315	-0.33264	-0.69349	-0.32392
Items 6/7/8/9/10	0.86206	0.87681	0.50596	0.34122	0.62720
Items 11/12/13/14/15	0.69535	0.76188	0.85735	0.83432	0.70300
Items 16/17/18/19/20	0.76295	0.75486	0.88974	0.82630	0.35347
Items 21/22/23/24/25	0.79588	0.32237	0.82739	0.80908	0.48091
Items 26/27/28/29/30	0.63106	0.68353	0.32146	0.44728	0.44507
Items 31/32/33/34/35	0.87793	0.82211	0.30362	0.54778	0.84997
Items 36/37/38/39/40	0.48303	0.76016	0.51589	0.61076	0.37337
Items 41/42/43/44/45	0.66817	0.75304	0.75242	0.79362	0.63180
Items 46/47/48/49/50	0.60789	0.66387	0.84616	0.77102	0.67730
Items 51/52/53/54/55	0.89215	0.30942	0.73816	0.83507	0.89776
Items 56/57/58/59/60	0.81602	0.54639	0.37223	0.66059	0.55291
Items 61/62/63/64/65	0.82567	0.63269	0.87703	0.50555	0.52368
Items 66/67/68/69/70	0.64907	0.42618	0.59951	0.75795	0.51461
Items 71/72/73/74/75	0.63231	0.87081	0.33040	0.78169	0.69302
Items 76/77/78/79/80	0.78752	0.80052	0.30027	0.71393	0.64363
Items 81/82/83/84/85	0.86396	0.56293	0.87640	0.43152	0.80592
Items 86/87/88/89/90	0.71069	0.35785	0.32074	0.32508	0.88538

### 5.2.4 Matrix of loadings $\Lambda$ (two negatively keyed items)

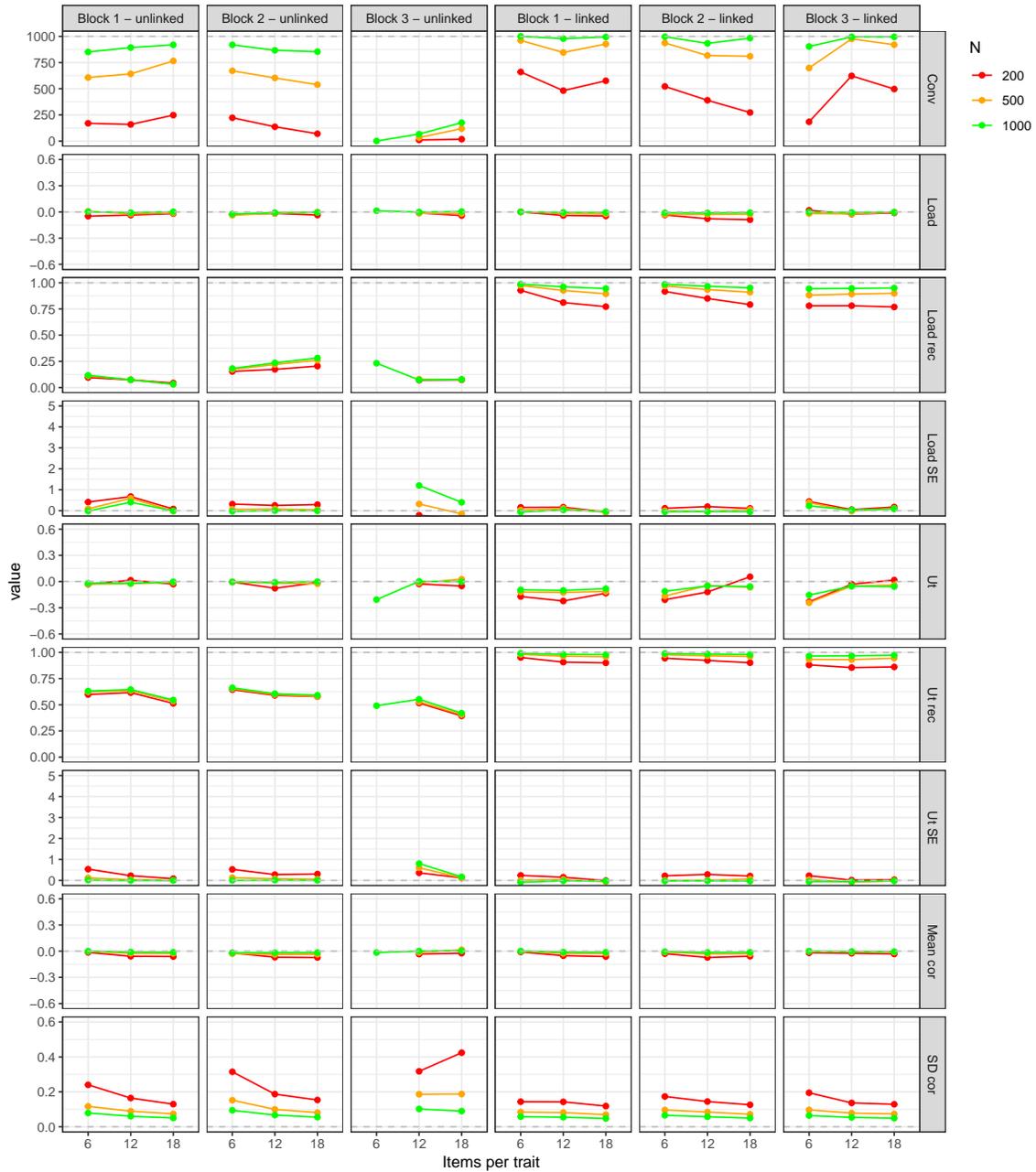
	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.81089	-0.58315	-0.33264	-0.69349	-0.32392
Items 6/7/8/9/10	-0.86206	-0.87681	-0.50596	-0.34122	-0.62720
Items 11/12/13/14/15	0.69535	0.76188	0.85735	0.83432	0.70300
Items 16/17/18/19/20	0.76295	0.75486	0.88974	0.82630	0.35347
Items 21/22/23/24/25	0.79588	0.32237	0.82739	0.80908	0.48091
Items 26/27/28/29/30	0.63106	0.68353	0.32146	0.44728	0.44507
Items 31/32/33/34/35	0.87793	0.82211	0.30362	0.54778	0.84997
Items 36/37/38/39/40	0.48303	0.76016	0.51589	0.61076	0.37337
Items 41/42/43/44/45	0.66817	0.75304	0.75242	0.79362	0.63180
Items 46/47/48/49/50	0.60789	0.66387	0.84616	0.77102	0.67730
Items 51/52/53/54/55	0.89215	0.30942	0.73816	0.83507	0.89776
Items 56/57/58/59/60	0.81602	0.54639	0.37223	0.66059	0.55291
Items 61/62/63/64/65	0.82567	0.63269	0.87703	0.50555	0.52368
Items 66/67/68/69/70	0.64907	0.42618	0.59951	0.75795	0.51461
Items 71/72/73/74/75	0.63231	0.87081	0.33040	0.78169	0.69302
Items 76/77/78/79/80	0.78752	0.80052	0.30027	0.71393	0.64363
Items 81/82/83/84/85	0.86396	0.56293	0.87640	0.43152	0.80592
Items 86/87/88/89/90	0.71069	0.35785	0.32074	0.32508	0.88538

## 5.3 Results

### 5.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 22

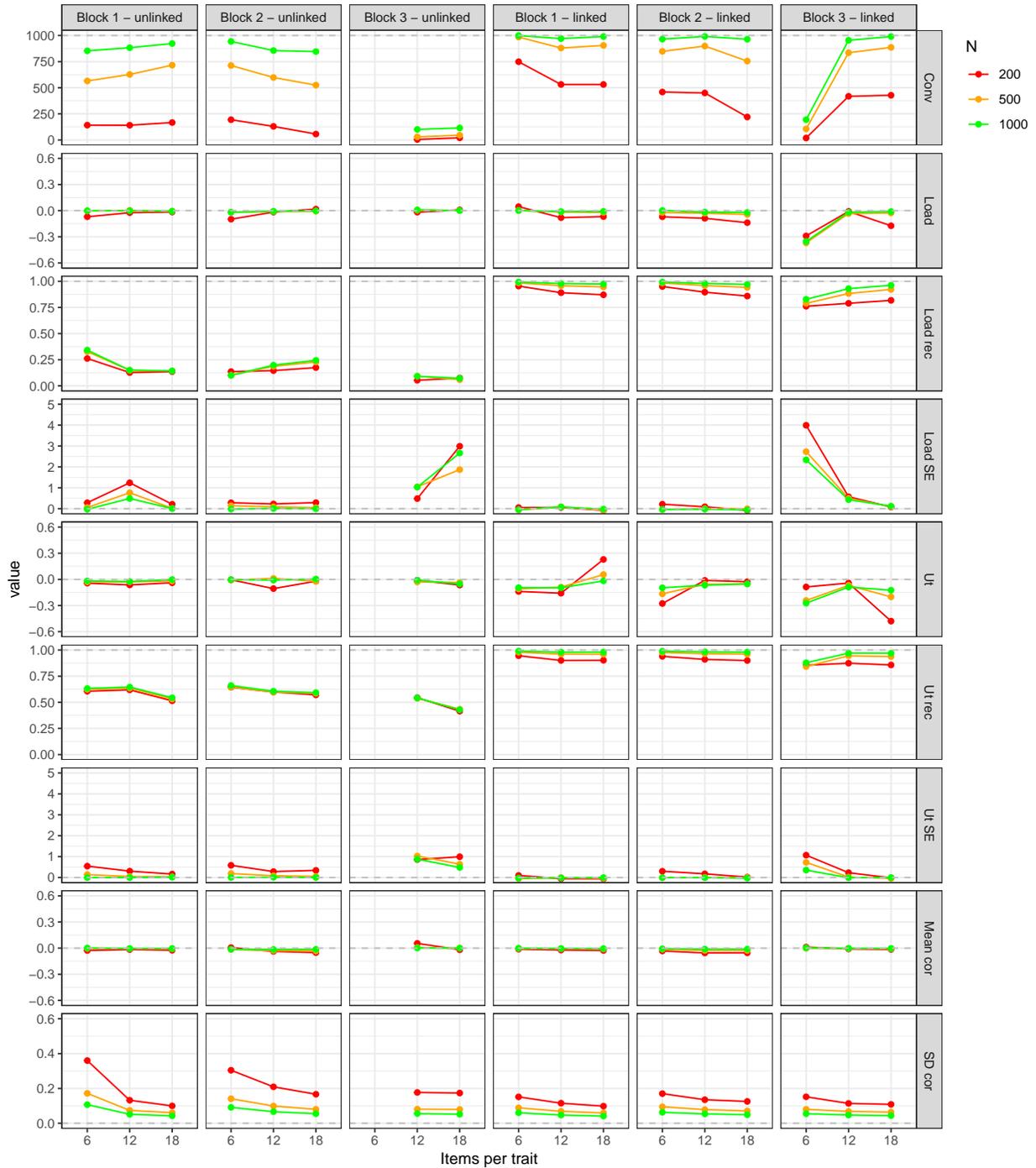
Bias results for the Thurstonian factor model with five uncorrelated traits. One negatively keyed item.



Note. Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 23**

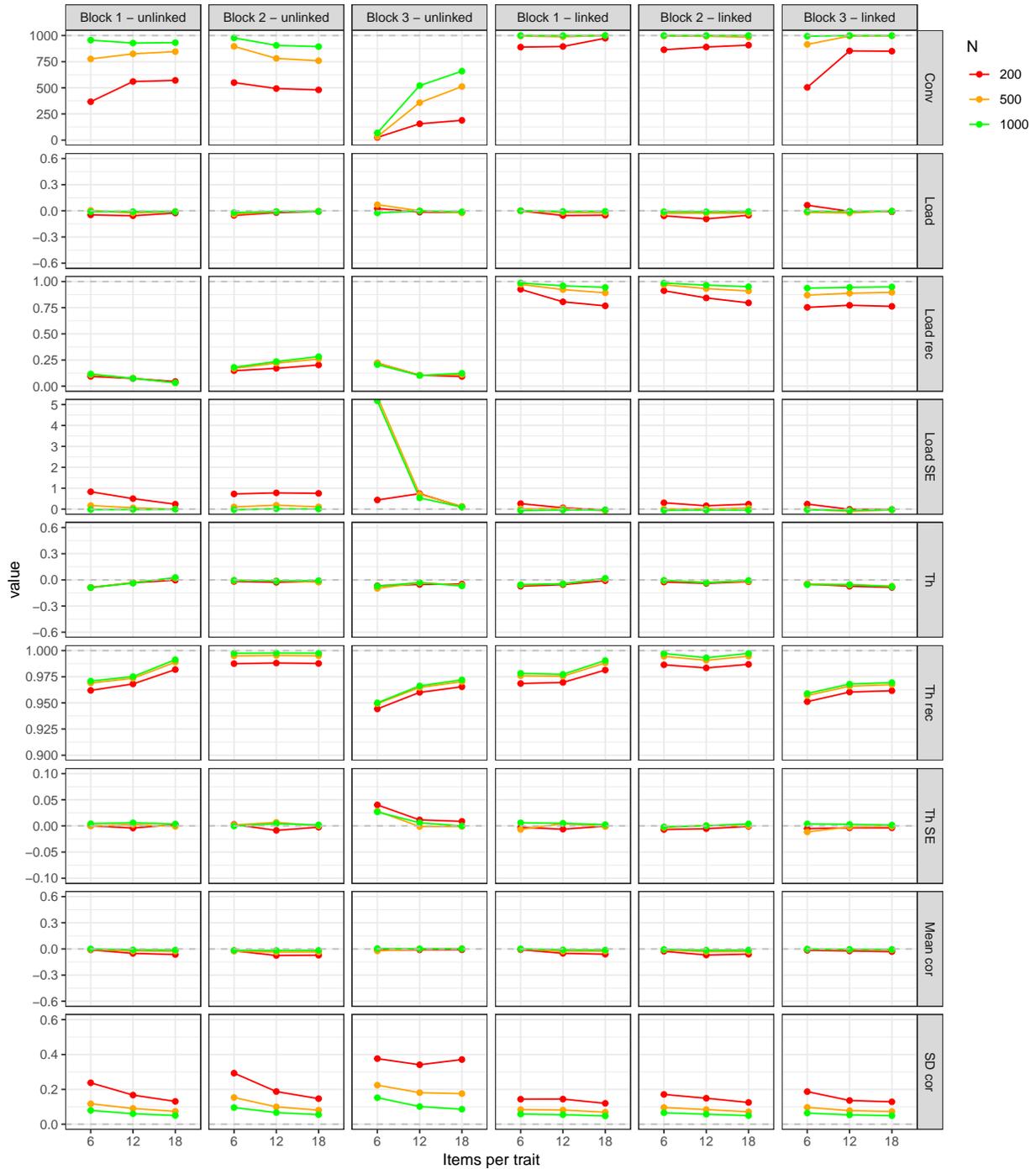
*Bias results for the Thurstonian factor model with five uncorrelated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 24**

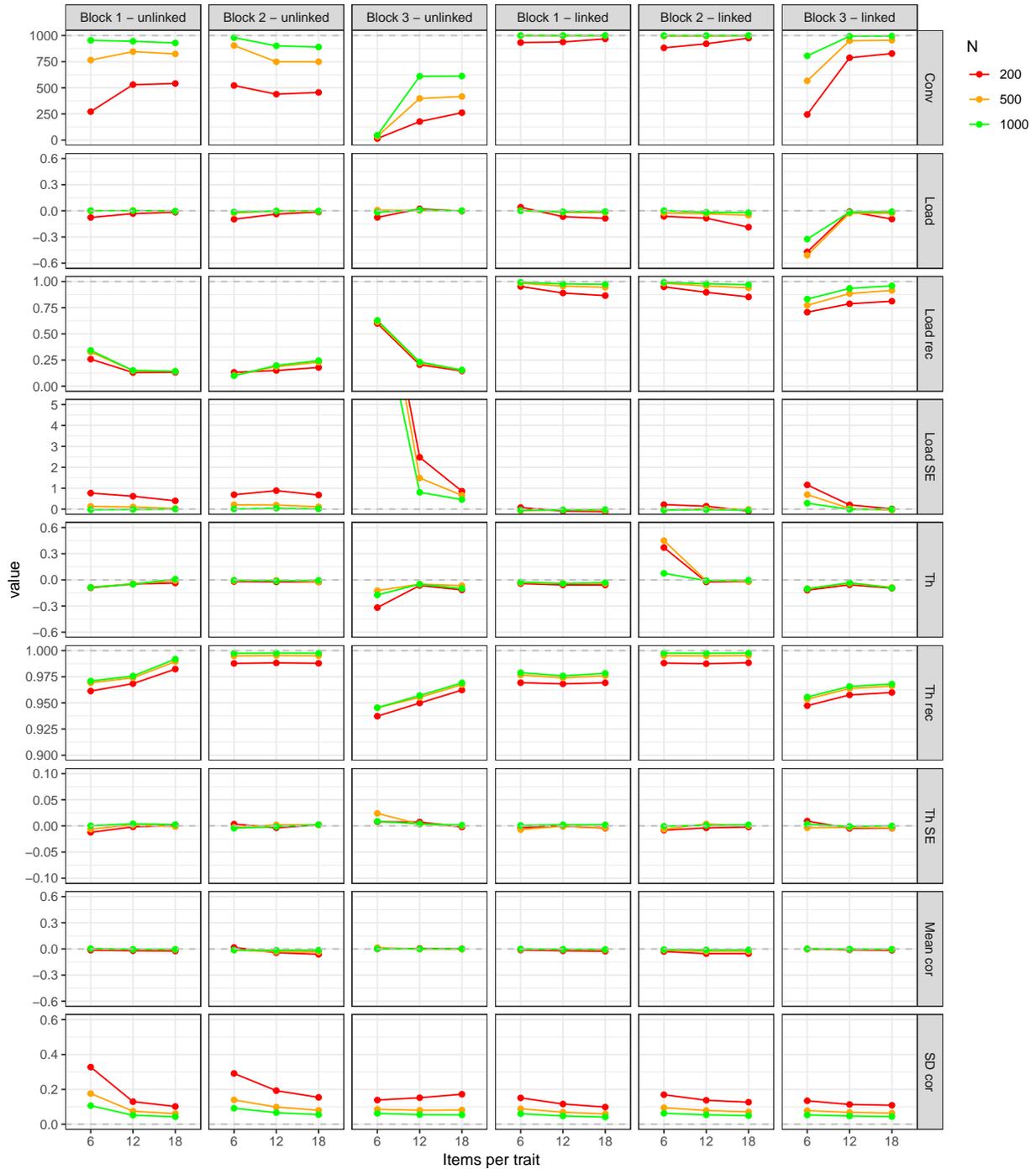
*Bias results for the Thurstonian IRT model with five uncorrelated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Figure 25**

*Bias results for the Thurstonian IRT model with five uncorrelated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 28**

*Bias results for the Thurstonian factor model with five uncorrelated traits. One negatively keyed item.*

	unlinked															linked								
	Block1					Block2					Block3					Block1			Block2			Block3		
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
N	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000
Conv	170	159	248	223	137	70	11	18	660	482	576	273	390	522	273	184	623	497	184	623	497	184	623	497
Load	-0.05	-0.04	-0.02	-0.03	-0.02	-0.04	-0.01	-0.04	0.00	-0.04	-0.05	-0.03	-0.03	-0.03	-0.03	0.02	-0.02	-0.01	0.02	-0.02	-0.01	0.02	-0.02	-0.01
Load rec	0.10	0.07	0.04	0.15	0.17	0.20	0.07	0.07	0.93	0.81	0.77	0.92	0.85	0.79	0.78	0.78	0.78	0.77	0.78	0.78	0.77	0.78	0.78	0.77
Load SE	0.41	0.68	0.08	0.32	0.25	0.29	-0.21	-0.38	0.16	0.17	-0.06	0.12	0.20	0.11	0.44	0.05	0.18		0.44	0.05	0.18	0.44	0.05	0.18
Ut	0.01	-0.03	-0.01	-0.01	-0.08	-0.01	-0.03	-0.05	-0.17	-0.22	-0.13	-0.21	-0.12	-0.12	-0.23	-0.03	0.02		-0.23	-0.03	0.02	-0.23	-0.03	0.02
Ut rec	0.60	0.62	0.51	0.64	0.59	0.58	0.52	0.39	0.95	0.91	0.90	0.94	0.92	0.90	0.88	0.85	0.86		0.88	0.85	0.86	0.88	0.85	0.86
Ut SE	0.54	0.22	0.08	0.52	0.28	0.30	0.36	0.12	0.24	0.15	-0.01	0.22	0.29	0.21	0.22	0.01	0.03		0.22	0.01	0.03	0.22	0.01	0.03
Mean cor	-0.01	-0.06	-0.06	-0.02	-0.07	-0.07	-0.03	-0.02	-0.01	-0.05	-0.06	-0.03	-0.07	-0.06	-0.02	-0.02	-0.03		-0.06	-0.02	-0.03	-0.06	-0.02	-0.03
SD cor	0.24	0.16	0.13	0.31	0.19	0.15	0.32	0.42	0.14	0.14	0.12	0.17	0.14	0.13	0.19	0.14	0.13		0.17	0.14	0.13	0.19	0.14	0.13

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 29**

*Bias results for the Thurstonian factor model with five uncorrelated traits. Two negatively keyed items.*

	N	unlinked															linked																																							
		Block1					Block2					Block3					Block1					Block2					Block3																													
		6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18																									
Conv	200	142	141	167	194	130	57	5	21	749	531	531	459	450	220	220	19	418	428	500	565	627	716	713	598	525	30	46	985	880	905	848	899	755	106	835	886	1000	853	883	923	943	856	846	101	115	999	970	990	965	991	964	194	954	990	
Load	200	-0.07	-0.02	-0.02	-0.10	-0.02	0.02	-0.02	0.01	0.05	-0.08	-0.07	-0.07	-0.09	-0.14	-0.29	-0.01	-0.17	-0.03	500	0.00	0.00	-0.01	-0.02	-0.01	0.00	0.01	0.00	0.00	-0.02	-0.02	-0.03	-0.04	-0.37	-0.03	-0.03	-0.03	1000	0.00	0.00	-0.01	-0.02	-0.01	0.00	0.01	0.00	0.00	-0.01	-0.01	0.00	-0.02	-0.02	-0.35	-0.02	-0.01	-0.01
Load rec	200	0.26	0.13	0.14	0.14	0.15	0.17	0.05	0.07	0.95	0.89	0.87	0.95	0.90	0.86	0.76	0.79	0.82	500	0.33	0.15	0.14	0.10	0.19	0.23	0.09	0.06	0.98	0.96	0.95	0.98	0.96	0.94	0.79	0.88	0.92	1000	0.34	0.15	0.14	0.10	0.20	0.24	0.09	0.07	0.99	0.98	0.97	0.99	0.98	0.97	0.83	0.93	0.96		
Load SE	200	0.29	1.24	0.21	0.29	0.23	0.29	0.49	2.99	0.06	0.06	-0.07	0.22	0.09	-0.08	3.99	0.57	0.09	500	0.06	0.77	0.02	0.14	0.09	0.05	1.05	1.87	-0.06	0.09	-0.06	-0.04	0.00	0.00	2.73	0.50	0.11	1000	-0.03	0.49	0.01	-0.02	0.01	0.00	1.03	2.66	-0.04	0.09	-0.01	-0.03	-0.02	-0.05	2.34	0.43	0.12		
Ut	200	-0.04	-0.06	-0.04	0.00	-0.11	-0.02	-0.01	-0.07	-0.14	-0.16	0.23	-0.28	-0.01	-0.03	-0.09	-0.04	-0.48	500	-0.03	-0.03	-0.02	-0.01	0.01	-0.02	-0.03	-0.04	-0.11	-0.09	0.06	-0.17	-0.06	-0.05	-0.24	-0.07	-0.20	1000	-0.02	-0.03	0.00	0.00	-0.01	0.00	-0.01	-0.05	-0.09	-0.10	-0.02	-0.10	-0.07	-0.05	-0.27	-0.09	-0.12		
Ut rec	200	0.61	0.62	0.51	0.64	0.60	0.57	0.54	0.42	0.95	0.90	0.90	0.94	0.91	0.90	0.85	0.87	0.86	500	0.62	0.64	0.53	0.65	0.60	0.59	0.54	0.43	0.98	0.96	0.96	0.98	0.96	0.96	0.84	0.94	0.94	1000	0.63	0.65	0.54	0.66	0.61	0.59	0.54	0.43	0.99	0.98	0.98	0.98	0.98	0.98	0.88	0.97	0.97		
Ut SE	200	0.54	0.30	0.16	0.58	0.28	0.34	0.85	0.99	0.10	-0.06	-0.07	0.30	0.17	0.02	1.07	0.23	-0.02	500	0.14	0.04	0.03	0.19	0.08	0.05	1.03	0.64	-0.06	-0.01	-0.04	-0.01	0.00	-0.01	0.72	0.02	-0.05	1000	-0.01	-0.01	0.01	0.00	0.01	0.00	0.87	0.48	-0.03	-0.02	-0.01	-0.01	-0.01	-0.03	0.35	-0.01	-0.01		
Mean cor	200	-0.03	-0.02	-0.02	0.01	-0.04	-0.05	0.05	-0.02	-0.01	-0.02	-0.03	-0.03	-0.06	-0.05	0.01	-0.01	-0.01	500	0.00	-0.01	-0.01	-0.02	-0.03	-0.03	0.01	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.03	0.00	0.00	-0.01	1000	0.00	0.00	0.00	-0.01	-0.02	-0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	
SD cor	200	0.36	0.13	0.10	0.30	0.21	0.17	0.18	0.17	0.15	0.12	0.10	0.17	0.14	0.13	0.15	0.11	0.11	500	0.17	0.07	0.06	0.14	0.10	0.08	0.08	0.08	0.09	0.07	0.06	0.09	0.08	0.07	0.08	0.07	0.06	1000	0.11	0.05	0.04	0.09	0.07	0.06	0.06	0.05	0.06	0.05	0.04	0.06	0.05	0.05	0.06	0.05	0.05	0.04	

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 30**

*Bias results for the Thurstonian IRT model with five uncorrelated traits. One negatively keyed item.*

	unlinked															linked														
	Block1					Block2					Block3					Block1					Block2					Block3				
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
N	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000
Conv	367	560	571	550	493	480	26	156	189	189	889	895	974	864	890	890	895	988	1000	995	993	985	985	985	985	985	985	985	985	985
Load	-0.05	-0.06	-0.02	-0.05	-0.02	-0.01	0.03	-0.01	-0.02	0.00	0.00	-0.05	-0.05	-0.06	-0.09	-0.06	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Load rec	0.09	0.07	0.04	0.15	0.17	0.20	0.22	0.10	0.09	0.93	0.81	0.77	0.91	0.84	0.80	0.84	0.80	0.75	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
Load SE	0.83	0.50	0.24	0.73	0.78	0.76	0.44	0.74	0.11	0.26	0.07	-0.04	0.31	0.16	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24	0.24
Th	0.17	0.07	0.00	0.11	0.19	0.11	5.39	0.71	0.13	0.02	0.03	-0.07	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Th rec	0.96	0.97	0.98	0.99	0.99	0.99	0.94	0.96	0.97	0.97	0.97	0.97	0.98	0.99	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Th SE	0.97	0.97	0.99	1.00	1.00	1.00	1.00	0.95	0.96	0.97	0.98	0.98	0.99	1.00	0.99	0.99	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.99	1.00	0.99	0.97	0.97
Mean cor	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.01	0.01	0.00	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
SD cor	0.24	0.17	0.13	0.29	0.19	0.15	0.38	0.34	0.37	0.14	0.14	0.12	0.17	0.15	0.13	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 31**

*Bias results for the Thurstonian IRT model with five uncorrelated traits. Two negatively keyed items.*

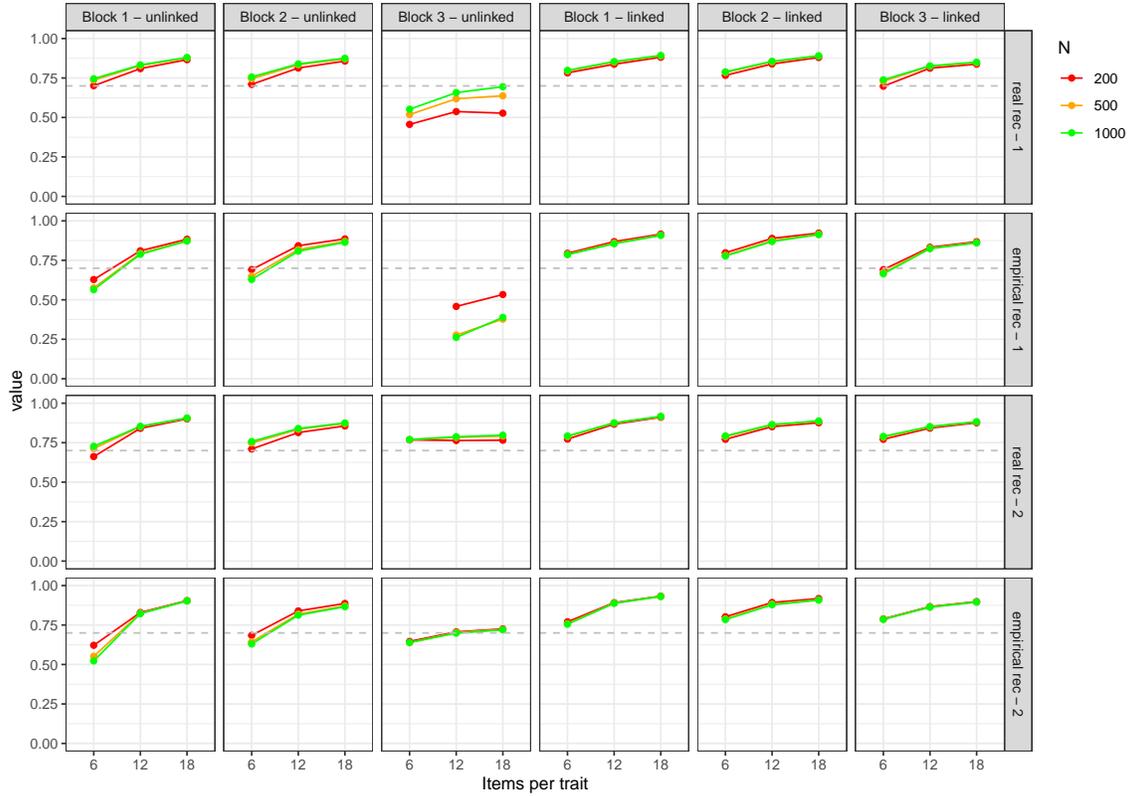
	unlinked															linked																																										
	Block1					Block2					Block3					Block1					Block2					Block3																																
	N	6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18																										
Conv	200	272	530	541	522	439	456	15	177	262	933	938	968	882	921	975	245	787	828	500	765	846	824	904	749	749	36	398	417	999	998	999	993	994	998	567	950	955	1000	954	945	929	980	901	890	47	610	612	1000	1000	1000	1000	1000	1000	1000	806	993	995
Load	200	-0.08	-0.03	-0.02	-0.10	-0.04	-0.01	-0.08	0.02	0.00	0.04	-0.07	-0.09	-0.06	-0.08	-0.19	-0.47	-0.01	-0.09	500	0.00	0.00	0.00	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	-0.03	-0.05	-0.51	-0.03	-0.03	1000	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	-0.02	0.01	0.00	0.00	-0.01	0.00	-0.02	-0.32	-0.01	-0.01				
Load rec	200	0.26	0.13	0.13	0.13	0.15	0.18	0.60	0.20	0.15	0.95	0.89	0.87	0.95	0.90	0.85	0.71	0.79	0.81	500	0.33	0.15	0.14	0.10	0.19	0.23	0.62	0.22	0.15	0.98	0.96	0.95	0.98	0.96	0.94	0.77	0.89	0.92	1000	0.34	0.15	0.14	0.10	0.20	0.24	0.63	0.23	0.15	0.99	0.98	0.97	0.99	0.98	0.97	0.83	0.93	0.96	
Load SE	200	0.77	0.62	0.40	0.69	0.89	0.68	13.21	2.48	0.86	0.08	-0.09	-0.12	0.22	0.14	-0.09	1.16	0.21	0.01	500	0.13	0.11	0.04	0.21	0.20	0.11	12.88	1.49	0.66	-0.08	-0.04	-0.07	-0.05	0.02	-0.01	0.70	0.03	-0.05	1000	-0.03	-0.01	0.00	0.01	0.05	0.02	10.50	0.81	0.46	-0.04	-0.04	-0.02	-0.05	-0.02	-0.05	0.29	-0.01	0.00	
Th	200	-0.09	-0.05	-0.04	-0.02	-0.02	-0.02	-0.32	-0.06	-0.11	-0.04	-0.06	-0.06	0.37	-0.02	-0.02	-0.12	-0.06	-0.10	500	-0.09	-0.05	-0.01	-0.01	-0.01	-0.02	-0.12	-0.05	-0.07	-0.03	-0.04	-0.04	0.45	-0.01	-0.02	-0.10	-0.04	-0.09	1000	-0.09	-0.05	0.01	0.00	-0.01	-0.01	-0.17	-0.05	-0.10	-0.03	-0.04	-0.03	0.07	-0.01	0.00	-0.10	-0.03	-0.09	
Th rec	200	0.96	0.97	0.98	0.99	0.99	0.99	0.94	0.95	0.96	0.97	0.97	0.97	0.99	0.99	0.99	0.95	0.96	0.96	500	0.97	0.97	0.99	0.99	1.00	1.00	0.95	0.96	0.97	0.98	0.97	0.98	1.00	1.00	1.00	0.95	0.96	0.97	1000	0.97	0.98	0.99	1.00	1.00	1.00	0.95	0.96	0.97	0.98	0.98	0.98	1.00	1.00	1.00	0.96	0.97	0.97	
Th SE	200	-0.01	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	500	-0.01	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	1000	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Mean cor	200	-0.01	-0.02	-0.02	0.02	-0.04	-0.06	0.00	0.01	0.00	-0.01	-0.02	-0.03	-0.03	-0.05	-0.05	0.00	-0.01	-0.02	500	0.00	-0.01	-0.01	-0.01	-0.03	-0.03	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.03	-0.03	0.00	0.00	-0.01	1000	0.00	0.00	0.00	0.00	-0.01	-0.02	-0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00
SD cor	200	0.33	0.13	0.10	0.29	0.19	0.15	0.14	0.15	0.17	0.15	0.12	0.10	0.17	0.14	0.13	0.13	0.11	0.11	500	0.18	0.07	0.06	0.14	0.10	0.08	0.09	0.08	0.08	0.09	0.07	0.06	0.09	0.08	0.07	0.08	0.07	0.06	1000	0.11	0.05	0.04	0.09	0.07	0.06	0.06	0.06	0.05	0.06	0.05	0.04	0.06	0.05	0.05	0.05	0.05	0.05	0.04

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 5.3.2 Results for latent trait recovery

Figure 26

*Latent trait recovery results with five uncorrelated traits.*



*Note.* Abbreviations: rec: recovery. The first two rows come from simulation with one (- 1), the second two rows from simulation with two (- 2) negatively keyed items per trait.

**Table 32**

*Latent trait recovery results with five uncorrelated traits.*

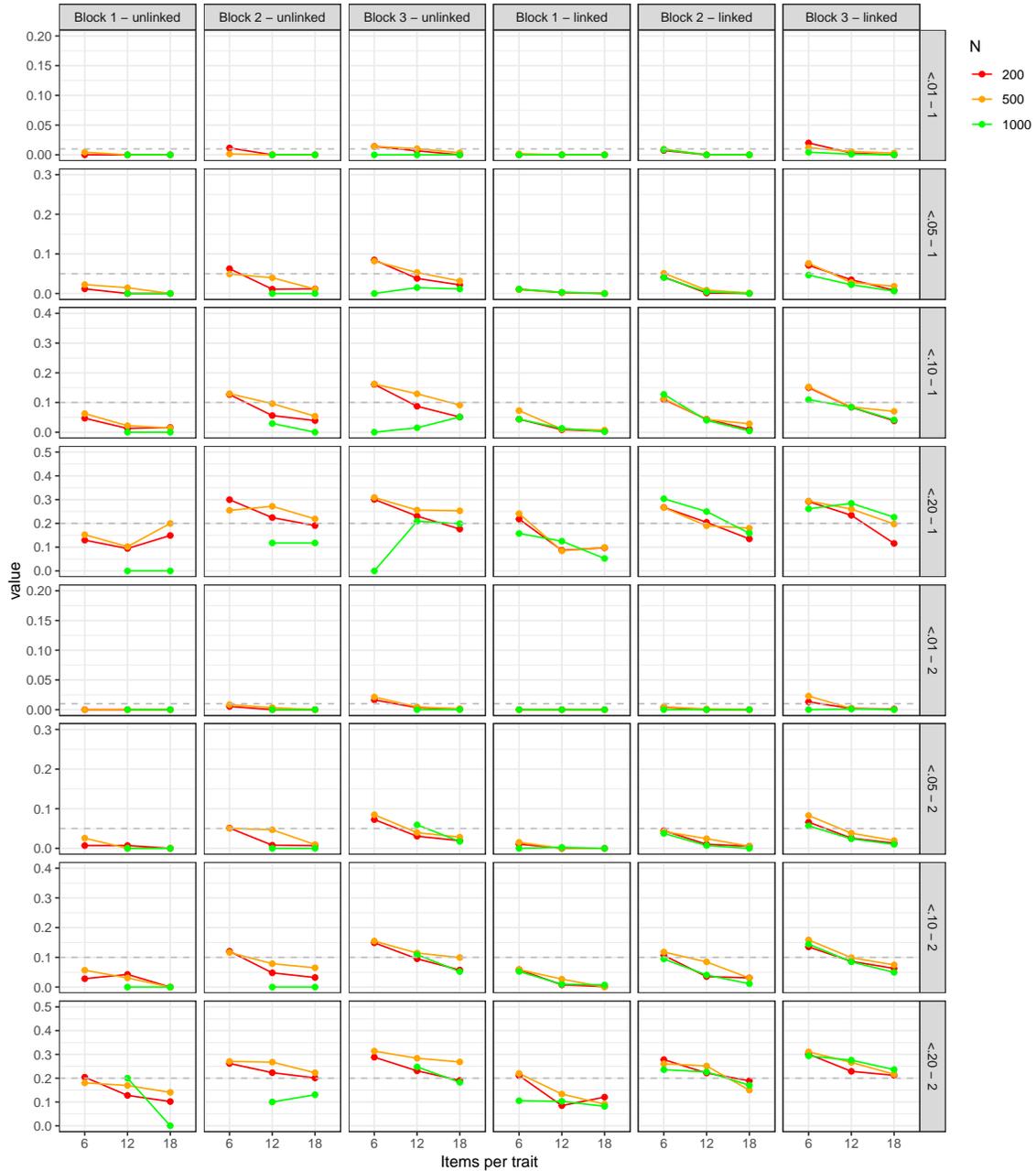
N	unlinked									linked									
	Block1			Block2			Block3			Block1			Block2			Block3			
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	
real rec - 1	200	0.701	0.809	0.866	0.710	0.813	0.857	0.456	0.537	0.527	0.783	0.837	0.882	0.767	0.839	0.880	0.698	0.812	0.838
	500	0.736	0.828	0.877	0.745	0.834	0.871	0.519	0.618	0.637	0.795	0.851	0.890	0.785	0.852	0.889	0.730	0.824	0.847
	1000	0.745	0.833	0.880	0.757	0.839	0.875	0.552	0.657	0.695	0.798	0.854	0.893	0.789	0.856	0.891	0.739	0.827	0.850
empirical rec - 1	200	0.628	0.811	0.883	0.692	0.843	0.886		0.458	0.533	0.795	0.868	0.916	0.798	0.889	0.923	0.691	0.833	0.868
	500	0.575	0.794	0.874	0.649	0.818	0.870		0.277	0.377	0.788	0.859	0.910	0.782	0.875	0.916	0.672	0.827	0.862
	1000	0.564	0.789	0.872	0.629	0.808	0.864		0.262	0.388	0.786	0.855	0.908	0.777	0.869	0.913	0.665	0.824	0.861
real rec - 2	200	0.662	0.841	0.900	0.710	0.814	0.856	0.768	0.764	0.766	0.773	0.867	0.911	0.771	0.851	0.876	0.771	0.842	0.876
	500	0.713	0.851	0.905	0.748	0.835	0.871	0.768	0.783	0.792	0.789	0.874	0.915	0.788	0.862	0.885	0.785	0.850	0.881
	1000	0.727	0.854	0.906	0.757	0.840	0.875	0.770	0.787	0.798	0.792	0.876	0.917	0.793	0.865	0.888	0.790	0.852	0.883
empirical rec - 2	200	0.622	0.829	0.905	0.684	0.839	0.887	0.646	0.707	0.725	0.770	0.892	0.933	0.802	0.893	0.918	0.789	0.866	0.898
	500	0.552	0.825	0.903	0.643	0.819	0.871	0.638	0.702	0.722	0.759	0.889	0.931	0.789	0.882	0.910	0.787	0.864	0.896
	1000	0.522	0.822	0.902	0.630	0.812	0.866	0.639	0.698	0.719	0.755	0.888	0.931	0.785	0.879	0.907	0.785	0.863	0.895

*Note.* Abbreviations: rec: recovery.

### 5.3.3 Results for empirical rejection rates

Figure 27

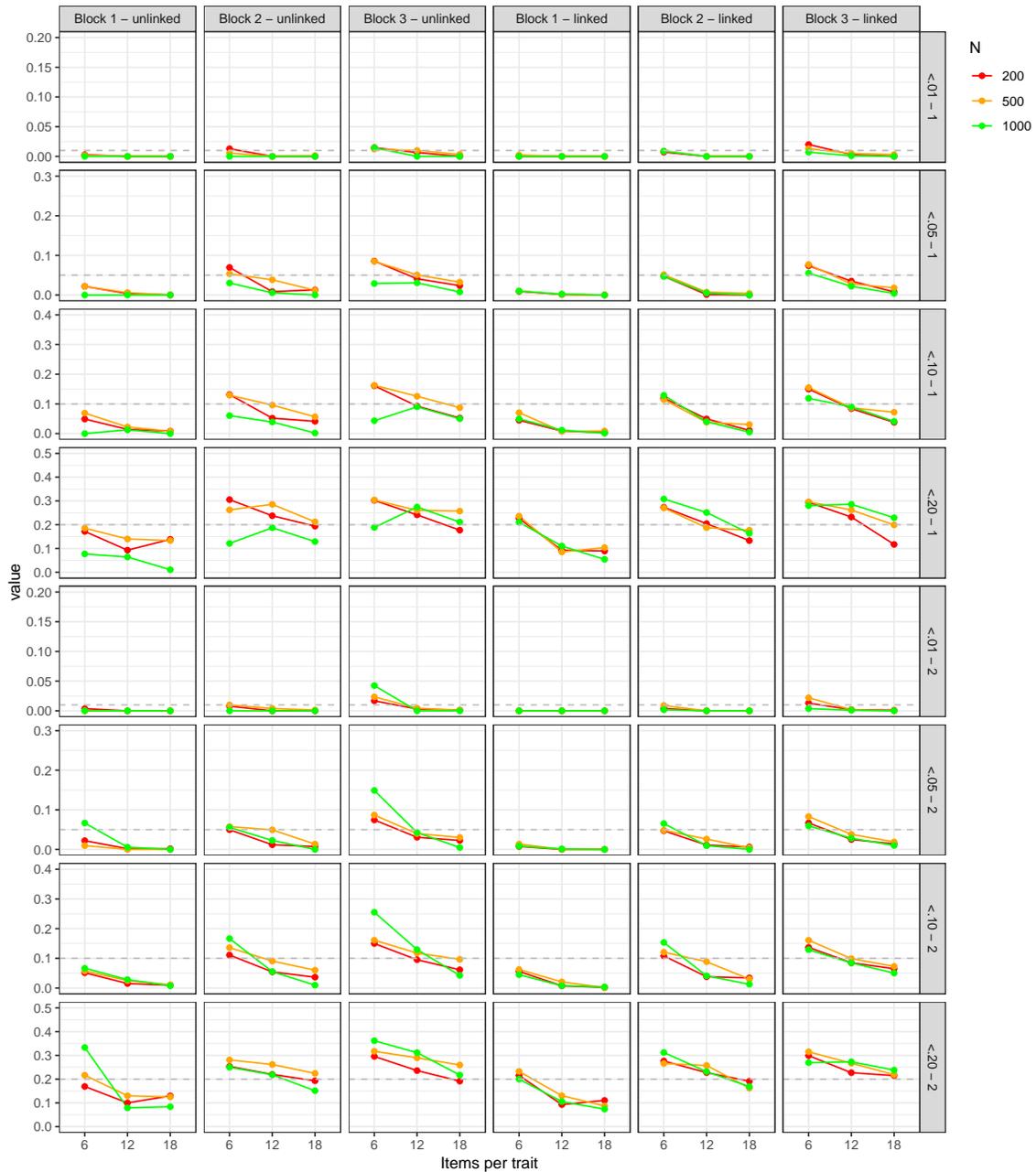
Empirical rejection rates for the Thurstonian factor model with five uncorrelated traits.



Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

Figure 28

*Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.



**Table 34**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five uncorrelated traits.*

	unlinked															linked																																										
	Block1					Block2					Block3					Block1					Block2					Block3																																
	N	6	12	18	24	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30																							
$\chi^2$ - 1	200	382	1680	3888	387	1685	3881	386	1682	3870	941	3911	8923	941	3917	8910	940	3913	8890	500	383	1682	3892	384	1687	3884	386	1683	3882	893	3907	8922	894	3914	8913	894	3914	8908	1000	1709	3906	1703	3913	381	1704	3918	937	3917	8921	944	3924	8925	938	3924	8927	938	3924	8927
$\chi^2$ SD - 1	200	15	20	31	21	31	43	24	40	52	22	28	34	30	41	54	37	55	69	500	19	23	28	21	35	45	25	43	57	24	30	39	31	43	59	36	56	76	1000	10	18	18	25	29	38	21	28	29	29	29	38	41	32	48	56	48	56	56
df - 1	200	385	1681	3875	385	1681	3875	385	1681	3875	937	3907	8902	937	3907	8902	937	3907	8902	500	385	1680	3875	385	1680	3875	385	1680	3875	892	3907	8902	892	3907	8902	892	3907	8902	1000	1700	3905	1700	3905	395	1700	3905	937	3907	8902	937	3907	8902	937	3907	8902	937	3907	8902
Corrected df - 1	200	375	1661	3845	375	1661	3845	375	1661	3845	922	3877	8857	922	3877	8857	922	3877	8857	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3877	8857	877	3877	8857	877	3877	8857	1000	1680	3875	1680	3875	385	1680	3875	922	3877	8857	922	3877	8857	922	3877	8857	922	3877	8857
$\chi^2$ - 2	200	383	1684	3884	386	1685	3881	385	1683	3872	941	3910	8924	941	3918	8923	940	3914	8915	500	382	1686	3887	385	1687	3885	386	1683	3882	894	3824	8919	895	3830	8912	896	3825	8909	1000	1693	3913	1701	3912	1709	3922	937	3917	8927	936	3922	8927	939	3921	8928	939	3921	8928	
$\chi^2$ SD - 2	200	16	25	27	21	30	42	24	39	52	22	29	37	30	41	54	35	53	69	500	17	24	31	22	36	44	25	42	57	23	34	38	30	50	57	36	58	76	1000	33	33	19	24	31	37	20	28	28	31	30	40	43	34	50	58	50	58	58
df - 2	200	385	1681	3875	385	1681	3875	385	1681	3875	937	3907	8902	937	3907	8902	937	3907	8902	500	385	1680	3875	385	1680	3875	385	1680	3875	892	3817	8902	892	3817	8902	892	3817	8902	1000	1700	3905	1700	3905	1700	3905	937	3907	8902	937	3907	8902	937	3907	8902	937	3907	8902	
Corrected df - 2	200	375	1661	3845	375	1661	3845	375	1661	3845	922	3877	8857	922	3877	8857	922	3877	8857	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3787	8857	877	3787	8857	877	3787	8857	1000	1680	3875	1680	3875	1680	3875	922	3877	8857	922	3877	8857	922	3877	8857	922	3877	8857	

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 35**

*Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits.*

	unlinked															linked														
	Block1					Block2					Block3					Block1					Block2					Block3				
	N	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12
<.01 - 1	200	0.003	0.000	0.000	0.013	0.000	0.000	0.000	0.000	0.015	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.020	0.003	0.000	
	500	0.002	0.000	0.000	0.006	0.000	0.001	0.012	0.010	0.003	0.002	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.013	0.005	0.003		
	1000	0.000	0.000	0.000	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.000	0.007	0.001	0.000		
<.05 - 1	200	0.022	0.004	0.000	0.070	0.008	0.013	0.086	0.041	0.024	0.009	0.001	0.000	0.000	0.047	0.001	0.000	0.000	0.000	0.000	0.047	0.001	0.000	0.000	0.074	0.035	0.008			
	500	0.022	0.006	0.000	0.054	0.038	0.012	0.085	0.051	0.032	0.010	0.001	0.000	0.051	0.007	0.004	0.000	0.000	0.000	0.051	0.007	0.004	0.000	0.077	0.028	0.018				
	1000	0.000	0.000	0.000	0.030	0.006	0.000	0.029	0.031	0.008	0.010	0.002	0.000	0.047	0.005	0.000	0.000	0.000	0.000	0.047	0.005	0.000	0.000	0.055	0.022	0.004				
<.10 - 1	200	0.049	0.014	0.009	0.131	0.052	0.041	0.161	0.093	0.053	0.045	0.009	0.003	0.120	0.050	0.011	0.150	0.084	0.038	0.120	0.050	0.011	0.150	0.084	0.038					
	500	0.069	0.022	0.008	0.129	0.096	0.057	0.163	0.126	0.087	0.071	0.008	0.009	0.115	0.038	0.030	0.155	0.087	0.072	0.115	0.038	0.030	0.155	0.087	0.072					
	1000	0.000	0.013	0.000	0.061	0.039	0.002	0.043	0.090	0.050	0.012	0.001	0.129	0.040	0.005	0.119	0.090	0.041	0.129	0.040	0.005	0.119	0.090	0.041						
<.20 - 1	200	0.172	0.093	0.138	0.305	0.238	0.194	0.302	0.241	0.177	0.226	0.093	0.089	0.273	0.204	0.133	0.294	0.232	0.117	0.273	0.204	0.133	0.294	0.232	0.117					
	500	0.185	0.140	0.133	0.262	0.286	0.212	0.304	0.260	0.257	0.236	0.085	0.104	0.270	0.187	0.177	0.296	0.262	0.199	0.270	0.187	0.177	0.296	0.262	0.199					
	1000	0.077	0.064	0.011	0.121	0.187	0.129	0.188	0.274	0.211	0.210	0.110	0.054	0.308	0.251	0.163	0.280	0.229	0.110	0.308	0.251	0.163	0.280	0.229						
<.01 - 2	200	0.004	0.000	0.000	0.008	0.000	0.000	0.017	0.003	0.001	0.000	0.000	0.000	0.004	0.000	0.000	0.000	0.001	0.001	0.000	0.004	0.000	0.000	0.000	0.013	0.002	0.001			
	500	0.000	0.000	0.000	0.010	0.004	0.001	0.023	0.004	0.001	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.000	0.000	0.009	0.000	0.000	0.000	0.022	0.002	0.000				
	1000	0.000	0.000	0.000	0.000	0.000	0.000	0.043	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000	0.000	0.004	0.001	0.000				
<.05 - 2	200	0.022	0.002	0.002	0.050	0.012	0.007	0.074	0.031	0.023	0.008	0.000	0.000	0.047	0.011	0.006	0.067	0.025	0.014	0.047	0.011	0.006	0.067	0.025	0.014					
	500	0.010	0.000	0.000	0.058	0.049	0.013	0.087	0.040	0.030	0.014	0.000	0.000	0.048	0.026	0.004	0.083	0.038	0.019	0.048	0.026	0.004	0.083	0.038	0.019					
	1000	0.067	0.006	0.000	0.056	0.023	0.000	0.149	0.043	0.005	0.008	0.001	0.000	0.065	0.009	0.000	0.060	0.028	0.010	0.065	0.009	0.000	0.060	0.028	0.010					
<.10 - 2	200	0.051	0.015	0.009	0.111	0.054	0.036	0.150	0.095	0.061	0.056	0.007	0.002	0.108	0.038	0.034	0.137	0.085	0.065	0.108	0.038	0.034	0.137	0.085	0.065					
	500	0.057	0.025	0.011	0.136	0.091	0.060	0.161	0.118	0.097	0.062	0.021	0.001	0.121	0.089	0.030	0.161	0.099	0.073	0.121	0.089	0.030	0.161	0.099	0.073					
	1000	0.067	0.028	0.008	0.167	0.055	0.010	0.255	0.130	0.042	0.045	0.008	0.004	0.153	0.041	0.013	0.129	0.085	0.050	0.153	0.041	0.013	0.129	0.085	0.050					
<.20 - 2	200	0.169	0.100	0.129	0.254	0.220	0.193	0.296	0.236	0.192	0.215	0.093	0.111	0.276	0.227	0.191	0.299	0.227	0.115	0.276	0.227	0.191	0.299	0.227	0.115					
	500	0.216	0.130	0.125	0.281	0.262	0.224	0.317	0.290	0.260	0.232	0.130	0.087	0.266	0.258	0.162	0.315	0.267	0.119	0.266	0.258	0.162	0.315	0.267	0.119					
	1000	0.333	0.079	0.084	0.250	0.219	0.151	0.362	0.311	0.217	0.200	0.105	0.074	0.312	0.232	0.169	0.269	0.273	0.119	0.312	0.232	0.169	0.269	0.273	0.119					

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 36**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five uncorrelated traits.*

	N	unlinked															linked																																								
		Block1					Block2					Block3					Block1					Block2					Block3																														
		6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30																					
$\chi^2 - 1$	200	374	1661	3859	378	1665	3852	376	1663	3841	925	3880	8875	925	3884	8864	924	3881	8844	500	373	1662	3855	375	1668	3854	376	1663	3852	879	3879	8874	879	3883	8868	879	3883	8863	1000	376	1681	3876	381	1685	3882	378	1688	3889	923	3886	8874	927	3892	8877	921	3893	8879
$\chi^2$ SD - 1	200	15	21	28	22	31	43	24	39	52	22	28	35	30	42	53	36	55	69	500	18	25	30	21	35	45	24	43	56	24	29	39	31	43	59	35	56	76	1000	15	18	20	18	27	30	22	34	39	21	27	29	29	38	42	33	48	56
df - 1	200	375	1661	3845	375	1661	3845	375	1661	3845	921	3875	8856	921	3875	8856	921	3875	8856	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3876	8856	877	3876	8856	877	3876	8856	1000	385	1680	3875	385	1680	3875	385	1680	3875	920	3875	8854	920	3875	8854	920	3875	8854
Corrected df - 1	200	365	1641	3815	365	1641	3815	365	1641	3815	906	3845	8811	906	3845	8811	906	3845	8811	500	365	1640	3815	365	1640	3815	365	1640	3815	862	3846	8811	862	3846	8811	862	3846	8811	1000	375	1660	3845	375	1660	3845	375	1660	3845	905	3845	8809	905	3845	8809	905	3845	8809
$\chi^2 - 2$	200	372	1661	3858	375	1666	3851	376	1663	3842	925	3879	8874	925	3887	8875	924	3882	8867	500	372	1663	3855	375	1667	3855	377	1663	3852	880	3794	8872	880	3800	8867	881	3795	8863	1000	391	1685	3887	385	1688	3885	389	1691	3888	921	3884	8875	926	3889	8877	920	3888	8879
$\chi^2$ SD - 2	200	17	22	28	21	30	42	24	39	52	22	29	37	30	41	54	35	53	70	500	18	24	30	22	36	45	25	42	57	23	32	37	30	50	57	36	58	77	1000	16	22	23	21	29	32	27	36	40	22	28	30	31	40	43	35	50	58
df - 2	200	375	1661	3845	375	1661	3845	375	1661	3845	921	3875	8854	921	3875	8854	921	3875	8854	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3787	8856	877	3787	8856	877	3787	8856	1000	385	1680	3875	385	1680	3875	385	1680	3875	920	3874	8852	920	3874	8852	920	3874	8852
Corrected df - 2	200	365	1641	3815	365	1641	3815	365	1641	3815	906	3845	8809	906	3845	8809	906	3845	8809	500	365	1640	3815	365	1640	3815	365	1640	3815	862	3757	8811	862	3757	8811	862	3757	8811	1000	375	1660	3845	375	1660	3845	375	1660	3845	905	3844	8807	905	3844	8807	905	3844	8807

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

## 6 Simulation Study with five correlated traits

### 6.1 Conditions

1. Correlation of traits: correlated traits
2. Number of traits: five traits
3. Number of items: 6 vs. 12 vs. 18 items per trait (30 vs. 60 vs. 90 total)

### 6.2 Definition of parameters

#### 6.2.1 Correlation matrix $\Phi$ of traits

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Trait 1	1.00	-0.21	0.02	-0.25	-0.53
Trait 2	-0.21	1.00	0.40	0.04	0.27
Trait 3	0.02	0.40	1.00	-0.02	-0.02
Trait 4	-0.25	0.04	-0.02	1.00	0.24
Trait 5	-0.53	0.27	-0.02	0.24	1.00

#### 6.2.2 Matrix of utilities $U$

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.34567	0.79553	0.53328	-0.96436	-0.74256
Items 6/7/8/9/10	0.13737	-0.99334	-0.59215	-0.42274	-0.23720
Items 11/12/13/14/15	-0.40507	-0.77530	0.23200	-0.38930	0.18594
Items 16/17/18/19/20	-0.97527	-0.78622	0.32019	0.98349	-0.11078
Items 21/22/23/24/25	0.95123	-0.37030	0.74589	0.93044	0.43302
Items 26/27/28/29/30	0.45010	-0.38733	-0.68014	0.20400	0.56176
Items 31/32/33/34/35	0.70070	-0.86070	0.66069	-0.65557	0.17820
Items 36/37/38/39/40	0.36273	0.77828	0.87510	0.52532	0.70419
Items 41/42/43/44/45	-0.64301	-0.67196	0.51213	0.60992	0.81855
Items 46/47/48/49/50	0.84748	0.98322	-0.57159	-0.26704	0.46579
Items 51/52/53/54/55	-0.20715	-0.44232	-0.03001	-0.59804	-0.14103
Items 56/57/58/59/60	-0.78247	-0.49488	-0.32916	0.49041	-0.50290
Items 61/62/63/64/65	-0.34482	-0.98712	0.47104	0.68767	-0.32055
Items 66/67/68/69/70	-0.70473	0.02411	0.97550	-0.02166	0.54123
Items 71/72/73/74/75	-0.61547	-0.49521	-0.89459	-0.08964	-0.36651
Items 76/77/78/79/80	-0.68779	-0.81978	0.00854	-0.69128	-0.08732
Items 81/82/83/84/85	0.18085	0.41053	0.02998	-0.99640	-0.64665
Items 86/87/88/89/90	-0.07979	0.66349	-0.02292	0.73334	0.06931

### 6.2.3 Matrix of loadings $\Lambda$ (one negatively keyed item)

	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.33858	-0.86637	-0.41863	-0.55710	-0.84268
Items 6/7/8/9/10	0.73961	0.57009	0.84544	0.72048	0.32819
Items 11/12/13/14/15	0.67896	0.74065	0.43174	0.81977	0.81771
Items 16/17/18/19/20	0.72322	0.75833	0.88978	0.72025	0.39667
Items 21/22/23/24/25	0.31312	0.40957	0.33955	0.45197	0.56050
Items 26/27/28/29/30	0.45731	0.69585	0.55313	0.62873	0.87730
Items 31/32/33/34/35	0.36407	0.30073	0.43693	0.60876	0.57798
Items 36/37/38/39/40	0.60245	0.72115	0.42822	0.52088	0.89467
Items 41/42/43/44/45	0.57392	0.68382	0.54867	0.41879	0.73512
Items 46/47/48/49/50	0.52211	0.35507	0.33417	0.83225	0.84946
Items 51/52/53/54/55	0.68593	0.47942	0.46723	0.34183	0.67193
Items 56/57/58/59/60	0.62670	0.73444	0.49247	0.58307	0.46557
Items 61/62/63/64/65	0.51801	0.60525	0.34321	0.70961	0.36983
Items 66/67/68/69/70	0.60035	0.67610	0.84991	0.36646	0.85169
Items 71/72/73/74/75	0.78619	0.32945	0.30743	0.75750	0.78287
Items 76/77/78/79/80	0.51326	0.62976	0.67271	0.48260	0.66641
Items 81/82/83/84/85	0.40595	0.37342	0.86181	0.61877	0.77765
Items 86/87/88/89/90	0.76570	0.46760	0.76167	0.89849	0.46765

### 6.2.4 Matrix of loadings $\Lambda$ (two negatively keyed items)

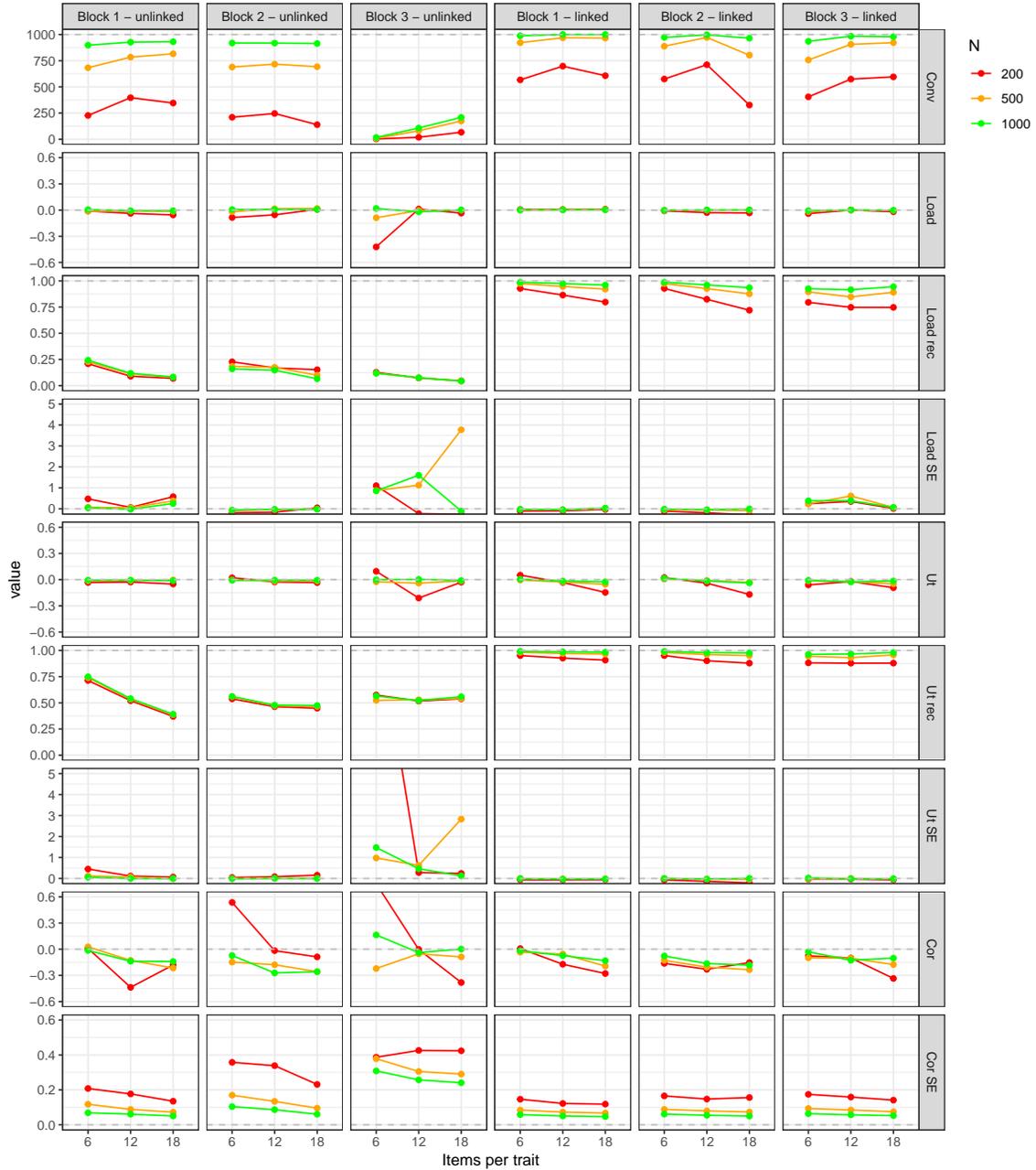
	Trait 1	Trait 2	Trait 3	Trait 4	Trait 5
Items 1/2/3/4/5	-0.33858	-0.86637	-0.41863	-0.55710	-0.84268
Items 6/7/8/9/10	-0.73961	-0.57009	-0.84544	-0.72048	-0.32819
Items 11/12/13/14/15	0.67896	0.74065	0.43174	0.81977	0.81771
Items 16/17/18/19/20	0.72322	0.75833	0.88978	0.72025	0.39667
Items 21/22/23/24/25	0.31312	0.40957	0.33955	0.45197	0.56050
Items 26/27/28/29/30	0.45731	0.69585	0.55313	0.62873	0.87730
Items 31/32/33/34/35	0.36407	0.30073	0.43693	0.60876	0.57798
Items 36/37/38/39/40	0.60245	0.72115	0.42822	0.52088	0.89467
Items 41/42/43/44/45	0.57392	0.68382	0.54867	0.41879	0.73512
Items 46/47/48/49/50	0.52211	0.35507	0.33417	0.83225	0.84946
Items 51/52/53/54/55	0.68593	0.47942	0.46723	0.34183	0.67193
Items 56/57/58/59/60	0.62670	0.73444	0.49247	0.58307	0.46557
Items 61/62/63/64/65	0.51801	0.60525	0.34321	0.70961	0.36983
Items 66/67/68/69/70	0.60035	0.67610	0.84991	0.36646	0.85169
Items 71/72/73/74/75	0.78619	0.32945	0.30743	0.75750	0.78287
Items 76/77/78/79/80	0.51326	0.62976	0.67271	0.48260	0.66641
Items 81/82/83/84/85	0.40595	0.37342	0.86181	0.61877	0.77765
Items 86/87/88/89/90	0.76570	0.46760	0.76167	0.89849	0.46765

## 6.3 Results

### 6.3.1 Results for convergence, mean relative bias and recovery of item parameters

Figure 29

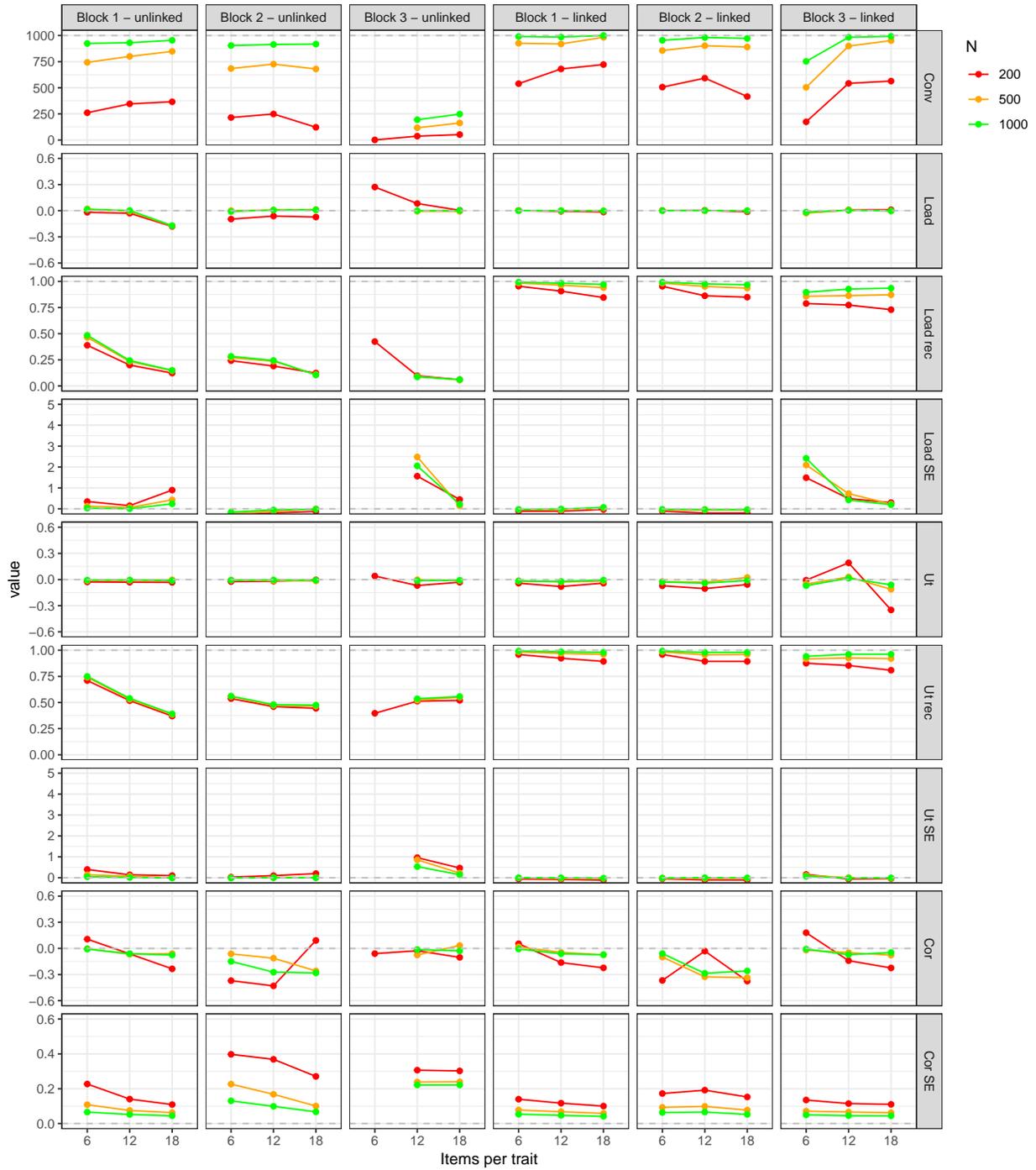
*Bias results for the Thurstonian factor model with five correlated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 30**

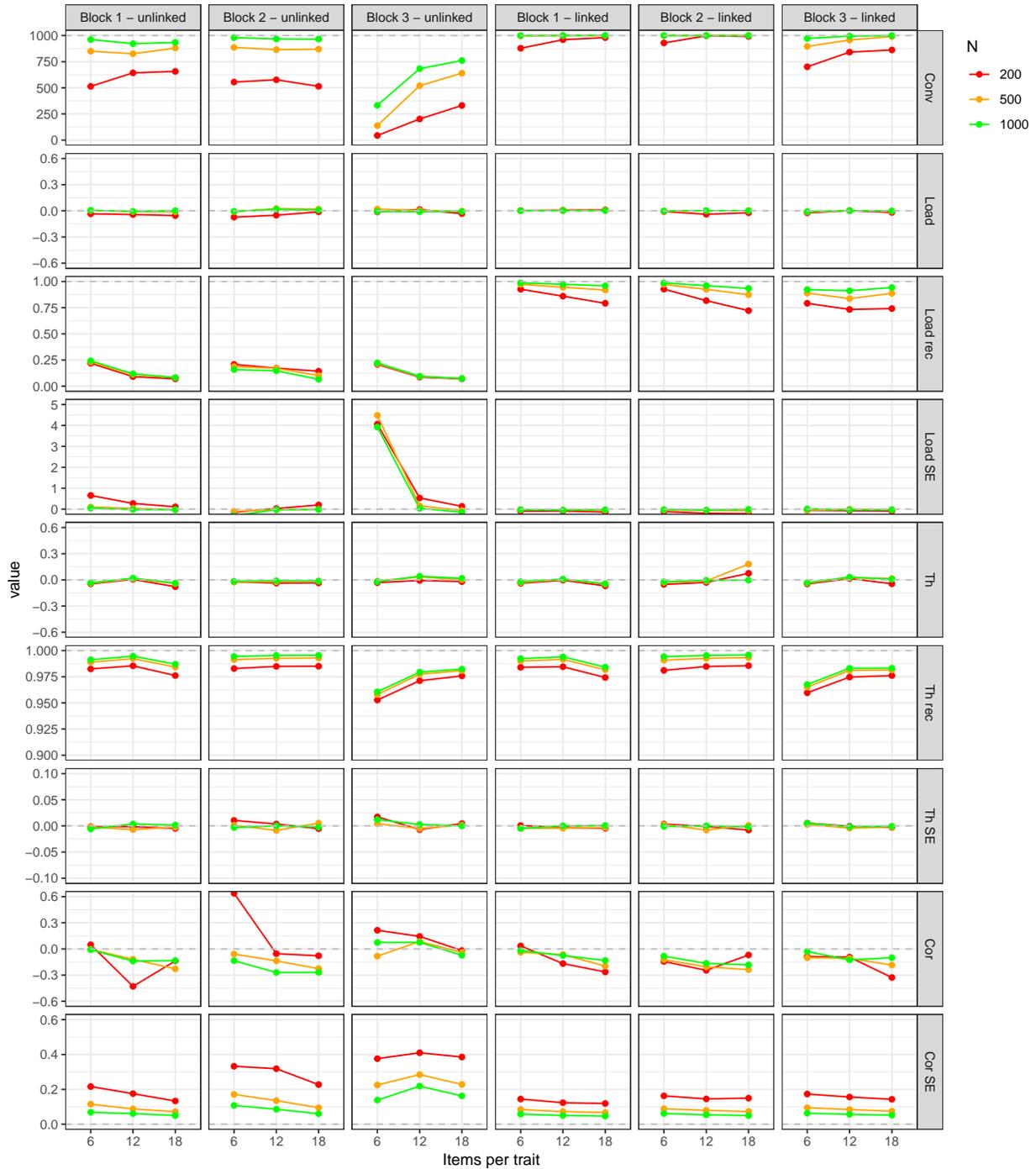
*Bias results for the Thurstonian factor model with five correlated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Figure 31**

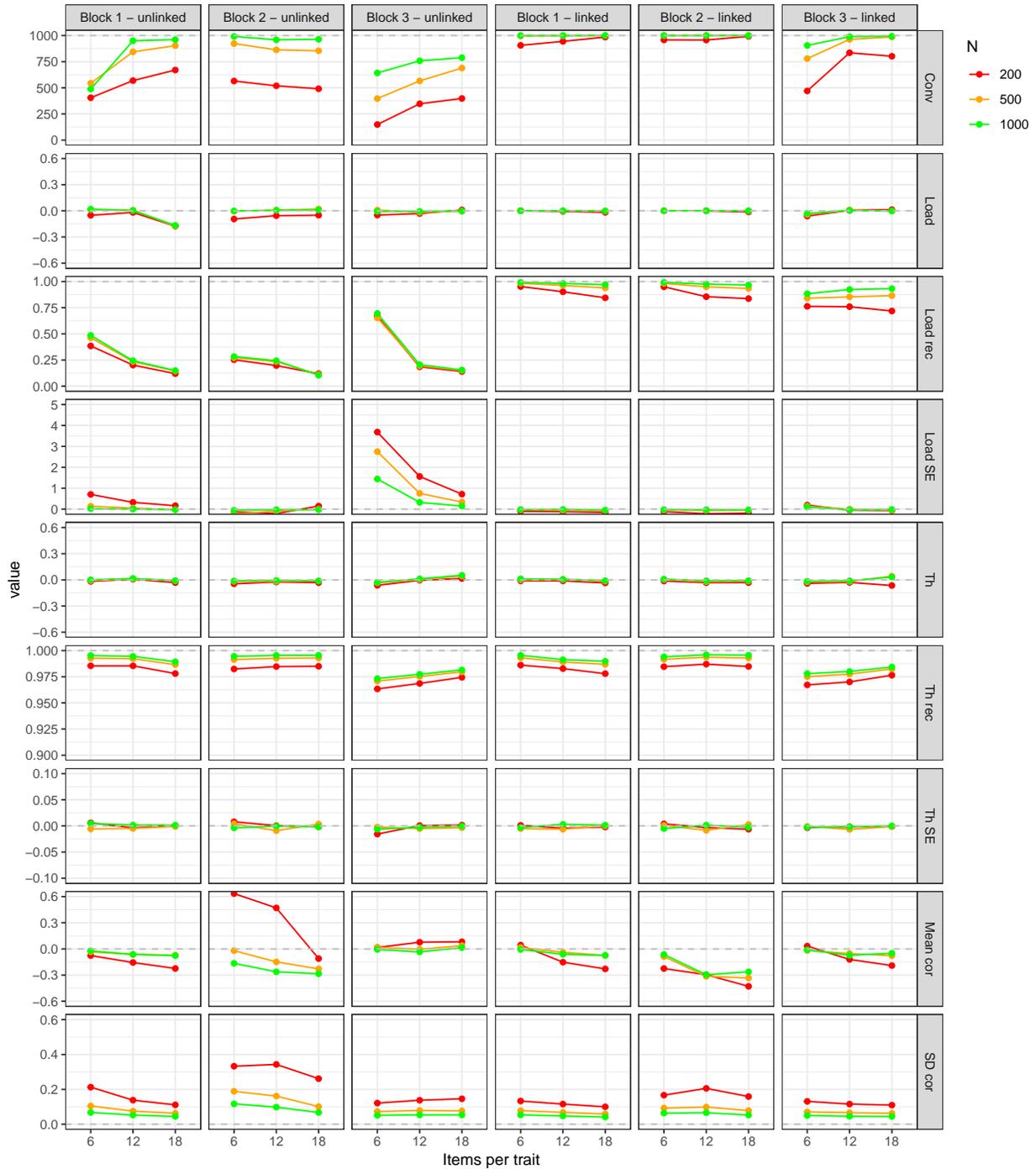
*Bias results for the Thurstonian IRT model with five correlated traits. One negatively keyed item.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Figure 32**

*Bias results for the Thurstonian IRT model with five correlated traits. Two negatively keyed items.*



*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

**Table 37**

*Bias results for the Thurstonian factor model with five correlated traits. One negatively keyed item.*

	unlinked															linked																																											
	Block1					Block2					Block3					Block1					Block2					Block3																																	
	N	6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18																											
Conv	200	227	397	346	210	246	139	3	19	67	567	698	607	575	712	327	405	574	596	500	683	784	816	690	717	693	8	81	174	922	969	966	888	972	803	757	906	922	1000	898	927	931	919	918	914	17	107	209	988	1000	1000	973	997	965	935	984	979		
Load	200	-0.01	-0.04	-0.06	-0.09	-0.05	0.01	-0.42	0.01	-0.03	0.01	0.01	0.01	0.01	-0.01	-0.03	-0.04	0.00	-0.02	500	-0.01	-0.01	-0.02	-0.02	0.02	0.02	-0.09	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	-0.02	0.00	0.00	1000	0.01	-0.01	-0.01	0.01	0.01	0.01	0.02	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00
Load rec	200	0.21	0.09	0.07	0.23	0.17	0.15	0.13	0.07	0.05	0.93	0.86	0.80	0.93	0.82	0.72	0.80	0.75	0.75	500	0.23	0.11	0.08	0.18	0.17	0.10	0.12	0.08	0.05	0.97	0.95	0.92	0.97	0.93	0.88	0.89	0.85	0.89	1000	0.24	0.12	0.08	0.16	0.15	0.07	0.12	0.08	0.04	0.99	0.97	0.96	0.99	0.96	0.94	0.93	0.92	0.94		
Load SE	200	0.48	0.07	0.58	-0.18	-0.16	0.05	1.10	-0.23	-0.26	-0.11	-0.10	-0.03	-0.11	-0.19	-0.29	0.25	0.35	0.02	500	0.08	0.05	0.38	-0.08	-0.03	-0.01	0.87	1.13	3.77	-0.02	-0.04	0.01	-0.04	-0.06	-0.08	0.24	0.62	0.07	1000	0.04	-0.02	0.25	-0.06	-0.03	-0.01	0.86	1.60	-0.10	-0.02	-0.04	0.03	-0.01	-0.04	0.00	0.38	0.39	0.07		
Ut	200	-0.03	-0.03	-0.05	0.02	-0.03	-0.03	0.10	-0.21	-0.03	0.05	-0.03	-0.15	0.02	-0.04	-0.17	-0.06	-0.02	-0.09	500	-0.01	-0.01	0.00	0.00	-0.02	-0.01	-0.02	-0.04	-0.01	0.00	-0.03	-0.05	0.01	-0.01	-0.03	-0.01	-0.02	-0.05	1000	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.00	-0.02	-0.03	0.01	-0.02	-0.04	-0.01	-0.03	-0.01		
Ut rec	200	0.71	0.52	0.37	0.54	0.46	0.45	0.57	0.52	0.54	0.95	0.93	0.91	0.95	0.90	0.88	0.88	0.88	0.88	500	0.74	0.53	0.39	0.56	0.47	0.47	0.52	0.53	0.54	0.98	0.97	0.97	0.98	0.96	0.95	0.94	0.93	0.96	1000	0.75	0.54	0.39	0.56	0.48	0.47	0.56	0.52	0.56	0.99	0.99	0.98	0.99	0.98	0.98	0.96	0.97	0.98		
Ut SE	200	0.45	0.11	0.06	0.04	0.08	0.16	12.79	0.28	0.24	-0.06	-0.07	-0.06	-0.07	-0.14	-0.21	-0.04	-0.07	500	0.12	0.06	-0.01	0.00	0.01	0.00	0.98	0.62	2.83	-0.01	-0.02	-0.02	0.00	-0.03	-0.05	-0.01	0.00	-0.02	1000	0.06	-0.01	0.00	-0.02	0.01	0.00	1.47	0.46	0.12	-0.01	-0.03	-0.02	0.00	-0.03	0.00	0.02	-0.02	-0.01			
Cor	200	0.01	-0.44	-0.18	0.54	-0.02	-0.09	0.75	0.00	-0.38	0.01	-0.17	-0.28	-0.16	-0.23	-0.15	-0.08	-0.10	-0.33	500	0.03	-0.13	-0.22	-0.15	-0.18	-0.26	-0.22	-0.05	-0.09	-0.03	-0.06	-0.19	-0.13	-0.21	-0.24	-0.10	-0.10	-0.18	1000	-0.01	-0.14	-0.14	-0.07	-0.27	-0.26	0.16	-0.04	0.00	-0.02	-0.07	-0.13	-0.08	-0.16	-0.18	-0.03	-0.13	-0.10		
Cor SE	200	0.21	0.18	0.13	0.36	0.34	0.23	0.39	0.43	0.42	0.15	0.12	0.12	0.17	0.15	0.16	0.17	0.16	0.14	500	0.12	0.09	0.07	0.17	0.13	0.10	0.38	0.31	0.29	0.08	0.07	0.07	0.09	0.08	0.07	0.09	0.08	0.07	1000	0.07	0.06	0.05	0.10	0.09	0.06	0.31	0.26	0.24	0.06	0.05	0.05	0.06	0.05	0.06	0.05	0.06	0.06	0.05	

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.

**Table 38**

*Bias results for the Thurstonian factor model with five correlated traits. Two negatively keyed items.*

	unlinked															linked																								
	Block1					Block2					Block3					Block1					Block2					Block3														
	N	6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18		6	12	18								
Conv	200	261	346	366	215	249	122	122	1	37	52	539	680	722	506	592	416	174	542	564	200	261	346	366	215	249	122	122	1	37	52	539	680	722	506	592	416	174	542	564
	500	743	800	848	684	726	680	680	117	163	926	919	983	856	902	890	504	899	951		500	743	800	848	684	726	680	680	117	163	926	919	983	856	902	890	504	899	951	
	1000	924	931	954	904	914	918	918	194	248	990	985	1000	954	981	972	752	983	992		1000	924	931	954	904	914	918	918	194	248	990	985	1000	954	981	972	752	983	992	
Load	200	-0.02	-0.03	-0.18	-0.10	-0.06	-0.07	0.27	0.08	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.01	0.01	200	-0.02	-0.03	-0.18	-0.10	-0.06	-0.07	0.27	0.08	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00		
	500	0.02	-0.01	-0.18	0.00	0.01	0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	500	0.02	-0.01	-0.18	0.00	0.01	0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
	1000	0.01	0.00	-0.17	-0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1000	0.01	0.00	-0.17	-0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Load rec	200	0.39	0.20	0.12	0.24	0.19	0.12	0.42	0.10	0.06	0.95	0.91	0.85	0.95	0.86	0.85	0.79	0.77	0.73	200	0.39	0.20	0.12	0.24	0.19	0.12	0.42	0.10	0.06	0.95	0.91	0.85	0.95	0.86	0.85	0.79	0.77	0.73		
	500	0.46	0.23	0.14	0.27	0.24	0.11	0.09	0.09	0.06	0.98	0.96	0.94	0.98	0.95	0.94	0.86	0.87	0.87	500	0.46	0.23	0.14	0.27	0.24	0.11	0.09	0.09	0.06	0.98	0.96	0.94	0.98	0.95	0.94	0.86	0.87	0.87		
	1000	0.48	0.24	0.15	0.28	0.24	0.10	0.09	0.09	0.06	0.99	0.98	0.97	0.99	0.98	0.97	0.90	0.93	0.93	1000	0.48	0.24	0.15	0.28	0.24	0.10	0.09	0.09	0.06	0.99	0.98	0.97	0.99	0.98	0.97	0.90	0.93	0.93		
Load SE	200	0.35	0.15	0.90	-0.31	-0.19	-0.12	1.56	0.45	-0.11	-0.11	-0.11	-0.03	-0.11	-0.20	-0.20	1.49	0.49	0.30	200	0.35	0.15	0.90	-0.31	-0.19	-0.12	1.56	0.45	-0.11	-0.11	-0.11	-0.03	-0.11	-0.20	-0.20	1.49	0.49	0.30		
	500	0.13	0.06	0.43	-0.22	-0.12	0.00	2.48	0.15	-0.03	-0.05	0.05	-0.05	-0.05	-0.06	-0.05	2.10	0.73	0.23	500	0.13	0.06	0.43	-0.22	-0.12	0.00	2.48	0.15	-0.03	-0.05	0.05	-0.05	-0.06	-0.05	-0.06	-0.05	2.10	0.73	0.23	
	1000	0.04	0.01	0.24	-0.15	-0.05	-0.02	2.05	0.23	-0.02	-0.01	0.07	-0.02	-0.03	-0.04	2.42	0.43	0.20	0.20	1000	0.04	0.01	0.24	-0.15	-0.05	-0.02	2.05	0.23	-0.02	-0.01	0.07	-0.02	-0.03	-0.04	2.42	0.43	0.20	0.20		
Ut	200	-0.03	-0.03	-0.03	-0.02	-0.02	0.00	0.04	-0.07	-0.03	-0.04	-0.08	-0.04	-0.07	-0.10	-0.06	-0.01	-0.35	200	-0.03	-0.03	-0.03	-0.02	-0.02	0.00	0.04	-0.07	-0.03	-0.04	-0.08	-0.04	-0.07	-0.10	-0.06	-0.01	-0.35				
	500	-0.01	-0.01	0.00	-0.01	-0.01	-0.02	0.00	-0.01	-0.01	-0.01	-0.03	-0.02	-0.02	-0.03	0.02	-0.05	-0.11	500	-0.01	-0.01	0.00	-0.01	-0.01	-0.02	0.00	-0.01	-0.01	-0.01	-0.03	-0.02	-0.02	-0.03	0.02	-0.05	-0.11				
	1000	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.03	-0.04	-0.01	-0.07	-0.06	1000	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.02	-0.02	-0.01	-0.03	-0.04	-0.01	-0.07	-0.06				
Ut rec	200	0.71	0.52	0.37	0.54	0.46	0.44	0.40	0.51	0.52	0.96	0.92	0.89	0.96	0.89	0.89	0.85	0.81	200	0.71	0.52	0.37	0.54	0.46	0.44	0.40	0.51	0.52	0.96	0.92	0.89	0.96	0.89	0.89	0.85	0.81				
	500	0.74	0.53	0.39	0.56	0.47	0.47	0.52	0.52	0.55	0.98	0.97	0.96	0.98	0.96	0.96	0.93	0.92	500	0.74	0.53	0.39	0.56	0.47	0.47	0.52	0.52	0.55	0.98	0.97	0.96	0.98	0.96	0.96	0.93	0.92				
	1000	0.75	0.54	0.39	0.56	0.48	0.47	0.54	0.54	0.56	0.99	0.99	0.98	0.99	0.98	0.98	0.96	0.96	1000	0.75	0.54	0.39	0.56	0.48	0.47	0.54	0.54	0.56	0.99	0.99	0.98	0.99	0.98	0.98	0.96	0.96				
Ut SE	200	0.39	0.14	0.10	0.03	0.10	0.20	0.96	0.46	-0.06	-0.08	-0.11	-0.05	-0.10	-0.11	0.15	-0.06	-0.05	200	0.39	0.14	0.10	0.03	0.10	0.20	0.96	0.46	-0.06	-0.08	-0.11	-0.05	-0.10	-0.11	0.15	-0.06	-0.05				
	500	0.14	0.07	-0.01	-0.01	0.01	0.01	0.85	0.22	-0.01	-0.04	-0.04	-0.03	-0.03	-0.03	0.10	0.00	-0.03	500	0.14	0.07	-0.01	-0.01	0.01	0.01	0.85	0.22	-0.01	-0.04	-0.04	-0.03	-0.03	-0.03	0.10	0.00	-0.03				
	1000	0.05	0.01	-0.01	-0.01	0.00	0.00	0.53	0.14	0.00	-0.01	-0.02	-0.01	0.00	-0.01	0.07	-0.02	-0.01	1000	0.05	0.01	-0.01	-0.01	0.00	0.00	0.53	0.14	0.00	-0.01	-0.02	-0.01	0.00	-0.01	0.07	-0.02	-0.01				
Cor	200	0.11	-0.06	-0.23	-0.37	-0.43	0.09	-0.06	-0.03	-0.10	0.05	-0.16	-0.22	-0.37	-0.03	-0.38	0.18	-0.14	-0.22	200	0.11	-0.06	-0.23	-0.37	-0.43	0.09	-0.06	-0.03	-0.10	0.05	-0.16	-0.22	-0.37	-0.03	-0.38	0.18	-0.14	-0.22		
	500	0.00	-0.07	-0.06	-0.06	-0.11	-0.26	-0.08	0.03	0.02	-0.05	-0.07	-0.10	-0.33	-0.34	-0.02	-0.05	-0.08	500	0.00	-0.07	-0.06	-0.06	-0.11	-0.26	-0.08	0.03	0.02	-0.05	-0.07	-0.10	-0.33	-0.34	-0.02	-0.05	-0.08				
	1000	-0.01	-0.06	-0.08	-0.15	-0.27	-0.28	-0.01	-0.03	-0.01	-0.06	-0.07	-0.06	-0.29	-0.26	-0.01	-0.07	-0.05	1000	-0.01	-0.06	-0.08	-0.15	-0.27	-0.28	-0.01	-0.03	-0.01	-0.06	-0.07	-0.06	-0.29	-0.26	-0.01	-0.07	-0.05				
Cor SE	200	0.23	0.14	0.11	0.40	0.37	0.27	0.31	0.30	0.14	0.14	0.12	0.10	0.17	0.19	0.15	0.14	0.12	0.11	200	0.23	0.14	0.11	0.40	0.37	0.27	0.31	0.30	0.14	0.14	0.12	0.10	0.17	0.19	0.15	0.14	0.12	0.11		
	500	0.11	0.08	0.06	0.23	0.17	0.10	0.24	0.24	0.08	0.07	0.06	0.06	0.09	0.10	0.08	0.07	0.06	0.06	500	0.11	0.08	0.06	0.23	0.17	0.10	0.24	0.24	0.08	0.07	0.06	0.06	0.09	0.10	0.08	0.07	0.06	0.06		
	1000	0.07	0.05	0.04	0.13	0.10	0.07	0.22	0.22	0.05	0.05	0.04	0.06	0.06	0.07	0.05	0.05	0.05	0.04	1000	0.07	0.05	0.04	0.13	0.10	0.07	0.22	0.22	0.05	0.05	0.04	0.06	0.06	0.07	0.05	0.05	0.05	0.04		

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Ut: utilities; Cor: correlation; rec: recovery.



**Table 40**

*Bias results for the Thurstonian IRT model with five correlated traits. Two negatively keyed items.*

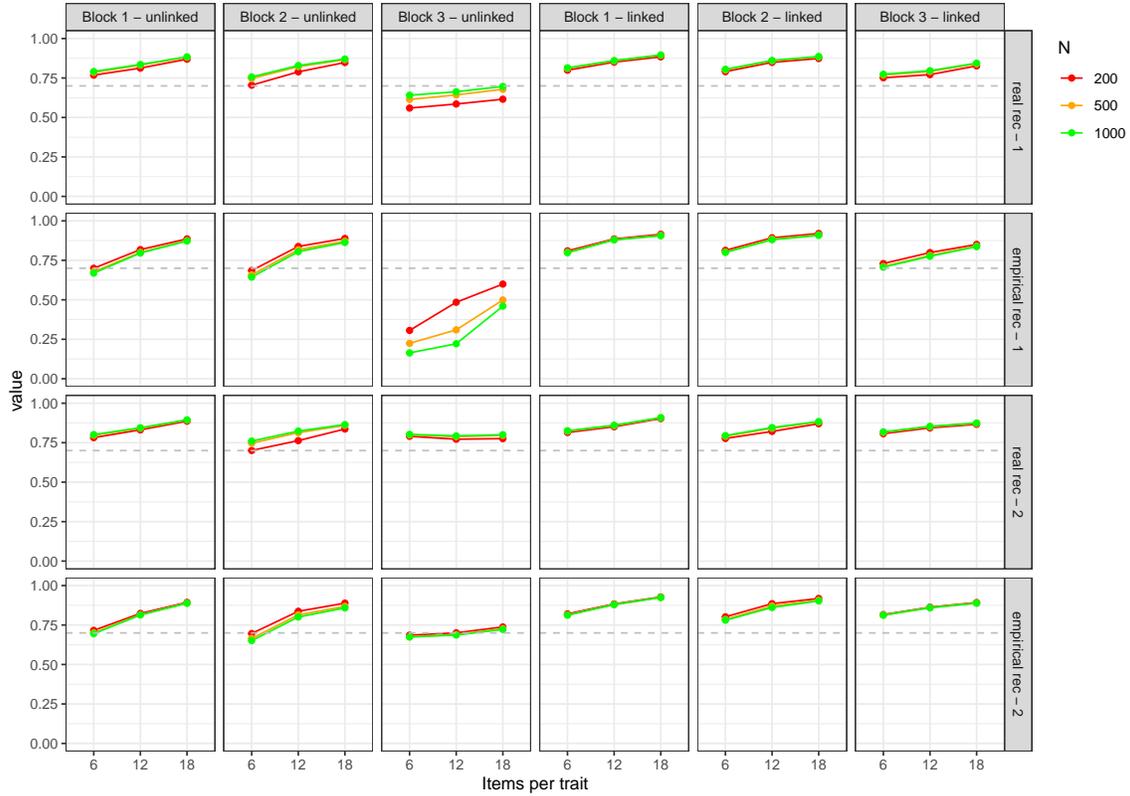
	unlinked															linked														
	Block1					Block2					Block3					Block1					Block2					Block3				
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
N	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000
Conv	405	569	670	565	519	491	491	397	566	689	995	995	906	944	985	985	958	957	992	992	1000	1000	1000	1000	470	835	802	835	963	986
Load	-0.05	-0.02	-0.18	-0.09	-0.06	-0.05	-0.05	-0.03	0.01	0.00	0.00	0.00	0.00	-0.01	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06	0.01	0.01	-0.06	0.01	0.01
Load rec	0.38	0.20	0.12	0.25	0.20	0.12	0.67	0.19	0.14	0.95	0.90	0.84	0.95	0.90	0.84	0.95	0.85	0.84	0.85	0.84	0.76	0.72	0.72	0.72	0.76	0.76	0.72	0.76	0.76	0.72
Load SE	0.71	0.33	0.17	-0.13	-0.21	0.15	3.68	1.56	0.72	-0.10	-0.12	-0.15	-0.12	-0.15	-0.12	-0.12	-0.22	-0.19	-0.19	-0.19	-0.05	-0.06	-0.05	-0.06	0.14	0.00	-0.05	0.14	0.00	-0.05
Th	0.02	0.00	-0.02	-0.05	-0.02	-0.02	1.44	0.33	0.15	-0.02	-0.01	-0.04	-0.02	-0.01	-0.04	-0.02	-0.03	-0.03	-0.03	-0.03	0.11	-0.03	-0.01	-0.01	-0.03	-0.01	-0.03	-0.03	-0.03	-0.01
Th rec	0.99	0.99	0.98	0.98	0.98	0.98	0.96	0.97	0.97	0.99	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.98	0.97	0.97	0.98
Th SE	0.01	0.00	0.00	0.01	0.00	0.00	-0.02	0.00	-0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cor	-0.08	-0.16	-0.22	0.64	0.47	-0.11	0.02	0.08	0.08	0.04	-0.15	-0.23	-0.22	-0.23	-0.23	-0.22	-0.30	-0.43	0.03	0.03	-0.12	-0.19	-0.19	-0.19	0.03	-0.12	-0.19	0.03	-0.12	-0.19
Cor SE	0.21	0.14	0.11	0.33	0.34	0.26	0.12	0.14	0.15	0.13	0.12	0.10	0.17	0.21	0.16	0.13	0.21	0.16	0.13	0.13	0.12	0.11	0.11	0.11	0.13	0.12	0.11	0.13	0.12	0.11

*Note.* Abbreviations: Conv: convergence rate; Load: loading; Th: thresholds; Cor: correlation; rec: recovery.

### 6.3.2 Results for latent trait recovery

Figure 33

*Latent trait recovery results with five correlated traits.*



*Note.* Abbreviations: rec: recovery. The first two rows come from simulation with one (- 1), the second two rows from simulation with two (- 2) negatively keyed items per trait.

**Table 41**

*Latent trait recovery results with five correlated traits.*

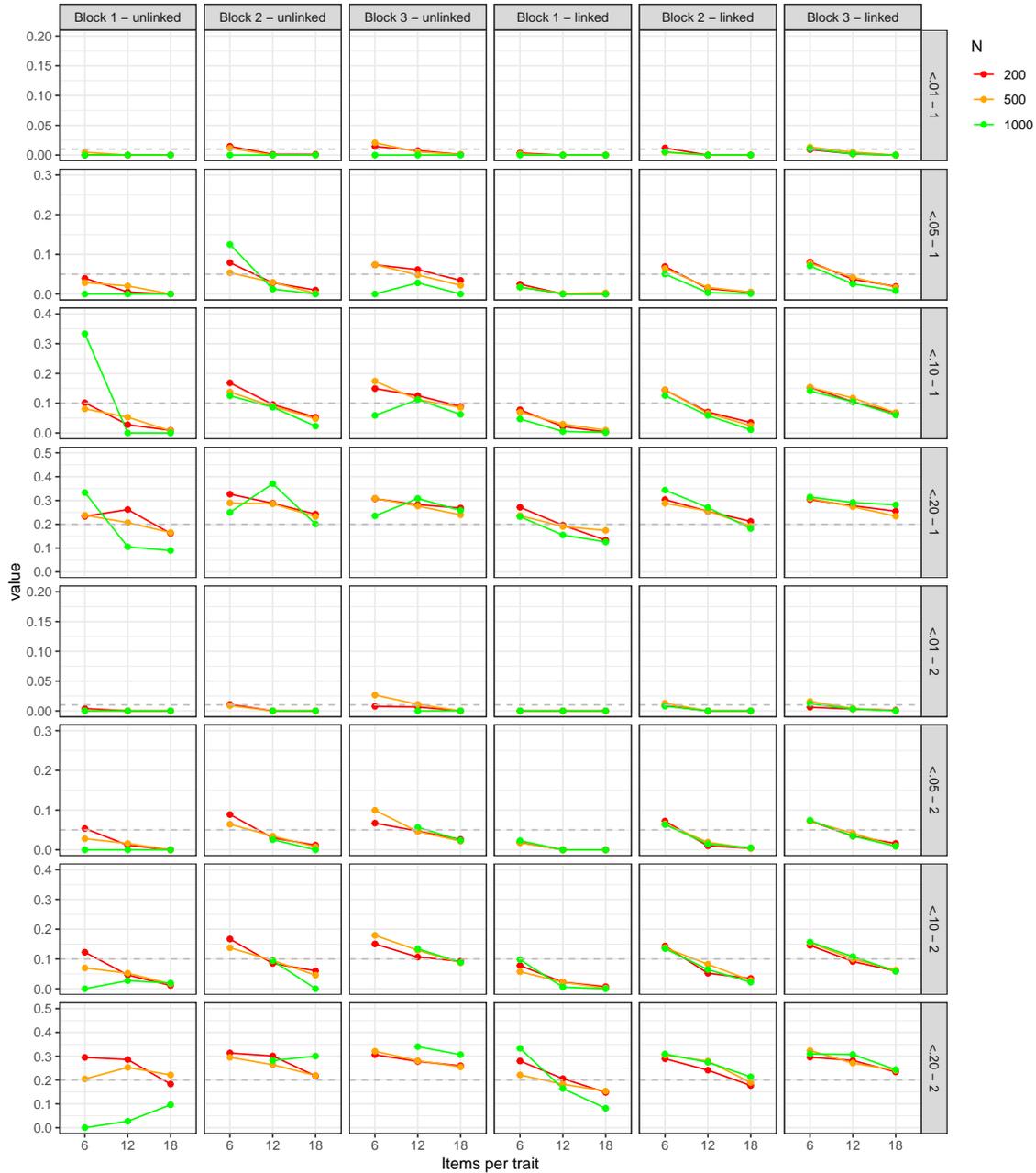
	unlinked															linked														
	Block1					Block2					Block3					Block1					Block2					Block3				
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18
N	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000	200	500	1000
real rec - 1	0.768	0.813	0.869	0.704	0.789	0.848	0.704	0.789	0.848	0.559	0.585	0.616	0.800	0.851	0.884	0.790	0.849	0.874	0.884	0.894	0.894	0.802	0.859	0.884	0.751	0.771	0.771	0.751	0.771	0.827
empirical rec - 1	0.785	0.831	0.881	0.747	0.822	0.866	0.747	0.822	0.866	0.614	0.643	0.679	0.811	0.858	0.894	0.802	0.859	0.884	0.894	0.894	0.894	0.802	0.859	0.884	0.769	0.791	0.791	0.769	0.791	0.840
real rec - 2	0.791	0.835	0.884	0.756	0.829	0.870	0.756	0.829	0.870	0.641	0.663	0.697	0.814	0.860	0.896	0.805	0.861	0.886	0.896	0.896	0.896	0.805	0.861	0.886	0.774	0.796	0.796	0.774	0.796	0.844
empirical rec - 2	0.700	0.818	0.886	0.686	0.838	0.889	0.686	0.838	0.889	0.306	0.485	0.599	0.810	0.886	0.915	0.813	0.893	0.920	0.915	0.915	0.915	0.813	0.893	0.920	0.729	0.799	0.799	0.729	0.799	0.851
real rec - 1	0.678	0.801	0.875	0.658	0.817	0.871	0.658	0.817	0.871	0.224	0.309	0.499	0.801	0.881	0.908	0.803	0.884	0.911	0.908	0.908	0.908	0.803	0.884	0.911	0.711	0.781	0.781	0.711	0.781	0.840
empirical rec - 1	0.669	0.796	0.872	0.644	0.805	0.863	0.644	0.805	0.863	0.164	0.222	0.460	0.798	0.880	0.906	0.800	0.881	0.908	0.906	0.906	0.906	0.800	0.881	0.908	0.707	0.776	0.776	0.707	0.776	0.837
real rec - 2	0.782	0.831	0.886	0.701	0.763	0.837	0.701	0.763	0.837	0.791	0.772	0.776	0.815	0.851	0.903	0.777	0.821	0.870	0.903	0.903	0.903	0.777	0.821	0.870	0.807	0.844	0.844	0.807	0.844	0.866
empirical rec - 2	0.797	0.842	0.893	0.747	0.814	0.860	0.747	0.814	0.860	0.799	0.788	0.796	0.823	0.858	0.907	0.791	0.841	0.881	0.907	0.907	0.907	0.791	0.841	0.881	0.815	0.851	0.851	0.815	0.851	0.872
real rec - 1	0.801	0.844	0.895	0.760	0.823	0.865	0.760	0.823	0.865	0.803	0.793	0.800	0.826	0.860	0.909	0.795	0.846	0.884	0.909	0.909	0.909	0.795	0.846	0.884	0.818	0.853	0.853	0.818	0.853	0.874
empirical rec - 1	0.716	0.823	0.893	0.696	0.837	0.888	0.696	0.837	0.888	0.685	0.701	0.738	0.820	0.883	0.928	0.802	0.885	0.918	0.928	0.928	0.928	0.802	0.885	0.918	0.816	0.863	0.863	0.816	0.863	0.892
real rec - 2	0.700	0.816	0.889	0.665	0.814	0.868	0.665	0.814	0.868	0.675	0.690	0.727	0.814	0.880	0.925	0.786	0.870	0.907	0.925	0.925	0.925	0.786	0.870	0.907	0.813	0.861	0.861	0.813	0.861	0.889
empirical rec - 2	0.695	0.814	0.888	0.651	0.801	0.858	0.651	0.801	0.858	0.674	0.687	0.722	0.812	0.879	0.924	0.781	0.861	0.903	0.924	0.924	0.924	0.781	0.861	0.903	0.812	0.859	0.859	0.812	0.859	0.888

*Note.* Abbreviations: rec: recovery.

### 6.3.3 Results for empirical rejection rates

Figure 34

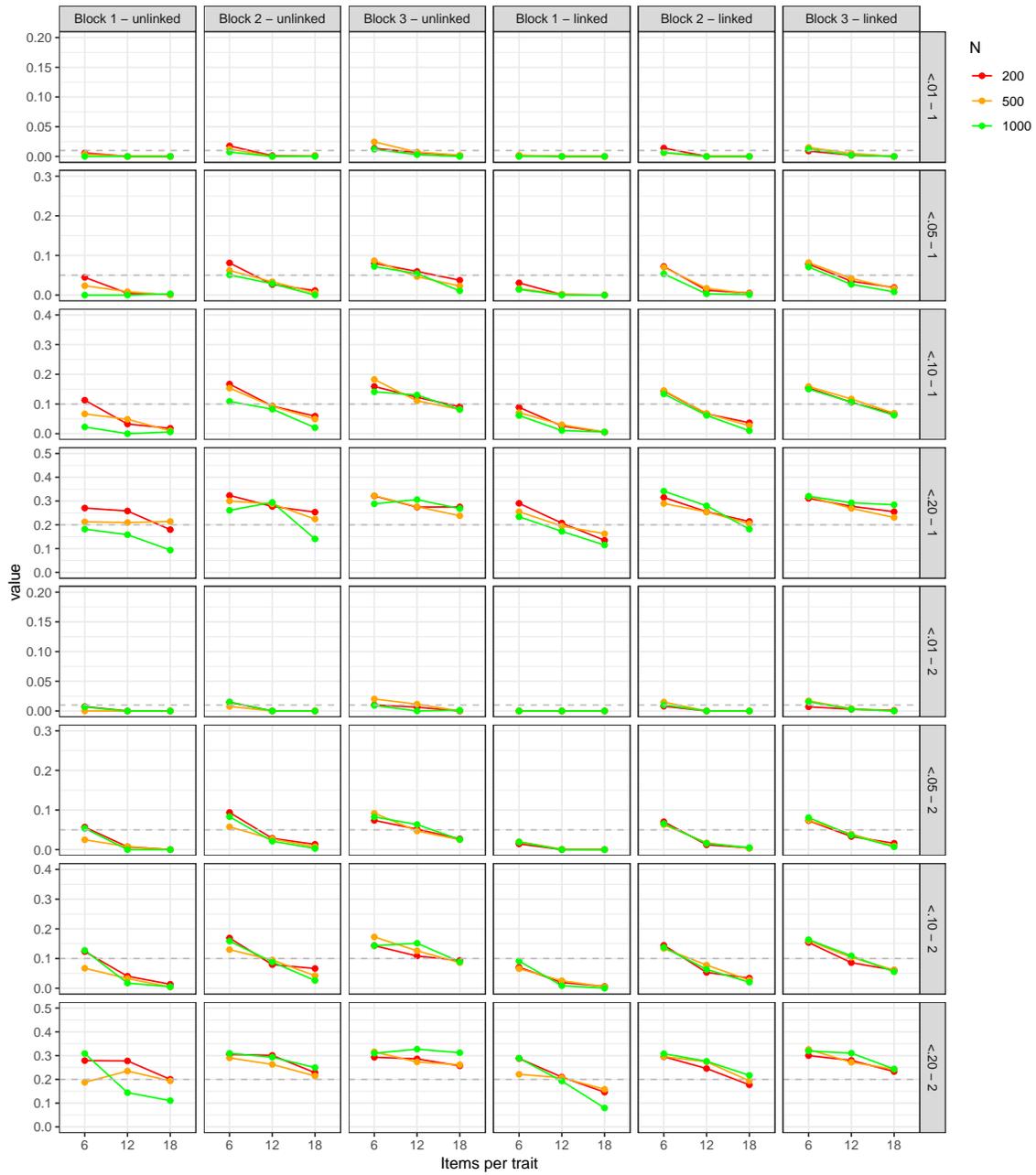
Empirical rejection rates for the Thurstonian factor model with five correlated traits.



Note. Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Figure 35**

*Empirical rejection rates for the Thurstonian IRT model with five correlated traits.*



*Note.* Degrees of freedom are adjusted by the number of redundancies per design. The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.



**Table 43**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five correlated traits.*

	N	unlinked															linked																																								
		Block1					Block2					Block3					Block1					Block2					Block3																														
		6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30																					
$\chi^2 - 1$	200	387	1693	3892	389	1688	3891	387	1684	3888	945	3927	8930	943	3920	8927	941	3915	8924	500	384	1688	3892	386	1687	3886	386	1685	3884	897	3835	8935	896	3829	8922	895	3827	8921	1000	401	1705	3915	405	1709	3920	394	1712	3922	942	3924	8930	946	3924	8932	942	3924	8934
$\chi^2$ SD - 1	200	17	23	27	23	35	42	24	43	52	23	29	34	32	45	53	35	60	68	500	19	27	29	22	34	42	26	42	52	23	32	37	32	46	53	36	58	71	1000	24	20	24	18	33	33	16	34	41	22	26	32	28	42	41	35	52	58
df - 1	200	385	1680	3876	385	1680	3876	385	1680	3876	937	3907	8902	937	3907	8902	937	3907	8902	500	385	1680	3875	385	1680	3875	385	1680	3875	892	3817	8902	892	3817	8902	892	3817	8902	1000	395	1700	3905	395	1700	3905	395	1700	3905	937	3907	8902	937	3907	8902	937	3907	8902
Corrected df - 1	200	375	1660	3846	375	1660	3846	375	1660	3846	922	3877	8857	922	3877	8857	922	3877	8857	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3787	8857	877	3787	8857	877	3787	8857	1000	385	1680	3875	385	1680	3875	385	1680	3875	922	3877	8857	922	3877	8857	922	3877	8857
$\chi^2 - 2$	200	389	1695	3894	388	1689	3889	386	1684	3887	944	3927	8933	944	3920	8923	940	3919	8918	500	384	1692	3895	385	1687	3885	387	1684	3884	940	3924	8928	943	3922	8921	941	3919	8919	1000	392	1705	3919	392	1712	3927	1709	3928	900	3925	8928	898	3923	8933	894	3922	8927	
$\chi^2$ SD - 2	200	18	24	27	23	34	42	23	41	52	22	30	35	31	43	53	34	54	68	500	17	25	30	23	36	42	26	43	54	23	30	37	33	46	55	37	57	70	1000	21	25	25	32	32	33	41	46	24	28	28	31	45	46	35	57	62	
df - 2	200	385	1680	3876	385	1680	3876	385	1680	3876	937	3907	8902	937	3907	8902	937	3907	8902	500	385	1680	3875	385	1680	3875	385	1680	3875	937	3907	8902	937	3907	8902	937	3907	8902	1000	395	1700	3905	395	1700	3905	395	1700	3905	892	3907	8902	892	3907	8902	892	3907	8902
Corrected df - 2	200	375	1660	3846	375	1660	3846	375	1660	3846	922	3877	8857	922	3877	8857	922	3877	8857	500	375	1660	3845	375	1660	3845	375	1660	3845	922	3877	8857	922	3877	8857	922	3877	8857	1000	385	1680	3875	385	1680	3875	385	1680	3875	877	3877	8857	877	3877	8857	877	3877	8857

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

**Table 44**

*Empirical rejection rates for the Thurstonian IRT model with five correlated traits.*

N	unlinked															linked																																																														
	Block1					Block2					Block3					Block1					Block2					Block3																																																				
	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18	6	12	18																																																
<.01 - 1	200	0.006	0.000	0.000	0.018	0.001	0.001	0.001	0.014	0.007	0.001	0.001	0.001	0.000	0.000	0.014	0.000	0.000	0.014	0.000	0.000	0.014	0.000	0.000	0.009	0.002	0.000	500	0.004	0.000	0.000	0.012	0.000	0.001	0.024	0.007	0.002	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.006	0.000	0.000	0.006	0.000	0.015	0.005	0.000	1000	0.000	0.000	0.000	0.007	0.000	0.000	0.012	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.007	0.000	0.000	0.007	0.000	0.012	0.002	0.000
<.05 - 1	200	0.045	0.005	0.002	0.081	0.027	0.011	0.080	0.060	0.037	0.031	0.001	0.000	0.000	0.072	0.012	0.005	0.078	0.035	0.019	500	0.023	0.009	0.000	0.062	0.034	0.006	0.087	0.047	0.023	0.016	0.002	0.000	0.070	0.017	0.004	0.082	0.042	0.017	1000	0.000	0.000	0.000	0.003	0.051	0.029	0.000	0.072	0.054	0.011	0.014	0.000	0.000	0.054	0.003	0.001	0.071	0.027	0.008																			
<.10 - 1	200	0.113	0.033	0.018	0.167	0.093	0.059	0.159	0.123	0.091	0.089	0.026	0.005	0.144	0.067	0.037	0.154	0.106	0.067	500	0.067	0.049	0.010	0.153	0.092	0.049	0.183	0.111	0.082	0.071	0.030	0.006	0.145	0.068	0.027	0.159	0.117	0.069	1000	0.023	0.000	0.000	0.006	0.109	0.083	0.020	0.141	0.130	0.081	0.061	0.011	0.006	0.134	0.062	0.010	0.150	0.107	0.062																				
<.20 - 1	200	0.270	0.258	0.180	0.324	0.277	0.253	0.321	0.274	0.275	0.290	0.207	0.135	0.315	0.255	0.214	0.311	0.278	0.255	500	0.213	0.210	0.214	0.300	0.288	0.224	0.322	0.276	0.238	0.255	0.195	0.162	0.290	0.254	0.205	0.319	0.269	0.231	1000	0.182	0.158	0.094	0.261	0.294	0.141	0.288	0.306	0.268	0.234	0.172	0.115	0.342	0.280	0.182	0.320	0.293	0.284																					
<.01 - 2	200	0.007	0.000	0.000	0.015	0.000	0.000	0.010	0.006	0.000	0.000	0.000	0.000	0.008	0.000	0.000	0.007	0.003	0.001	500	0.000	0.000	0.000	0.008	0.000	0.000	0.020	0.011	0.000	0.000	0.000	0.000	0.015	0.000	0.000	0.017	0.004	0.000	1000	0.007	0.000	0.000	0.015	0.000	0.000	0.009	0.000	0.001	0.000	0.000	0.000	0.010	0.000	0.000	0.015	0.003	0.000																					
<.05 - 2	200	0.057	0.007	0.000	0.094	0.028	0.013	0.074	0.052	0.027	0.014	0.000	0.000	0.070	0.012	0.004	0.073	0.033	0.016	500	0.025	0.008	0.000	0.057	0.027	0.007	0.092	0.047	0.025	0.020	0.001	0.000	0.063	0.017	0.003	0.074	0.039	0.009	1000	0.054	0.000	0.000	0.083	0.021	0.003	0.083	0.063	0.025	0.019	0.000	0.000	0.065	0.016	0.005	0.081	0.036	0.007																					
<.10 - 2	200	0.123	0.040	0.013	0.169	0.079	0.066	0.143	0.108	0.094	0.071	0.019	0.006	0.145	0.053	0.034	0.154	0.086	0.061	500	0.067	0.033	0.004	0.130	0.095	0.042	0.173	0.126	0.086	0.066	0.025	0.005	0.134	0.078	0.027	0.160	0.104	0.061	1000	0.128	0.017	0.005	0.159	0.088	0.026	0.143	0.152	0.089	0.091	0.008	0.000	0.137	0.063	0.020	0.164	0.109	0.055																					
<.20 - 2	200	0.279	0.278	0.200	0.305	0.301	0.228	0.293	0.286	0.257	0.288	0.210	0.146	0.295	0.246	0.177	0.300	0.280	0.233	500	0.188	0.235	0.193	0.290	0.263	0.214	0.316	0.273	0.261	0.221	0.207	0.158	0.296	0.274	0.194	0.326	0.272	0.243	1000	0.309	0.144	0.111	0.310	0.293	0.250	0.310	0.327	0.312	0.289	0.193	0.080	0.308	0.276	0.217	0.319	0.310	0.244																					

*Note.* Degrees of freedom are adjusted by the number of redundancies per design.

**Table 45**

*Mean and standard deviations of  $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five correlated traits.*

	N	unlinked															linked																																								
		Block1					Block2					Block3					Block1					Block2					Block3																														
		6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30	6	12	18	24	30																					
$\chi^2 - 1$	200	378	1672	3864	379	1667	3863	377	1664	3858	928	3895	8883	927	3887	8879	924	3882	8875	500	374	1670	3865	376	1668	3856	377	1664	3855	882	3805	8887	881	3799	8877	880	3796	8875	1000	379	1687	3886	386	1689	3888	385	1689	3894	926	3892	8881	929	3892	8884	925	3891	8886
$\chi^2$ SD - 1	200	17	23	28	22	35	42	23	43	53	23	30	33	32	45	52	35	60	68	500	17	25	29	22	34	43	26	42	52	23	31	35	31	46	53	36	58	71	1000	16	20	25	21	33	30	23	39	42	22	25	31	28	42	41	35	52	58
df - 1	200	375	1660	3846	375	1660	3846	375	1660	3846	920	3874	8853	920	3874	8853	920	3874	8853	500	375	1660	3845	375	1660	3845	375	1660	3845	877	3787	8856	877	3787	8856	877	3787	8856	1000	385	1680	3875	385	1680	3875	385	1680	3875	920	3874	8854	920	3874	8854	920	3874	8854
Corrected df - 1	200	365	1640	3816	365	1640	3816	365	1640	3816	905	3844	8808	905	3844	8808	905	3844	8808	500	365	1640	3815	365	1640	3815	365	1640	3815	862	3757	8811	862	3757	8811	862	3757	8811	1000	375	1660	3845	375	1660	3845	375	1660	3845	905	3844	8809	905	3844	8809	905	3844	8809
$\chi^2 - 2$	200	379	1674	3866	378	1669	3860	376	1665	3856	927	3894	8886	927	3888	8878	923	3887	8872	500	373	1670	3863	375	1667	3855	376	1664	3854	924	3895	8884	926	3891	8876	925	3888	8873	1000	389	1688	3890	389	1690	3895	386	1690	3896	883	3894	8878	882	3891	8884	879	3890	8877
$\chi^2$ SD - 2	200	18	23	28	22	34	42	23	42	52	22	30	35	30	43	53	34	54	68	500	17	25	28	22	35	42	25	43	54	24	31	37	32	46	55	36	56	70	1000	19	20	23	22	32	34	24	40	47	23	28	29	31	45	45	35	57	62
df - 2	200	375	1660	3846	375	1660	3846	375	1660	3846	920	3875	8856	920	3875	8856	920	3875	8856	500	375	1660	3845	375	1660	3845	375	1660	3845	921	3876	8856	921	3876	8856	921	3876	8856	1000	385	1680	3875	385	1680	3875	385	1680	3875	876	3875	8852	876	3875	8852	876	3875	8852
Corrected df - 2	200	365	1640	3816	365	1640	3816	365	1640	3816	905	3845	8811	905	3845	8811	905	3845	8811	500	365	1640	3815	365	1640	3815	365	1640	3815	906	3846	8811	906	3846	8811	906	3846	8811	1000	375	1660	3845	375	1660	3845	375	1660	3845	861	3845	8807	861	3845	8807	861	3845	8807

*Note.* The first four rows come from simulation with one (- 1), the second four rows from simulation with two (- 2) negatively keyed items per trait.

## 7 List of Figures

### List of Figures

1	Bias results for the Thurstonian factor model with one trait. One negatively keyed item. . . . .	9
2	Bias results for the Thurstonian factor model with one trait. Two negatively keyed items. . . . .	10
3	Bias results for the Thurstonian IRT model with one trait. One negatively keyed item. . . . .	11
4	Bias results for the Thurstonian IRT model with one trait. Two negatively keyed items. . . . .	12
5	Latent trait recovery results with one trait. . . . .	17
6	Empirical rejection rates for the Thurstonian factor model with one trait. . .	19
7	Empirical rejection rates for the Thurstonian IRT model with one trait. . . .	20
8	Bias results for the Thurstonian factor model with three uncorrelated traits. One negatively keyed item. . . . .	27
9	Bias results for the Thurstonian factor model with three uncorrelated traits. Two negatively keyed items. . . . .	28
10	Bias results for the Thurstonian IRT model with three uncorrelated traits. One negatively keyed item. . . . .	29
11	Bias results for the Thurstonian IRT model with three uncorrelated traits. Two negatively keyed items. . . . .	30
12	Latent trait recovery results with three uncorrelated traits. . . . .	35
13	Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits. . . . .	37
14	Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits. . . . .	38
15	Bias results for the Thurstonian factor model with three correlated traits. One negatively keyed item. . . . .	45
16	Bias results for the Thurstonian factor model with three correlated traits. Two negatively keyed items. . . . .	46
17	Bias results for the Thurstonian IRT model with three correlated traits. One negatively keyed item. . . . .	47
18	Bias results for the Thurstonian IRT model with three correlated traits. Two negatively keyed items. . . . .	48
19	Latent trait recovery results with three correlated traits. . . . .	53
20	Empirical rejection rates for the Thurstonian factor model with three correlated traits. . . . .	55
21	Empirical rejection rates for the Thurstonian IRT model with three correlated traits. . . . .	56
22	Bias results for the Thurstonian factor model with five uncorrelated traits. One negatively keyed item. . . . .	63

23	Bias results for the Thurstonian factor model with five uncorrelated traits. Two negatively keyed items. . . . .	64
24	Bias results for the Thurstonian IRT model with five uncorrelated traits. One negatively keyed item. . . . .	65
25	Bias results for the Thurstonian IRT model with five uncorrelated traits. Two negatively keyed items. . . . .	66
26	Latent trait recovery results with five uncorrelated traits. . . . .	71
27	Empirical rejection rates for the Thurstonian factor model with five uncorrelated traits. . . . .	73
28	Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits. . . . .	74
29	Bias results for the Thurstonian factor model with five correlated traits. One negatively keyed item. . . . .	81
30	Bias results for the Thurstonian factor model with five correlated traits. Two negatively keyed items. . . . .	82
31	Bias results for the Thurstonian IRT model with five correlated traits. One negatively keyed item. . . . .	83
32	Bias results for the Thurstonian IRT model with five correlated traits. Two negatively keyed items. . . . .	84
33	Latent trait recovery results with five correlated traits. . . . .	89
34	Empirical rejection rates for the Thurstonian factor model with five correlated traits. . . . .	91
35	Empirical rejection rates for the Thurstonian IRT model with five correlated traits. . . . .	92

## 8 List of Tables

### List of Tables

1	Bias results for the Thurstonian factor model with one trait. One negatively keyed item. . . . .	13
2	Bias results for the Thurstonian factor model with one trait. Two negatively keyed items. . . . .	14
3	Bias results for the Thurstonian IRT model with one trait. One negatively keyed item. . . . .	15
4	Bias results for the Thurstonian IRT model with one trait. Two negatively keyed items. . . . .	16
5	Latent trait recovery results with one trait. . . . .	18
6	Empirical rejection rates for the Thurstonian factor model with one trait. . .	21
7	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with one trait. . . . .	22
8	Empirical rejection rates for the Thurstonian IRT model with one trait. . . .	23
9	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with one trait. . . . .	24
10	Bias results for the Thurstonian factor model with three uncorrelated traits. One negatively keyed item. . . . .	31
11	Bias results for the Thurstonian factor model with three uncorrelated traits. Two negatively keyed items. . . . .	32
12	Bias results for the Thurstonian IRT model with three uncorrelated traits. One negatively keyed item. . . . .	33
13	Bias results for the Thurstonian IRT model with three uncorrelated traits. Two negatively keyed items. . . . .	34
14	Latent trait recovery results with three uncorrelated traits. . . . .	36
15	Empirical rejection rates for the Thurstonian factor model with three uncorrelated traits. . . . .	39
16	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three uncorrelated traits. . . . .	40
17	Empirical rejection rates for the Thurstonian IRT model with three uncorrelated traits. . . . .	41
18	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with three uncorrelated traits. . . . .	42
19	Bias results for the Thurstonian factor model with three correlated traits. One negatively keyed item. . . . .	49
20	Bias results for the Thurstonian factor model with three correlated traits. Two negatively keyed items. . . . .	50

21	Bias results for the Thurstonian IRT model with three correlated traits. One negatively keyed item. . . . .	51
22	Bias results for the Thurstonian IRT model with three correlated traits. Two negatively keyed items. . . . .	52
23	Latent trait recovery results with three correlated traits. . . . .	54
24	Empirical rejection rates for the Thurstonian factor model with three correlated traits. . . . .	57
25	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with three correlated traits. . . . .	58
26	Empirical rejection rates for the Thurstonian IRT model with three correlated traits. . . . .	59
27	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with three correlated traits. . . . .	60
28	Bias results for the Thurstonian factor model with five uncorrelated traits. One negatively keyed item. . . . .	67
29	Bias results for the Thurstonian factor model with five uncorrelated traits. Two negatively keyed items. . . . .	68
30	Bias results for the Thurstonian IRT model with five uncorrelated traits. One negatively keyed item. . . . .	69
31	Bias results for the Thurstonian IRT model with five uncorrelated traits. Two negatively keyed items. . . . .	70
32	Latent trait recovery results with five uncorrelated traits. . . . .	72
33	Empirical rejection rates for the Thurstonian factor model with five uncorrelated traits. . . . .	75
34	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five uncorrelated traits. . . . .	76
35	Empirical rejection rates for the Thurstonian IRT model with five uncorrelated traits. . . . .	77
36	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five uncorrelated traits. . . . .	78
37	Bias results for the Thurstonian factor model with five correlated traits. One negatively keyed item. . . . .	85
38	Bias results for the Thurstonian factor model with five correlated traits. Two negatively keyed items. . . . .	86
39	Bias results for the Thurstonian IRT model with five correlated traits. One negatively keyed item. . . . .	87
40	Bias results for the Thurstonian IRT model with five correlated traits. Two negatively keyed items. . . . .	88
41	Latent trait recovery results with five correlated traits. . . . .	90
42	Empirical rejection rates for the Thurstonian factor model with five correlated traits. . . . .	93

43	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian factor model with five correlated traits. . . . .	94
44	Empirical rejection rates for the Thurstonian IRT model with five correlated traits. . . . .	95
45	Mean and standard deviations of $\chi^2$ -values, degrees of freedom, and degrees of freedom corrected by redundancies for the Thurstonian IRT model with five correlated traits. . . . .	96

## 9 References

Jansen, M. T. (2023). ThurMod: A R package for Thurstonian modeling. Manuscript and package in preparation.